TEST OF THE LINEARITY OF QUANTUM MECHANICS

BY RF SPECTROSCOPY OF THE $^{9}BE^{+}$ GROUND STATE*

D.J. Heinzen, J.J. Bollinger, Wayne M. Itano,
S.L. Gilbert, and D.J. Wineland

National Institute of Standards and Technology
Boulder, CO 80303

The agreement between quantum-mechanical predictions and experiments has so firmly established the validity of quantum mechanics that we might question the need for further tests. For example, the observed energy levels of hydrogen are in excellent agreement with quantum-mechanical predictions. However, to a large extent this agreement can be regarded more as a test of the specific Hamiltonian used, than of quantum mechanics itself. It should be possible to test the basic framework of quantum mechanics independently of and to much greater accuracy than that provided by such specific predictions. Recently, Weinberg[1] has formulated a general framework which introduces nonlinear corrections to quantum mechanics and enables such a test. He suggested that a sensitive test could be made for the presence of such a nonlinearity by slowly driving a resonant transition which transfers a quantum system from one state to another. The effect of the nonlinearity is to make the effective resonance frequency a function of the state probabilities. Thus, the resonance frequency changes as the quantum system is driven from one state to the other, and the applied perturbation (assumed to be monochromatic) cannot stay in resonance through the entire transition. The system will never be driven completely to the other state. This effect would not be observed, that is, the transition could still be driven, if the maximum frequency shift were much less than 1/T. Here, T is the time required to drive the transition if there is no nonlinearity. Since such transitions have been observed with T as long as ~ 1 s, we can set a limit $\sim 10^{-15}$ eV on the magnitude of such nonlinear corrections to quantum mechanics.[1] In this work, we report an experiment which improves this limit by 5 orders of magnitude.

In the formalism developed in Ref. 1, the equation which describes the time evolution of the wave function $\psi(t)$ is nonlinear and derivable from a Hamiltonian function $h(\psi,\psi^*)$. For a discrete system, it takes the form

$$i\hbar\frac{d\psi_k}{dt} = \frac{\partial h(\psi,\psi^*)}{\partial \psi_k^*} , \tag{1}$$

where ψ_k is the amplitude of state k. In general, h is not a bilinear function of ψ and ψ^* as in ordinary quantum mechanics, but the property of homogeneity is retained. Homogeneity guarantees that if $\psi(t)$ is a solution

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of Eq. (1), $\lambda\psi(t)$ is also a solution representing the same physical state, where λ is an arbitrary complex number.

Consider a two-level system which in the absence of nonlinear corrections has eigenvalues E_k, $k = 1, 2$. Because any nonlinear corrections to quantum mechanics are expected to be small, we write the Hamiltonian function as the sum of the bilinear term $h_0(\psi,\psi^*) = \sum_{k=1,2} E_k\psi_k^*\psi_k$ of ordinary quantum mechanics and a term $h_{n\ell}(\psi,\psi^*)$ that is not bilinear and contains the small nonlinear corrections. A form of $h_{n\ell}$ appropriate for the work discussed here is $h_{n\ell} = n\bar{h}(a)$, where $n \equiv |\psi_1|^2 + |\psi_2|^2$, $a \equiv |\psi_2|^2/n$, and $\bar{h}$ is a real function.[1] The nonlinear wave equation (1) then has solutions

$$\psi_k(t) = c_k\, e^{-i\omega_k(a)t},\ k = 1,2, \tag{2}$$

where a and the c_k's can be parameterized by $c_1 = \sin(\theta/2)$ and $c_2 = a^{\frac{1}{2}} = \cos(\theta/2)$. The relative phase of the two components of the wave function (specifically, the time dependence of the coherence $\psi_1\psi_2^*$) evolves with a frequency

$$\omega_p \equiv \omega_1(a) - \omega_2(a) = \omega_0 - \frac{d\bar{h}}{da}/\hbar, \tag{3}$$

where $\omega_0 = (E_1-E_2)/\hbar$ is the atomic transition frequency in the absence of nonlinearities. We thus see that the effect of the nonlinearity is to introduce a dependence of the precession frequency of a two-level system on the admixture of states (i.e., on the "tipping angle" θ of the equivalent fictitious spin-$\frac{1}{2}$ system).

In this work, we look directly for such a change in the precession frequency of a two-level system as a function of the admixture of upper and lower states. $^9Be^+$ ions are stored in a Penning trap and sympathetically cooled and compressed by laser-cooled $^{26}Mg^+$ ions. The $(m_I, m_J) = (-1/2, +1/2) \rightarrow (-3/2, +1/2)$ "clock" transition[2] of the $^9Be^+$ ground state is chosen as the two level system; its transition frequency of $\nu \cong 303$ MHz is magnetic field-independent to first order at the field $B \cong 0.8194$ T. The transition frequency is probed using Ramsey's method of separated oscillatory fields in the time domain, with free precession times as long as 550 s, giving a linewidth of 900 μHz and a line Q of 3.3×10^{11}. The second Ramsey RF pulse has an angle of $\pi/2$, as usual, but the angle of the first RF pulse alternates between $\theta_A = 0.325\pi$ and $\theta_B = 0.675\pi$, thus preparing differing initial admixtures. The frequency of a reference oscillator is then locked to the $^9Be^+$ precession frequency, and the data are analyzed for a difference in the precession frequency with initial pulse angle θ_A from that with initial pulse angle θ_B. A typical recording of oscillator frequency as a function of time is shown in Fig. 1, with the periods for which the pulse angle was θ_A or θ_B denoted. A weighted average over a number of runs similar to that shown in Fig. 1 yields, for the difference in precession frequencies

$$[\omega_p(\theta_B) - \omega_p(\theta_A)]/2\pi = 3.8(8.3)\ \mu\text{Hz}. \tag{4}$$

With $\psi_1 \equiv \psi(m_I = -3/2,\ m_J = +1/2)$ and $\psi_2 \equiv \psi(-1/2, +1/2)$, the simplest nonbilinear addition to the Hamiltonian on the free $^9Be^+$ nucleus is[1] $\bar{h}(a) = 2\varepsilon a^2$, where ε is a measure of the strength of the nonlinear correction. This gives rise to a dependence of ω_p on θ of $\omega_p = \omega_0 - 4(\varepsilon/\hbar)\cos^2(\theta/2)$. Thus, given our limit [Eq.(4)] on the difference in precession frequency, and a loss of 28% in sensitivity due to a finite servo time constant, we are able to set a limit of $\varepsilon = 1.8(4.0)$ μHz, or

$$|\varepsilon| < 2.4 \times 10^{-20} \text{ eV } (5.8\ \mu\text{Hz}) \tag{5}$$

on a possible nonlinear correction to the $^9Be^+$ nuclear Hamiltonian. This is less than 4 parts in 10^{27} of the binding energy per nucleon of the $^9Be^+$ nucleus and improves the limit quoted in Ref. 1 by roughly 5 orders of magnitude.

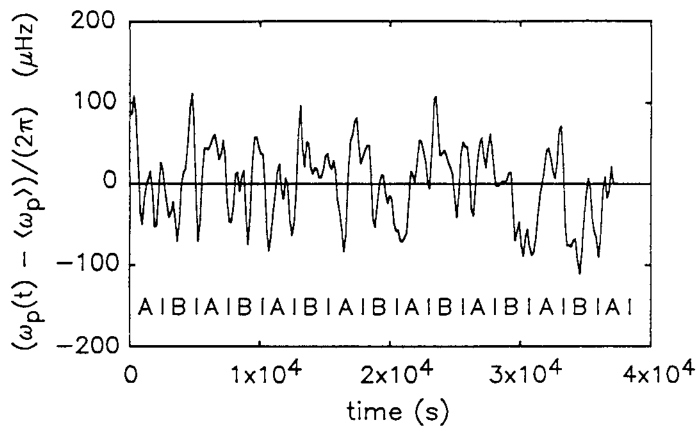

Fig. 1. $^9Be^+$ precession frequency $\omega_p(t)$, referred to a passive hydrogen maser, as a function of time. $\langle\omega_p\rangle$ is the average value of $\omega_p(t)$ over the run. Also indicated are the periods A(B) during which the initial pulse angle was $\theta_A(\theta_B)$.

Our experimental result is limited by statistics due to the instability of the reference oscillator (commercial cesium beam clock or passive hydrogen maser) used to drive the RF synthesizer. With a better reference oscillator, such as an active hydrogen maser, it should be possible to improve our limit on $|\varepsilon|$ by more than an order of magnitude.

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