QUANTUM OPTICS OF SINGLE, TRAPPED IONS*

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INTRODUCTION

Single ions in ion traps can be localized in small volumes and held for long periods of time. This makes it easier to observe certain nonclassical properties of the electromagnetic field, such as photon antibunching and sub-Poissonian photon statistics, which are reduced when large numbers of atoms are present. Such properties can be observed in atomic beams so dilute that the probability of having more than one atom in the observation volume is low.[1,2] Trapped ions can be studied for much longer times. This makes it possible, for example, to observe repeated quantum jumps of the *same* atom.[3-5] When there are only a few (or one) ions in the trap, their number can be known and kept fixed. Thus, sub-Poissonian photon statistics can be observed[6,7] *without* the time gating which is necessary with atomic beams.[2] Another advantage of trapped ions is that, thanks to their isolation from collisions and other perturbations, they can be laser-cooled to low temperatures and studied spectroscopically with great precision. A single ion has even been cooled to the ground energy level of the harmonic well of the trap, so its motion must be treated quantum mechanically.[8]

EXPERIMENT

Hg^+ ions were confined in a Paul trap that consists of a toroidal ring electrode, with an inner diameter slightly less than 1 mm, and two endcap electrodes, which are placed symmetrically on opposite sides of the hole through the ring.[9] A combination of static and rf potentials applied between the electrodes effectively creates a three-dimensional harmonic well. The classical motion consists of a small-amplitude oscillation, at the frequency of the applied rf potential, superimposed on a large-amplitude, harmonic motion, called the secular motion. The frequencies of the secular motion were 1–4 MHz.

A cw, tunable 194 nm radiation source[10] with a power of about 5 μW was used to excite the $^2S_{1/2}$ to $^2P_{1/2}$ first resonance line of Hg^+. In order to cool the ions, the frequency of the 194 nm source was tuned slightly lower than the atomic resonance. The radiation pressure force increases when the ion's velocity is opposed to the direction of 194 nm propagation, because the Doppler shift brings the light frequency closer to resonance. This form of laser cooling is called Doppler cooling. The 194 nm fluorescence

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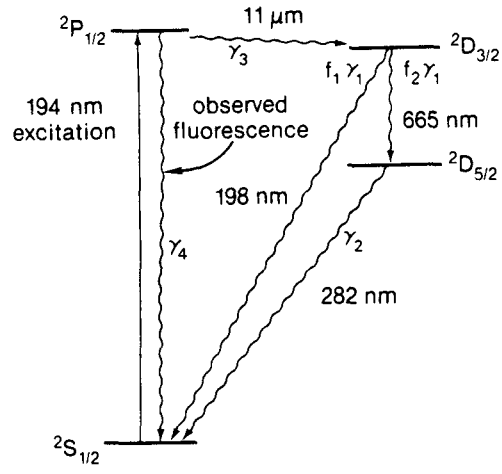

Figure 1 The lowest four energy levels of Hg^+.

from a single, cooled ion was easily observed. The photon detection efficiency was about 5×10^{-4}. Approximately 50 000 photons/s from a single ion were detected with a photomultiplier tube.

QUANTUM JUMPS

Quantum jumps of single, trapped ions have been observed in Ba^+,[3,4] Hg^+,[5] and Mg^+.[11] In this context, the term "quantum jump" refers to a sudden change in the fluorescence intensity when the atom makes a transition to or from a metastable level. We have observed photon antibunching and sub-Poissonian statistics in the system shown in Fig. 1. Measurements of the radiative decay rates were reported previously.[7] The 194 nm source drives the Hg^+ ion from the ground $^2S_{1/2}$ level to the $^2P_{1/2}$ level, which decays at a rate of 4×10^8 s^{-1}. Usually, the ion decays back to the ground state, emitting a 194 nm photon. However, about once in 10^7 times, it decays instead to the metastable $^2D_{3/2}$ level, and emits an 11 μm photon. The $^2D_{3/2}$ level decays with a total rate of $\gamma_1 = 109 \pm 5$ s^{-1}. It decays directly to the ground state with probability $f_1 = 0.491 \pm 0.015$ and to the metastable $^2D_{5/2}$ level with probability $f_2 = 1 - f_1$. The $^2D_{5/2}$ level decays to the ground state at a rate $\gamma_2 = 11.6 \pm 0.4$ s^{-1}.

Figure 2 shows the 194 nm fluorescence from a single Hg^+ ion as a function of time. When the ion makes transitions between the $^2S_{1/2}$ level and the $^2P_{1/2}$ level, the fluorescence has a high, steady level. When the ion makes a transition (quantum jump) to the $^2D_{3/2}$ level and emits an 11 μm photon, the fluorescence drops to the background level. When the ion returns to the ground state, the fluorescence goes back to the high level. We call the sudden drops in fluorescence "on-to-off" quantum jumps and the sudden increases "off-to-on" quantum jumps.

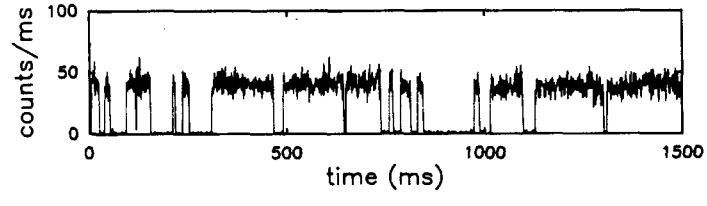

Figure 2. 194 nm fluorescence of a single Hg^+ ion as a function of time.

Table 1. Table of observed and calculated values of $Np(n)$ and Q. Values of $Np(n)$ and Q for Poissonian distributions having the same $\langle n \rangle$ are shown for comparison.

		$p(n)$								Q
	$n =$	0	1	2	3	4	5	6	7	
1-ion	Obs.	350	785	572	222	58	12	0	1	-0.253 ± 0.025
	Calc.	360.3	771.5	568.3	228.2	59.2	10.9	1.5	0.2	-0.243
	Poiss.	470.5	680.9	492.6	237.6	86.0	24.9	6.0	1.2	0.000
2-ion	Obs.	796	1503	1118	445	99	35	2	2	-0.194 ± 0.019
	Calc.	775.0	1558.1	1077.1	436.8	122.3	25.8	4.3	0.6	-0.198
	Poiss.	969.0	1373.8	973.9	460.2	163.1	46.3	10.9	2.2	0.000

Let $g^{(2)}(\tau)$ be the normalized intensity correlation function of the 11 μm field generated by the radiative decay from the $^2P_{1/2}$ level to the $^2D_{3/2}$ level. Reference 7 used a solution of the density-matrix equations of the four-level atomic system, valid for times long compared to the lifetime of the $^2P_{1/2}$ level, to show that

$$g^{(2)}(\tau) = 1 - C_+ e^{-\gamma_+ \tau} - C_- e^{-\gamma_- \tau}, \tag{1}$$

where

$$\gamma_\pm \equiv \frac{1}{2}\{(\gamma_0 + \gamma_1 + \gamma_2) \pm [(\gamma_0 + \gamma_1 + \gamma_2)^2 - 4f_2\gamma_0\gamma_1 - 4\gamma_1\gamma_2 - 4\gamma_0\gamma_2]^{\frac{1}{2}}\}, \tag{2}$$

$$C_\pm \equiv \pm\gamma_\mp(f_1\gamma_\pm - \gamma_2)/[\gamma_2(\gamma_+ - \gamma_-)]. \tag{3}$$

Here, γ_0 is the transition rate from the "on" to the "off" state. It depends on the intensity and frequency detuning of the 194 nm source. Since $g^{(2)}(\tau) \to 0$ as $\tau \to 0$, the 11 μm light is antibunched. The calculated and measured $g^{(2)}(\tau)$ are in good agreement [see Fig. 2(a) of Ref. 7]. Photon antibunching was also observed with two Hg^+ ions separated by about 3 μm.[7] The observed $g^{(2)}(\tau)$ was consistent with the assumption that the two ions were independent.

Given $g^{(2)}(\tau)$, we can compute the complete photon counting distribution. Let $p(m)$ be the probability that m, 11 μm photons are emitted in an interval of length T. It can be shown that

$$p(m) = \frac{1}{m!}\sum_{r=0}^{\infty}\frac{(-1)^r}{r!}\langle n^{(m+r)}\rangle, \tag{4}$$

where the rth factorial moment of n is defined for $r = 1, 2, 3\ldots$, by

$$\langle n^{(r)}\rangle \equiv \langle n(n-1)\ldots(n-r+1)\rangle \equiv \sum_{n=0}^{\infty} n(n-1)\ldots(n-r+1)p(n) \tag{5}$$

and $\langle n^{(0)}\rangle = 1$.[13] With the assumption that the ion loses memory of its previous history after each 11 μm photon emission, the factorial moments are given by

$$\langle n^{(r)}\rangle = \langle\hat{I}\rangle^r r! \int_0^T dt_r \ldots \int_0^{t_2} dt_1\, g^{(2)}(t_r - t_{r-1})\ldots g^{(2)}(t_2 - t_1), \quad r = 2, 3, \ldots,$$

$$\langle n^{(1)}\rangle = \langle n\rangle = \langle\hat{I}\rangle T, \qquad \langle n^{(0)}\rangle = 1, \tag{6}$$

where $\langle\hat{I}\rangle$ is the average number of 11 μm photons emitted per unit of time.[13] The normalized second factorial moment Q is given by Eq. (11a) of Ref. 13. This parameter measures the departure of the variance of a distribution from that of a Poissonian distribution. Negative values of Q indicate sub-Poissonian statistics.

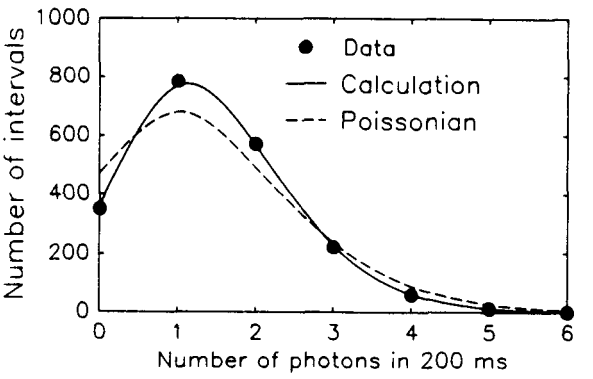

Figure 3. Observed and calculated photon count distributions. The uncertainties are less than or approximately equal to the radii of the dots. A Poissonian distribution is shown for comparison.

Observed and calculated values of $Np(n)$ and Q for one and two ions are shown in Table 1. Here, $Np(n)$ is the number of intervals in which n photons were detected, where N is the total number of intervals. For the one-ion data, $N = 2000$, $T = 200$ ms, $\gamma_0 = 12.55\ \mathrm{s}^{-1}$. For the two-ion data, $N = 4000$, $T = 100$ ms, $\gamma_0 = 12.1\ \mathrm{s}^{-1}$. The observed and calculated values are in good agreement with each other and clearly differ from the values for a Poissonian distribution. The statistical uncertainties of $Np(n)$ are approximately the square roots of the measured values. The values of Q depend on the value of T. For example, Q was measured to be -0.242 ± 0.025 for $T = 200$ ms.[7] Values of $p(n)$ were calculated by computing $\langle n^{(r)} \rangle$ for $r = 0, \ldots, 11$ from Eq. (6) and using Eq. (4). Figure 3 shows the observed and calculated values of $Np(n)$ for one ion. The dots represent the experimental data. The solid line is a smooth curve connecting the calculated values.

If the atom loses all memory after each quantum jump, then the lengths of successive intervals between jumps should show no correlation. Let $T_1, T_2, \ldots, T_n$ be the successive intervals between off-to-on or on-to-off quantum jumps. A nonrandom pattern in the scatter plot of T_n vs T_{n+1} would be an indication that the seemingly random sequence of quantum jumps was actually governed by a low-dimensional chaotic attractor. Such patterns have been observed, for example, in the time sequence of drips from a leaky faucet.[14] Plots of T_n vs T_{n+1} for both the off-to-on and the on-to-off quantum jumps are shown in Fig. 4. No nonrandom structure is apparent.

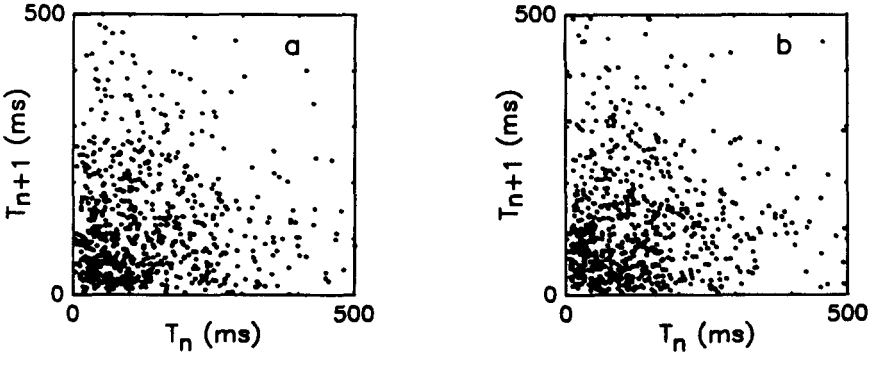

Figure 4. Scatter plot of T_{n+1} vs T_n, where T_n is the nth interval between successive (a) on-to-off quantum jumps and (b) off-to-on quantum jumps.

QUANTUM HARMONIC OSCILLATOR

The wavefunctions of a single charged particle in a Paul trap have been calculated in Refs. 15 and 16. Since the Hamiltonian is time dependent, stationary states do not exist. However, quasi-stationary states, obtained by solving for the eigenvalues of the Floquet operator, do exist. They correspond in some sense to the stationary states of the secular harmonic oscillator potential.

The absorption spectrum of a narrow optical resonance of an ion in a Paul trap consists of an unshifted carrier, surrounded by discrete motional sidebands. The sidebands are spaced by multiples and combinations of the secular frequencies. We have observed these sidebands in the absorption spectrum of the $^2S_{1/2}$-to-$^2D_{5/2}$ transition of a single trapped Hg^+ ion.[9] The carrier results from transitions in which the vibrational quasi-energy is unchanged. The upper and lower sidebands correspond to transitions which increase or decrease the quasi-energy.

Recently, we have cooled a single Hg^+ ion almost to the lowest ($n_v = 0$) vibrational quasi-energy state, by a method called sideband cooling.[8] First, the ion was Doppler cooled to about 2 mK with the $^2S_{1/2}$-to-$^2P_{1/2}$ transition, so that the mean value of n_v was about 12 at a secular frequency of 2.96 MHz. Laser radiation tuned to the first lower sideband of the narrow $^2S_{1/2}$-to-$^2D_{5/2}$ transition was then applied to the ion, lowering n_v by one for each photon absorbed. After the sideband cooling period, laser radiation of saturating intensity was applied at the lower sideband frequency. Absence of absorption, detected by optical double resonance,[9] indicated that the ion was in the $n_v = 0$ state. The ion was found to be in the ($n_v = 0$) state about 95% of the time.

Starting from the $n_v = 0$ state, other quantum harmonic oscillator states could be prepared by manipulation of the electric potentials applied to the trap. A coherent state could be prepared by quickly shifting the static potential of one endcap relative to the other, thus shifting the center of the potential well. A squeezed state could be prepared by shifting the static potential on both of the endcaps relative to the ring.

Acknowledgments

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