Search for Anomalous Spin-Dependent Forces Using Stored-Ion Spectroscopy

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Resonances in atomic ions can be used to search for new, weak, spin-dependent interactions. Upper limits on anomalous dipole-monopole and dipole-dipole couplings for the neutron and electron are determined by examining hyperfine resonances in stored $^9Be^+$ ions. These experiments also place strict limits on anomalous weights of spinning gyroscopes.

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The existence of weakly interacting bosons has been suggested previously [1-10]. Laboratory experiments might detect scalar or pseudoscalar couplings of such particles to photons [11-15] or matter [6,7,16-30]. In the latter case, new spin-dependent forces would occur. Here, we report the use of stored atomic ion spectroscopy to search for anomalous potentials having a dipole-monopole or dipole-dipole character. The first is expected to include terms like [7,9,16,31]

$$V_{AB}^{D}=\hbar^{2}D\mathbf{S}_{A}\cdot\hat{\mathbf{r}}(1/\lambda_{\phi}r+1/r^{2})\exp(-r/\lambda_{\phi})\,, \tag{1}$$

where the spin $\mathbf{S}_A$ (in units of $\hbar$) of particle A couples to particle B, r is the distance between particles, λ_ϕ is the range of the force, and D is a coupling constant with units of $(\text{mass})^{-1}$ [see Fig. 1(a)]. In 1968, observation of an interaction like Eq. (1) was reported [22] where particles A were proton spins and particles B comprised the Earth. Subsequent measurements [23,24] contradicted this measurement and found null results (see Table I).

A dipole-dipole interaction [Fig. 1(b)] would be expected to include terms like [7,16,31]

$$V_{AB}^{T}=(\hbar^{3}/c)T\exp(-r/\lambda_{\phi})[(1/\lambda_{\phi}r^{2}+1/r^{3})\mathbf{S}_{A}\cdot\mathbf{S}_{B}-(1/\lambda_{\phi}^{2}r+3/\lambda_{\phi}r^{2}+3/r^{3})(\mathbf{S}_{A}\cdot\hat{\mathbf{r}})(\mathbf{S}_{B}\cdot\hat{\mathbf{r}})]\,, \tag{2}$$

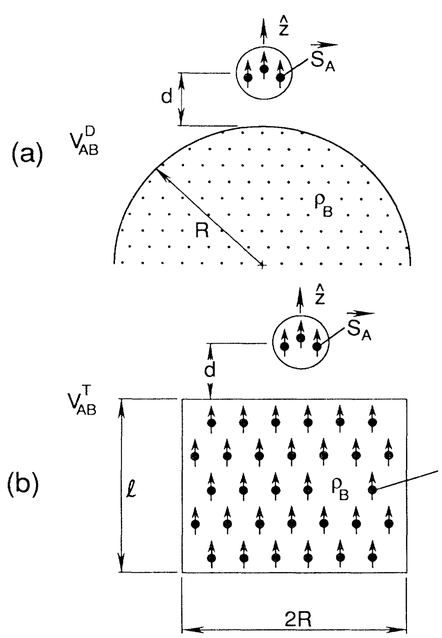

FIG. 1. Experimental configurations sensitive to (a) V_{AB}^{D} and (b) V_{AB}^{T} [refer to Eqs. (1) and (2) of the text]. In both parts, we assume the size of sample A is small compared to d, R, and l. In (a), B is assumed to be spherical with density of B particles ρ_B. In (b), B is assumed to be a cylinder of radius R and height l and spin density ρ_B. Experimentally, V_{AB}^{D} and V_{AB}^{T} are sensed as an anomalous change in energy when $\mathbf{S}_A\cdot\hat{\mathbf{z}}$ is changed. In the experiments reported here, $\mathbf{S}_A\cdot\hat{\mathbf{z}}$ for the electron and neutron are changed by driving a hyperfine transition in the ground state of atomic $^9Be^+$ ions.

where the spin of particle A interacts with that of particle B. T has units of $(\text{mass})^{-2}$ and characterizes the strength of the interaction [see Fig. 1(b)]. This type of interaction is sought in acceleration [7,16,19,20], resonance [16,22-27,29], and induced magnetism [17,18] experiments.

Such forces mediated by axions [4] have received the most attention because the axion emerges in schemes attempting to resolve the strong *CP* problem [4,10]. The mass and coupling of axions and related particles to matter can be severely constrained by arguments based on observed energy-loss rates of stellar objects [32]. As discussed below, these constraints on axions appear to be much stronger than those derived from current laboratory experiments. Nevertheless, the laboratory experiments are still useful because they can search for interactions outside the scope of the axion-type models.

TABLE I. Values of the coupling strength D [Eq. (1)] evaluated from laboratory experiments. Here, we assume $\lambda_\phi \gg R = R_{\text{Earth}}$.

Experiment	Measured quantity	Result (kg^{-1})
Ref. [22]	D(proton)	$\simeq 0.02$
Ref. [23]	D(proton)	$< 2\times10^{-4}$
Ref. [24]	D(deuteron)	$< 6.7\times10^{-8}$
Ref. [28]	D(electron)	$< 3.7\times10^{5}$
This work	$D(^9\text{Be})$	$< 9.0\times10^{-9}$
This work	D(electron)	$< 4.5\times10^{-5}$
This work	D(neutron)	$< 2.7\times10^{-8}$
Ref. [30]	$D(^{201}\text{Hg})-D(^{199}\text{Hg})$	$\lesssim 3\times10^{-10}$

For the assumed configurations shown in Fig. 1, either V_{AB}^D or V_{AB}^T could be observed as an anomalous dependence of energy on $\mathbf{S}_A\cdot\hat{\mathbf{z}}$. In the experiments reported here, we looked for such a dependence in the energy $2\pi\hbar\nu_0$ of the atomic $^9\mathrm{Be}^+$ $^2S_{1/2}$ ground-state $(F=1,m_F=0)\rightarrow(F=1,m_F=-1)$ hyperfine transition [33] in a magnetic field $B_0\simeq0.8194$ T (transition frequency $\nu_0\simeq303$ MHz). Here, m_F is the sum of the electron spin $\mathbf{J}$ $(=\frac{1}{2})$ and nuclear spin $\mathbf{I}$ $(=\frac{3}{2})$ projections along the magnetic-field axis, taken to be along $\hat{\mathbf{z}}$. Measurements of ν_0 were made on about 5000 $^9\mathrm{Be}^+$ ions stored in a Penning ion trap [34]. The $^9\mathrm{Be}^+$ ions are trapped and laser cooled (to reduce Doppler shifts), then optically pumped into the $(F=1,m_F=0)$ level using a combination of laser and radio-frequency (rf) radiation. Additional rf radiation is then used to drive the ion population from the $(F=1,m_F=0)$ to the $(F=1,m_F=-1)$ state. Changes in the population of the two states are sensed by looking for changes in laser light scattered by the ions [34]. The frequency ν_0 is the frequency of radiation which drives the above transition with maximum probability. The reference for ν_0 is an ensemble of hydrogen maser and cesium atomic clocks. Particles A are assumed to be either the ^{9}Be nucleus or the unpaired outer electron in $^9\mathrm{Be}^+$. For the above hyperfine transition, the changes in $\mathbf{S}_A\cdot\hat{\mathbf{z}}$ are given by $|\Delta(\mathbf{S}_A\cdot\hat{\mathbf{z}})|\equiv|\Delta S_{Az}|\simeq1$ or $|\Delta S_{Az}|\simeq2\times10^{-4}$ for the ^{9}Be nuclear spin and electron spin, respectively. (We assume that V_{AB} affects either the electron or ^{9}Be nucleus but not both at the same time.)

In the search for V_{AB}^D, particles B were taken to be the nucleons in the Earth. We looked for a change in ν_0 between the cases where $\mathbf{B}_0$ was parallel or antiparallel to the vertical direction in the laboratory. (Since the hyperfine transition is a $|\Delta m_F|=1$ transition, we had to subtract the effects of the Earth's rotation.) In the search for V_{AB}^T, particles B were taken to be the electron spins in the iron pole faces of an electromagnet. We compared ν_0 when B_0 was created by this electromagnet with ν_0 when B_0 was created by a superconducting solenoid ($\mathbf{S}_B$ spins absent). The value of B_0 was chosen such that $\partial\nu_0/\partial B_0=0$. B_0 was set by measuring a magnetic-field-dependent transition in $^9\mathrm{Be}^+$ [34]. (The inaccuracy in our measurement of B_0 due to V_{AB} is negligible on this field-dependent transition.) For $\partial\nu_0/\partial B_0=0$, we could rule out any positive results which might arise from small changes in B_0 between the different configurations. The largest systematic error in the experiment was due to a background-gas pressure shift of ν_0 [34].

For the geometries of Figs. 1(a) and 1(b) we integrate over the volumes containing the B particles to find the anomalous energy E_A of a single A particle. Assuming $\mathbf{S}_A=S_{Az}\hat{\mathbf{z}}$ and $\mathbf{S}_B=S_{Bz}\hat{\mathbf{z}}$, we find [Fig. 1(a)]

$$E_A^D=C^DL_D=C^D\lambda_\phi(R+d)^{-2}\{[R(R+d)+\lambda_\phi(2R+d)+\lambda_\phi^2]\exp[-(2R+d)/\lambda_\phi]+[R(R+d)-\lambda_\phi d-\lambda_\phi^2]\exp(-d/\lambda_\phi)\}$$
$$=2C^DR^3/3(R+d)^2\quad(\text{for }\lambda_\phi\gg R,d)\tag{3}$$

and [Fig. 1(b)]

$$E_A^T=C^T\xi_T=C^T\{(d/r_1)\exp(-r_1/\lambda_\phi)-[(d+l)/r_2]\exp(-r_2/\lambda_\phi)-\exp(-d/\lambda_\phi)+\exp[-(d+l)/\lambda_\phi]\}$$
$$=C^T\{d/r_1-(d+l)/r_2\}\quad(\text{for }\lambda_\phi\gg R,d,l)\,,\tag{4}$$

where $C^D=2\pi\hbar^2D\rho_BS_{Az}$, $C^T=2\pi(\hbar^3/c)T\rho_BS_{Az}S_{Bz}$, $r_1=(R^2+d^2)^{1/2}$, $r_2=[R^2+(d+l)^2]^{1/2}$, and ρ_B is the number density of particles B (assumed here to be nucleons for V_{AB}^D and electron spins for V_{AB}^T).

In our experiment, we found the shift $\Delta\nu_0$ of ν_0 due to V_{AB}^D (where the magnetic field is upward) to be $\Delta\nu_0=-6.4\pm2.9\pm6.4$ μHz for $d\simeq1.5$ m [35]. The first error is the random uncertainty, the second, an estimate of the pressure shift variation between field-up and -down measurements. The estimate of the pressure shift variation was made by measuring the fluctuations in ν_0 over a period of many months with the magnetic field in one direction. Adding these errors in quadrature, we find $|\Delta\nu_0|<13.4$ μHz $(<5.5\times10^{-20}$ eV). Therefore we find for the coupling constant defined in Eq. (1), $D(^9\mathrm{Be})<3.8/L_D$ kg^{-1} or $D(e^-)<1.9\times10^4/L_D$ kg^{-1}, where L_D [Eq. (3)] is in centimeters. If we assume $\lambda_\phi\gg R,d$ and $R=R_{\text{Earth}}\simeq6.36\times10^8$ cm [$\rho_B=\rho(\text{Earth})\simeq3.33\times10^{24}$ nucleons/cm^3] we can make the estimates of D shown in Table I. To obtain D(neutron) we argue as follows: In the simplest version of the shell model, the spin of the ^{9}Be nucleus is due to the odd neutron, which is in the $1p_{3/2}$ shell. In this model, the expectation value of a component of the neutron spin is $\frac{1}{3}$ of the expectation value of the corresponding component of the total nuclear spin. From this we obtain $D(\text{neutron})<2.7\times10^{-8}$ kg$^{-1}\simeq4.8\times10^{-35}$ GeV^{-1}. This constrains one model proposed in Ref. [9] where D might be as large as 10^{-34} GeV^{-1}.

We also found $\Delta\nu_0=-13\pm30\pm170$ μHz between ν_0 measured in an electromagnet [36] and a superconducting magnet system. The first error is the uncertainty in the reference oscillator for the time between measurements ($\simeq5$ yr), the second, relatively large error is due to the uncertainty in an estimate of the pressure shift [34] in different apparatuses. Adding the errors in quadrature, we obtain $|\Delta\nu_0|<186$ μHz $(<7.7\times10^{-19}$ eV). For the electromagnet, $d=2.7$ cm, $l=33$ cm, $R=15$ cm (V_{AB}^T is enhanced by 2 because of the two pole faces of the magnet). We neglect the magnet yoke which will increase the effects of V_{AB}^T slightly. From $\Delta\nu_0$ and Eq. (4), we can put an upper limit on T [Eq. (2)] for assumed values of λ_ϕ. Here, for brevity, we will assume that $\lambda_\phi\gg d,l,R$ in which case V_{AB}^T has the same form as normal magnetic coupling. In Table II, we list the ratio α of E_A^T [Eq. (4)] to the normal magnetic interaction for various experi-

TABLE II. Experimental determinations of $\alpha(A,B)$, the ratio of E_A^T [Eq. (4)] to normal magnetic coupling for particles A and B as indicated in Fig. 1. Here, e^- denotes electron, p denotes proton, n denotes neutron, and d denotes deuteron.

Experiment	$\alpha(A,B)$	Method
Ref. [25]	$\alpha(p,p)<5\times10^{-5}$	Molecular spectra
Ref. [26]	$\alpha({}^{201}\mathrm{Hg},e^-)<10^{-10}$–$10^{-11}$	NMR
Ref. [26]	$\alpha({}^{199}\mathrm{Hg},e^-)<10^{-10}$–$10^{-11}$	NMR
Ref. [27]	$\alpha(p,e^-)-\alpha(d,e^-)<2\times10^{-10}$	NMR
Ref. [20]	$\alpha(e^-,e^-)\lesssim3\times10^{-10}$	Acceleration
Ref. [19]	$\alpha(e^-,e^-)<0.9\times10^{-11}$	Acceleration
Ref. [17]	$\alpha(e^-,e^-)<0.005\times10^{-11}$	Induced magnetism
Ref. [18]	$\alpha(e^-,e^-)<0.00085\times10^{-11}$	Induced magnetism
This work	$\alpha({}^9\mathrm{Be},e^-)<3.8\times10^{-11}$	NMR
This work	$\alpha(n,e^-)<2.3\times10^{-11}$	NMR
This work	$\alpha(e^-,e^-)<4.1\times10^{-11}$	NMR

ments. From Ref. [32], $\lambda_\phi\simeq10$ cm is within a "window" on the axion which has not been ruled out by stellar energy-loss rate arguments. If we take $T=g_Ag_B/16\pi \times M_AM_B$, where g_A and g_B are coupling strengths and M_A and M_B are the masses of the elementary particles responsible for V_{AB}^T, then from Refs. [7] and [32] we estimate theoretically for axions $\alpha(A,B)\approx10^{-26}$ for $\lambda_\phi \simeq10$ cm, considerably smaller than the sensitivity of the current laboratory experiments.

So far we have assumed that $\mathbf{S}_A$ is due to intrinsic spin. From Eq. (1), if $\mathbf{S}_A$ were orbital angular momentum rather than intrinsic spin, we would expect spinning gyroscopes to have a weight dependent on $\mathbf{S}_A\cdot\mathbf{g}/|\mathbf{g}|$, where $\mathbf{g}$ is the acceleration of gravity. Although an effect like this (for one sign of $\mathbf{S}_A\cdot\mathbf{g}$) has been reported [37], more accurate subsequent experiments [38–40] found a null result (an earlier experiment on the intrinsic spin of the neutron was reported in Ref. [21]). The authors of Ref. [40] found $\Delta m<9\times10^{-8}$ kg for a 0.143 kg rotor whose angular momentum was approximately equal to $7.2 \times10^{32}\hbar$. If we assume a gyroscopic weight is due to V_{AB}^D, and $\lambda_\phi\gg R_{\mathrm{Earth}}$, then the force derived from Eq. (3) gives $D<4.0\times10^{-3}$ kg^{-1} for the experiment of Ref. [40]. In our experiment, if we consider the orbital motion of the unpaired neutron in ^{9}Be to act like a gyroscope with orbital angular momentum $\mathbf{L}$, we obtain $D<5\times10^{-9}$ kg^{-1}, about 10^6 times more stringent a limit on an $\mathbf{S}_A\cdot\mathbf{g}/|\mathbf{g}|$ effect for angular momentum [4]. From Refs. [37–40], for our experiment, we might expect to replace $\mathbf{S}_A$ in Eq. (1) by $m_{\mathrm{neutron}}r_{\mathrm{eq}}\boldsymbol{\omega}\simeq\mathbf{L}/r_{\mathrm{eq}}$, where r_{eq} is defined in Ref. [37] (taken to be 10^{-13} cm here). If we assume $\lambda_\phi\gg R,d$ in Eq. (3), then the resonance test reported here is approximately 10^{18} times more sensitive than the acceleration test of Nitschke and Wilworth [40,41].

It is interesting to compare the sensitivity of resonance experiments (such as the type of experiment reported here) with acceleration tests. From Eqs. (3) and (4), we calculate the accelerations $a_D=-(\partial E_A^D/\partial d)/m_A$ and $a_T =-(\partial E_A^T/\partial d)/m_A$, where m_A is the unit of mass associated with each spin in A. If we use parameters of the current resonance and acceleration [16] experiments, the limits placed on C^D and C^T are roughly equal when λ_ϕ is on the order of the size of laboratory experiments. Resonance (acceleration) tests are more sensitive for larger (smaller) values of λ_ϕ.

Significant improvements in the sensitivity of the $^9\mathrm{Be}^+$ experiment could be expected. For 100% detection efficiency [34], a resonance time T_R (time taken to drive the ν_0 transition), N stored ions, and total measurement time τ, the statistical uncertainty in a frequency measurement is $\delta\nu=(4\pi^2NT_R\tau)^{-1/2}$ [34]. In future experiments $N=10^7$, $T_R=100$ s, and $\tau=30$ days do not seem unreasonable, whence $\delta\nu_0\simeq3\times10^{-9}$ Hz. In the $^9\mathrm{Be}^+$ experiments, the pressure shift can be made negligible by using cryogenic pumping. By operating on other field-independent transitions in $^9\mathrm{Be}^+$ or other ions, we could make $|\Delta\sigma_{Sz}|\simeq1$ for the electron and be much more sensitive to anomalous couplings to electron spin. The characteristic dimensions of the ion experiments can also be made very small and therefore can investigate relatively short ranges ($\lambda_\phi<1$ mm).

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