

# A New Cavity Configuration for Cesium Beam Primary Frequency Standards

ANDREA DE MARCHI, JON SHIRLEY, DAVID J. GLAZE, AND ROBERT DRULLINGER

**Abstract**—In the design of cesium beam frequency standards, the presence of distributed cavity-phase-shifts (associated with residual running waves) in the microwave cavity, due to the small losses in the cavity walls, can become a significant source of error. To minimize such errors in future standards, it has been proposed that the long Ramsey excitation structure be terminated with ring-shaped cavities in place of the conventional shorted waveguide. The ring cavity will minimize distributed cavity-phase-variations at the position of the atomic beam, provided only that the two sides of the ring and the  $T$ -junction feeding the ring are symmetric. In this paper, a model is developed to investigate the validity of this concept in the presence of the small asymmetries that inevitably accompany the fabrication of such a cavity. The model, partially verified by laboratory tests, predicts that normal tolerances will allow the frequency shifts due to distributed cavity-phase-variations to be held at the  $10^{-15}$  level for a beam tube with a  $Q$  of  $10^8$ .

## I. INTRODUCTION

THE TRAJECTORIES of atoms in atomic beam frequency standards can give rise to a number of systematic errors [1], [2]. In this paper we shall discuss two: residual first-order Doppler shift and distributed cavity-phase-shift. In principle, it is possible to eliminate both effects by exciting the atoms with an ideal plane standing wave. In conventional designs utilizing the Ramsey separated oscillatory field method, a standing wave is created by reflecting an incoming wave from the shorted end of a waveguide cavity.

However, the finite conductivity of metal walls introduces losses which cause the reflected wave to be somewhat smaller than the incoming wave. The resulting cavity field can be described as a superposition of the ideal standing wave and a position-dependent residual travelling wave, characterized by its Poynting vector  $P$ . The Poynting vector indicates at each point the magnitude and direction of net power flow inside the cavity. Its component  $P \cdot V$  along the atomic trajectory must average to zero to avoid first-order Doppler effects. Furthermore, the  $2\pi/\lambda$  phase gradient associated with the travelling wave introduces a phase variation from point to point within the cavity. As a consequence, the average phase experienced by the atoms in the two cavities may be different. A frequency shift of the standard then results from this effective end-to-end phase difference.

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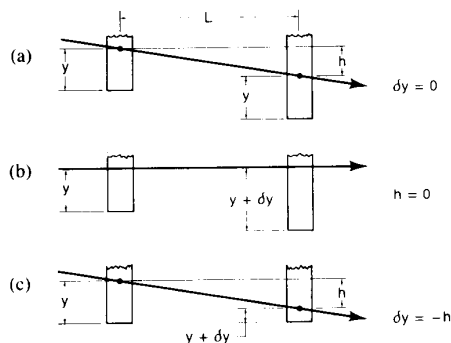


Fig. 1. Schematic representation of possible types of alignment errors in a traditional Ramsey cavity. In each case the arrow shows the trajectory of the atomic beam's center. In Case A there is no (phase) shift due to distributed phase, in Case B there is no first-order Doppler shift, and in Case C they are both present.

The occurrence of the two effects is distinct as illustrated by the geometries shown in Fig. 1 for a typical Ramsey cavity. Assume for the moment that perfect symmetry exists between the two long arms of the Ramsey structure, so that there is no true end-to-end phase shift. There are three cases. In Case A the beam's center of gravity passes at the same distance  $y$  from the short circuit in both cavities and therefore experiences the same average phase, but the inclination of the trajectory results in a first-order Doppler effect. That is, the radiation frequency as seen by the atoms is shifted by

$$\frac{\delta\nu'}{\nu} = -\zeta \frac{\lambda}{\lambda_g} \frac{\langle h \rangle \langle v \rangle}{Lc} \quad (1)$$

where  $\zeta$  is the relative travelling wave amplitude imbalance at the beam position,  $\langle v \rangle$  is the average longitudinal atomic velocity, and  $\langle h \rangle$  is the average displacement of the beam over the distance  $L$  due to its inclination. But with the same average phase at both ends of the cavity, no Doppler shift occurs in the drift region between the cavity ends. Hence, the Doppler shift (1) for the interaction zones of length  $l_c$  is reduced by the factor  $l_c/L$  when the Ramsey resonance is observed. The resulting shift, in a clock where the frequency is servo controlled, is

$$\frac{\delta\nu}{\nu} \approx -\frac{l_c}{L} \frac{\delta\nu'}{\nu} \quad (2)$$

In Case B the beam is perpendicular to the residual running wave in both cavities, but passes at different dis-

tances from the short circuits. The first-order Doppler shift is eliminated, but the average phase seen by the atoms is different at the two ends. The usual formula for the end-to-end phase shift then applies

$$\frac{\delta\nu}{\nu} = \frac{\langle \delta\phi \rangle}{\pi Q} \quad (3)$$

where  $Q$  is the quality factor of the central Ramsey fringe and  $\langle \delta\phi \rangle$  is the end-to-end phase difference caused by the phase gradient and the beam misalignment.

In Case C we show a trajectory more typical of the beam optics in existing standards. Both the first-order Doppler shift and the end-to-end phase shift are present. Although the two effects are physically different, if  $\langle h \rangle = -\langle \delta y \rangle$  (1) and (3) give equal shifts. However, the additional  $l_c/L$  reduction coming from the Ramsey separated oscillatory fields method usually makes first-order Doppler shifts negligible. For this reason, only phase shifts will be considered in the following.

To measure the end-to-end phase shift in a standard, the direction of the atomic beam is reversed. The intended result of this "beam reversal" is to accumulate the phase difference between the ends in the opposite sense, thereby producing a frequency shift of the opposite sign. The observed phase shift, however, is the difference in the trajectory-averaged phases experienced by the beam atoms as they traverse each end of the cavity. As pointed out above, the travelling wave component in the radiation field results in a phase change with position within the cavity and across the atomic beam window. For this reason, beam reversal is an accurate measure of the true end-to-end phase shift only to the degree that the forward and reversed beams experience the same average of the distributed phase at each cavity end. The distributed cavity phase-shift is not easily measured in a direct way. Most standards rely on precise retrace of the reversed beam to minimize its effects. This operational mode sets limits on the required retrace for a given cavity type and desired clock accuracy. As an example, for a standard with a  $Q \approx 10^8$  and a phase slope of  $50 \mu\text{rad}/\text{mm}$  (as one expects half a wavelength from the end of a shorted copper waveguide [4], [7]), a misalignment  $\langle \delta y \rangle$  of 1 mm produces a shift of about  $2 \times 10^{-13}$ .

In the future, new cesium beam frequency standards will be designed with overall accuracy goals of at least  $10^{-14}$ . It will be desirable to reduce distributed phase shift effects in these standards to not more than a few parts in  $10^{15}$ . Using atomic beams with thermal velocities, it is unlikely that these new devices will have line  $Q$ 's much higher than the  $10^8$ . Equation (3) implies that the uncompensated distributed cavity phase shift should not be greater than  $1 \mu\text{rad}$ . It seems unrealistic to think that the 10- or 20- $\mu\text{m}$  beam retrace precision required by these effects in a traditional shorted waveguide cavity can be guaranteed.

This paper analyzes a ring-shaped cavity [3] that promises to solve this problem. Section II introduces the cavity

concept and its potential symmetry problems. Section III treats the case where some small asymmetry is introduced into the cavity feed Tee during fabrication and Section IV looks at the effect of possible asymmetries in the ring itself. Section V describes an experimental look at a prototype cavity that confirms the model predictions.

## II. THE RING CAVITY CONCEPT

In place of the conventional shorted waveguide at each end of the Ramsey structure, we propose using the ring cavity illustrated in Fig. 2. In an ideally symmetric ring with a perfect  $T$ -junction feed, two equal counter-propagating waves combine to form the desired stationary phase point directly opposite the point where the ring is fed. The phase variation around this point has a parabolic minimum of the form  $\alpha\beta y^2$  ( $\beta = 2\pi/\lambda_g$  is the imaginary part of the propagation constant  $\gamma = \alpha + j\beta$ ) due solely to the waveguide attenuation in the ring. Phase variations in the other two directions also occur with parabolic minima about the center of the waveguide. The coefficients of  $x^2$  and  $z^2$  are somewhat smaller than  $\alpha\beta$ , however [2].

The phase variation in the  $y$  direction is compared with that found in conventional cavity designs in Fig. 3. Curve  $a$  shows variations about a position  $\lambda_g/2$  away from the short, where the atomic beam passes in most primary standards. Curve  $b$  shows variations close to the short, where the beam is passed in commercial standards. Curves  $a$  and  $b$  were presented long ago by Lacey [4], but are now shown in Fig. 3 on a much expanded scale. Curve  $c$  shows variations about the symmetry point in the ring cavity. For a beam of 4-mm diameter centered about the stationary phase point, no atomic trajectory can experience a phase more than  $7 \mu\text{rad}$  different from that of any other and the average phase is  $\approx 3 \mu\text{rad}$ . An error of 10 percent in the phase retrace, corresponding to about half a millimeter beam retrace imprecision, still meets the  $1\text{-}\mu\text{rad}$  specification desired for a new standard. It appears feasible to maintain mechanical uncertainties in the cavity and beam parameters (e.g., optical pumping laser beam positioning) within this tolerance. The distributed cavity phase shift problem, therefore, seems to be solved.

However, a difficulty arises from the consideration of possible electromagnetic asymmetries in the ring structures. An imbalance in the amplitudes of the counter-propagating waves launched in the ring by the junction, or a propagation asymmetry in the two arms of the ring, can cause the two waves to be unbalanced at  $y = 0$ . As a result of their oppositely directed propagation loss, they will then be balanced somewhere else and the phase minimum will be displaced.

If the phase minima in the two rings are displaced by  $d_1$  and  $d_2$  with respect to the centers of their respective beam holes, the average phase difference is  $\langle \delta\phi \rangle \approx \alpha\beta(d_1^2 - d_2^2)$  for a uniform beam (centered on the holes). Additionally, if the centers of the forward and reversed beams pass instead at an average distance  $y_i$  from the hole centers, and  $\delta y_i$  from each other in the  $i$ th cavity, then

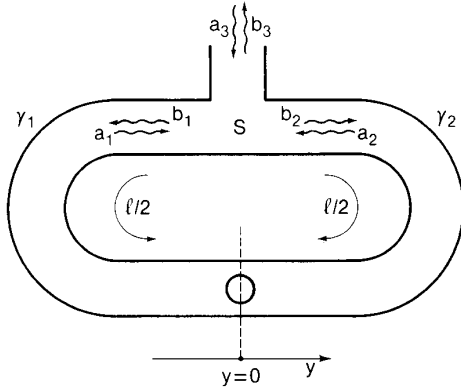


Fig. 2. Geometry of the proposed ring cavity with schematic definition of symbols.

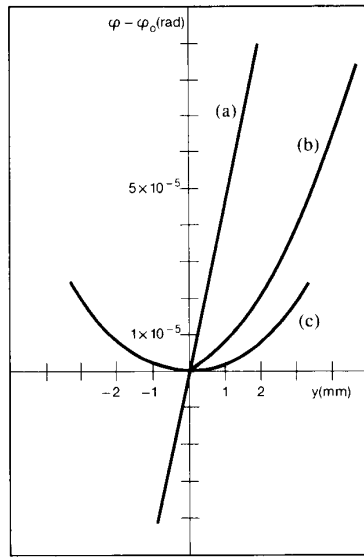


Fig. 3. Spatial phase variations for standard X-band copper waveguide in three different configurations: (a) beam passing  $\lambda_g/2$  from a waveguide short, (b) beam passing near the short, and (c) beam passing at the phase minimum in a ring cavity.

$\langle \delta\phi \rangle \approx 2\alpha\beta[(d_1 - y_1)\delta y_1 - (d_2 - y_2)\delta y_2]$ . The  $1\text{-}\mu\text{rad}$  condition on  $\langle \delta\phi \rangle$  imposes  $(d_1 - d_2) - (y_1 - y_2) < 0.5\text{ mm}$  if both  $\delta y_i$  are smaller than  $0.5\text{ mm}$ . In the worst case this would require that both values of  $d_i$  be  $< 1\text{ mm}$ . We will take this as our guideline for the specification on mode definition.

The concept of phase minimum displacement can be more precisely described in terms of the standing wave mode excited in the cavity. In the absence of an obvious mode-defining feature in the ring, the resonant mode is free to rotate and settle in any position guided only by very small and unpredictable irregularities and asymmetries. In the ring of Fig. 2, however, the  $T$ -junction is a feature which clearly breaks cylindrical symmetry. How effective is it in defining the desired mode in the presence of mechanical imperfections? The rest of this paper will discuss how much imbalance or mode rotation can be ac-

cepted, and how much should be expected from reasonable fabrication techniques.

The question of overall ring cavity symmetry is separable into two parts: the  $T$ -junction, which will be addressed in Section III, and the ring itself, which will be addressed in Section IV.

### III. $T$ -JUNCTION ASYMMETRY ANALYSIS

An intuitive understanding of how asymmetries come into play can be grasped more easily using the standing-wave cavity mode approach, rather than a running-wave description. The basic field pattern in a ring can be described by a linear combination of two orthogonal fundamental modes. We shall call these two modes  $D$  and  $U$ , for desirable and undesirable. The natural basis in our case is determined by the  $T$ -junction. We choose to use an  $E$ -plane Tee so that mode  $D$  has a magnetic field maximum at the  $T$ -junction and at the atomic-beam-passing hole. Mode  $U$  is rotated  $\lambda_g/4$  from this position. Mode  $D$  has the correct orientation to couple strongly to the feed Tee, while mode  $U$  couples only through asymmetric imperfections in the  $T$ -junction. To first order, all of the effects of asymmetry are then isolated in mode  $U$ .

As a result of its weak coupling, mode  $U$  has a high  $Q$  while mode  $D$  does not. Since the two modes couple differently to the feed arm, they experience a slightly different electrical length across the Tee. Hence, they resonate at different frequencies. As will be seen, these differences in resonant frequency and  $Q$  values for the two modes make it possible to reduce sensitivity to asymmetry to acceptable levels.

We begin the analysis of the ring structure by introducing the running-wave amplitudes indicated schematically in Fig. 2. The outgoing amplitudes  $b_i$  from port  $i$  are related to the incoming amplitudes  $a_i$  at the reference planes by the scattering matrix for the junction. We shall neglect losses in the junction; hence the scattering-matrix is unitary. But the losses in the waveguide arms will be retained since they are the origin of the phase variations we are studying.

The scattering matrix of an  $E$ -plane Tee with small asymmetries can be represented by  $S + E$  [3], where

$$S = \begin{pmatrix} R & M & T \\ M & R & -T \\ T & -T & K \end{pmatrix} \quad \text{and} \quad E = \begin{pmatrix} \epsilon_R & 0 & \epsilon_T \\ 0 & -\epsilon_R & \epsilon_T \\ \epsilon_T & \epsilon_T & 0 \end{pmatrix} \quad (4)$$

are the symmetric and antisymmetric parts of the total scattering matrix. In general all elements of  $S$  and  $E$  are complex. For convenience, we choose reference planes such that  $T$  is real. The unitarity conditions on  $S$  then impose (corrections second order in  $E$  are neglected)

$$M + R = e^{-j\delta} \quad \text{and} \quad M - R = K^* \quad (5)$$

where  $\delta$  is defined by this equation. Its meaning will be discussed below. A first-order relationship between  $\epsilon_R$  and  $\epsilon_T$  is also found from the unitarity condition on  $S + E$ .

The ring geometry causes the wave leaving port 1 to re-enter the junction at port 2, and vice-versa. For a ringing of length  $l$ , we thus obtain the feedback relations

$$a_1 = b_2 e^{-\gamma l} \quad \text{and} \quad a_2 = b_1 e^{-\gamma l} \quad (6)$$

at the junction reference planes. These relations, combined with the scattering matrix relations, allow a solution to be found for all the travelling wave amplitudes in terms of the input amplitude  $a_3$ .

At a distance  $y$  into arm 2 from the geometric symmetry point where the beam is passed, the travelling wave amplitudes are

$$b_1(y) = b_1 e^{-\gamma(l/2+y)} \quad \text{and} \quad b_2(y) = b_2 e^{-\gamma(l/2-y)}. \quad (7)$$

The relevant magnetic field at the beam is  $H(y) = b_1(y) - b_2(y)$ . Dropping the fixed phase factor  $e^{-\gamma l/2}$  we can write

$$H(y) = (b_1 e^{-\gamma y} - b_2 e^{\gamma y}). \quad (8)$$

When the junction is symmetric  $b_1 = -b_2 = b$  and  $H(y) = 2b \cosh \gamma y$ , the desired cosine standing-wave mode  $D$ . For  $\beta y \ll 1$  its phase variation is  $\alpha \beta y^2$  as already stated.

When the  $T$ -junction is slightly asymmetric, the resulting mode can be represented as a superposition of the  $D$  and  $U$  modes

$$H(y) = (b_1 - b_2) \cosh \gamma y - (b_1 + b_2) \sinh \gamma y \quad (9)$$

or, introducing  $-b_2 = b - \epsilon_b$  and  $b_1 = b + \epsilon_b$ , as

$$H(y) = 2b(\cosh \gamma y - \epsilon_b/b \sinh \gamma y). \quad (10)$$

The presence of mode  $U$  causes the actual field pattern in the cavity to rotate and shift its phase with respect to mode  $D$ . To find how much rotation and phase shift occur, the real and imaginary parts of  $\epsilon_b/b$  must be found. After some algebra involving (4)-(6), (10) and the relationship between  $\epsilon_R$  and  $\epsilon_T$ , we find the relative amplitude for mode  $U$  to be

$$\frac{\epsilon_b}{b} = j \frac{\text{Im } \epsilon_T}{T} + \frac{\text{Re } \epsilon_T}{T} \left( \frac{\alpha l}{2} - j \sin x \cos x \right) / \left( \sin^2 x + \left( \frac{\alpha l}{2} \right)^2 \right) \quad (11)$$

where  $\alpha l \ll 1$  and

$$x = \frac{1}{2}(\beta l + \delta). \quad (12)$$

Equation (11) shows that mode  $U$  has a sharp resonance ( $Q_U = \beta/\alpha \approx 10^4$  for copper) centered about a frequency  $\nu_U$  lower than  $\nu_D$ , the center frequency of the broader mode  $D$ . Mode  $D$  is defined by  $\beta l = 2n\pi$  when  $l$  corresponds to that position of the reference planes which makes  $K$  real. For standard  $X$ -band waveguide at the ce-

sium frequency

$$\nu_U/\nu_D = 1 - \frac{1}{2} \frac{\delta}{2n\pi} \quad (13)$$

where  $n$  is the number of wavelengths around the ring. The meaning is clear if one interprets  $\delta$  as the electrical length (in radians) of the Tee between the selected reference planes. Since mode  $U$  couples only very weakly to the outside, it is resonant in a ring of length  $l + \delta/\beta$ ; where as, mode  $D$  resonates in a ring of length  $l$ . Hence, if  $n$  is small,  $\nu_D - \nu_U$  may be substantial. For example, for  $n = 2$ , and  $\delta = 0.14$  radians as calculated in [3], the fractional separation between the two modes turns out to be about  $6 \times 10^{-3}$ . This separation can be used to reduce the effect of asymmetries. If we operate the cavity centered on resonance for mode  $D$  we are  $\approx 100$  half line-widths away from mode  $U$ , and coupling to mode  $U$  is therefore reduced by an additional factor of  $\approx 10^4$ .

The effect of a small junction asymmetry can now be easily calculated from (10) with the approximation  $|\epsilon_b/b| \ll 1$ . In this case the mode becomes  $\cosh(\gamma y - \epsilon_b/b)$ . If the argument in parentheses is small, the phase is simply the product of its real and imaginary parts. Equating to zero the derivative of the phase with respect to  $y$ , one finds the position  $y_m$  of the stationary phase point and its phase  $\phi_m$

$$y_m = \frac{1}{2} \left( \frac{1}{\alpha} \frac{\text{Re } \epsilon_b}{b} + \frac{1}{\beta} \frac{\text{Im } \epsilon_b}{b} \right) = \frac{1}{2\alpha} \frac{\text{Re } \epsilon_b}{b} \quad (14)$$

$$\phi_m = -\frac{1}{4} \frac{\alpha}{\beta} \left( \frac{\beta \text{Re } \epsilon_b}{\alpha} \frac{\epsilon_b}{b} - \frac{\text{Im } \epsilon_b}{b} \right)^2 \approx -\frac{1}{4} \frac{\beta}{\alpha} \left( \frac{\text{Re } \epsilon_b}{b} \right)^2. \quad (15)$$

Equation (14) quantifies the result found in Section II from a straightforward discussion of the travelling wave imbalance. Both (14) and (15) set the condition  $\text{Re } \epsilon_b/b < 2 \times 10^{-5}$  in order to have  $y_m < 1$  mm and  $|\phi_m| < 10^{-6}$  radians. Assuming that  $\text{Re } \epsilon_b/b < 10^{-5}$  must be assured because contributions from the two cavities may add, the condition  $\text{Re } \epsilon_T/T < 8 \times 10^{-5}$  is found from (11) evaluated at  $x = \frac{1}{2}\delta$ . Recalling the arguments on the averaging of distributed phase shift effects in beam reversal which were made in Section II above, if the end-to-end phase shift,  $\phi_{m1} - \phi_{m2}$ , is corrected by a beam reversal, we may relax the condition to  $\text{Re } \epsilon_T/T < 2 \times 10^{-4}$ .

The specification above is obtained assuming that mode  $D$  is centered at the atomic resonance frequency. However, this mode is very broad and can be excited with little intensity loss on the high side of center frequency. In this way, rejection of mode  $U$  can be easily increased, thereby relaxing the symmetry specification to the order of  $\text{Re } \epsilon_T/T < 10^{-3}$ .

We next need to relate fabrication imperfections to electromagnetic asymmetries. The three possible angle errors in manufacture of the Tee may be treated as related

to junction asymmetries while all errors in linear dimensions will be treated as part of the ring. It can be shown that a rotation of the feed arm about its axis does not break the symmetry of the junction, nor does a tilt out of the plane of the ring. However, a tilt  $\theta$  in the plane of the ring will introduce an asymmetry. It can be calculated by the approach of [5], that an upper limit for the dependence of  $\text{Re } \epsilon_T/T$  on the angle  $\theta$  is  $\text{Re } \epsilon_T/T \approx 0.2\theta$ , with  $\theta$  in radians [6]. If care is taken in dimensioning a two-wavelength ring, it is therefore sufficient that  $\theta$  be less than  $0.5 \times 10^{-2}$  radians in order to guarantee that phase-shift effects on the measured cesium frequency will be lower than  $3 \times 10^{-15}$ .

#### IV. RING ASYMMETRY

A similar displacement of the stationary phase point can be caused by propagation asymmetry in the two arms of the ring even if the Tee is perfectly symmetric. If we let  $b_1 = -b_2 = b_0 e^{\gamma/2}$  for a symmetric junction, but insert different propagation constants  $\gamma_1$  and  $\gamma_2$  into (7), we obtain the magnetic field at the atomic beam

$$H(y_1) = 2b_0 \cosh(\gamma y - \delta\gamma l/4). \quad (16)$$

Here  $\gamma = \frac{1}{2}(\gamma_1 + \gamma_2)$  is the average propagation constant,  $\delta\gamma = \gamma_2 - \gamma_1 = \delta\alpha + j\delta\beta$  represents the difference between propagation constants for the two arms of the ring, and  $\delta\gamma y$  has been neglected. In analogy to (14) and (15) we find

$$y_m = \frac{l}{8} \left( \frac{\delta\alpha}{\alpha} + \frac{\delta\beta}{\beta} \right) \quad (17)$$

$$\phi_m = -\alpha\beta \left( \frac{l}{8} \right)^2 \left( \frac{\delta\alpha}{\alpha} - \frac{\delta\beta}{\beta} \right)^2. \quad (18)$$

If both  $\delta\alpha/\alpha$  and  $\delta\beta/\beta$  are less than  $10^{-2}$ ,  $y_m$  is less than 0.2 mm for a two wavelength ring and  $\phi_m$  is unimportant. This condition on the uniformity of the ring waveguide seems quite easy to satisfy.

#### V. EXPERIMENTAL MEASUREMENTS

To check the validity of the model just developed, a prototype ring cavity was made with  $l = 4\lambda_g$ . The cavity consists of two half-shells milled in copper and held together by guide pins and bolts. The  $T$ -junction is an open  $E$  plane Tee in standard  $X$ -band waveguide. This tightens the symmetry requirements with respect to other coupling configurations (a small iris or  $H$  slit would make symmetry easier) but maximizes mode separation.

A coaxial probe was inserted into the center of the wide side of the waveguide, 1 mm from the center of the atomic beam passing hole ( $y = 1$  mm). This probe couples to the  $E$ -field with a sensitivity which is much higher for mode  $U$  than for mode  $D$ . The results of the  $E$ -field measurements made with the probe are shown in Fig. 4.

In the same way followed to obtain (10), the  $E$  field can be written

$$E(y) = -2b(\sinh \gamma y - \epsilon_b/b \cosh \gamma y). \quad (19)$$

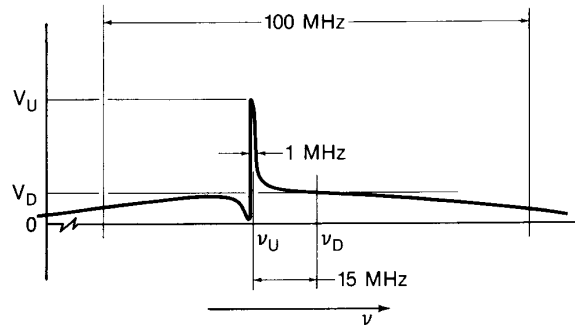


Fig. 4. Experimental measurement of the relative intensity ( $E^2$ ) of modes  $U$  and  $D$  in the test cavity as a function of frequency. The probe was 1 mm from  $y = 0$  so that the sensitivity was maximized for mode  $U$  and attenuated about a factor of 50 for mode  $D$ .

The signal observed with a square law detector connected to the probe is (in the limit  $\alpha y \ll 1$ ) proportional to

$$|E|^2 = 4b^2 \left[ \sin^2 \beta y + \left| \frac{\epsilon_b}{b} \right|^2 \cos^2 \beta y - \frac{\text{Re } \epsilon_b}{b} 2\alpha y - \frac{\text{Im } \epsilon_b}{b} \sin 2\beta y \right]. \quad (20)$$

The first term with the null at  $y = 0$  is mode  $D$ . The following two terms give the peak corresponding to mode  $U$ , and the last term produces the dispersive aspect of the peak shown in Fig. 4. The maximum and minimum in Fig. 4 can be found from  $\tan 2\beta y = 2 \text{Im } \epsilon_b/b$ . At resonance for mode  $U$ , (20) and (11) give

$$|E|_{\text{max}}^2 = V_U = 4b^2 \left[ \sin^2 \beta y + \left( \frac{\text{Re } \epsilon_T/T}{\alpha l/2} \right)^2 \cos^2 \beta y + \frac{\text{Re } \epsilon_T/T}{\alpha l/2} 2\alpha y \right]. \quad (21)$$

At resonance for mode  $D$  we have simply  $V_D = 4b^2 \sin^2 \beta y$ . The magnitude of  $\text{Re } \epsilon_T/T$  can be estimated from the ratio between measured peak values  $V_U$  and  $V_D$  of the two modes

$$|\text{Re } \epsilon_T/T| \approx \alpha\beta |y| l/2 \sqrt{(V_U/V_D - 1)}. \quad (22)$$

The resonance curve of Fig. 4 shows a measured ratio  $V_U/V_D \approx 3$ , which yields, through (22), a value  $\text{Re } \epsilon_T/T \approx 1 \cdot 10^{-4}$  for the asymmetry of the  $T$ -junction. This corresponds to an inclination of the Tee  $\theta \approx 0.5$  mrad. This value is the minimum that could be experimentally obtained by varying the penetration depth of the probe into the guide. In this instance the probe was flush with the waveguide wall. Even in this situation, the probe itself might have been disturbing the symmetry enough to produce a significant part of the observed  $U$  mode. Our result should be taken only as an upper limit to the asymmetry of the structure. Since this still meets the specifications outlined above, we have concluded that the fabrication technique used is adequate for our needs.

In the light of our experimental results, there seems no reason to doubt the model developed here. However, the

ultimate verification will have to await actual use in a standard where an atomic beam of reduced diameter can be used to probe the actual distributed phase shift [7].

## VI. CONCLUSIONS

In this paper we have considered the accuracy-limiting effects of distributed cavity-phase-shift in primary cesium beam frequency standards. We conclude that this effect imposes too tight a beam reversal retrace requirement for accuracies in the  $10^{-15}$  range to be attainable with traditional cavities.

A ring cavity concept was introduced to reduce the phase variation across the atomic beam by creating a parabolic phase minimum. In this way, distributed phase uncertainties become negligible if the beam is reasonably centered on the stationary phase point.

It was also shown that asymmetries in the ring structure can unbalance the two running waves, displacing the phase minimum from the atomic beam passage point. Small asymmetries were analyzed in a standing wave for-

malism which localizes the effects in a single mode and provides a good intuitive picture. Asymmetries can be controlled to the desired degree in a suitable design. The proposed cavity should reduce uncertainties from distributed phase effects to the low  $10^{-15}$  range for devices with  $Q \approx 10^8$ .

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