readjustment of the constants by Cohen and Taylor[16] uses the $N_A\Lambda^3$ results of Henins and Bearden,[10] and of Bearden,[17] together with the two most recent μ_p/μ_n measurements (which can be arranged to yield Avogadro's constant, N_A) to obtain a value of Λ. In terms of Cu $K\alpha_1$ =1.537 400 kxu, their result is 1.002 0772 Å/kxu (5.3 ppm) which differs by 3.0 ppm from our direct measurement.

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[1]R. D. Deslattes and W. C. Sauder, in *Atomic Masses and Fundamental Constants 4,* edited by J. H. Sanders and A. H. Wapstra (Plenum, New York, 1972), p. 337.

[2]Static versions of x-ray interferometers were first demonstrated by U. Bonse and M. Hart, Appl. Phys. Lett. 7, 99 (1965). Several efforts to combine scanning x-ray interferometers with optical ones were subsequently begun. These efforts are reviewed by R. D. Deslattes, in *Precision Measurement and Fundamental Constants,* edited by D. N. Langenberg and B. N. Taylor, National Bureau of Standards Special Publication No. 343 (U.S. GPO, Washington, D.C., 1971), p. 265; U. Bonse, E. teKaat, and P. Spieker, *ibid.,* p. 291; I. Curtis, I. Morgan, M. Hart, and A. D. Milne, *ibid.,* p. 285.

[3]This sample was produced several years ago by Dow Chemical and is of the type which was marketed under the trade name Perfex.

[4]H. Kogelnik and T. Li, Proc. IEEE 54, 1312 (1966).

[5]W. G. Schweitzer, Jr., E. G. Kessler, Jr., R. D. Deslattes, H. P. Layer, and J. R. Whetstone, to be published.

[6]R. L. Barger and J. L. Hall, Appl. Phys. Lett. 22, 196 (1973); K. M. Evenson, J. S. Wells, F. R. Petersen, B. L. Danielson, and G. W. Day, Appl. Phys. Lett. 22, 192 (1973).

[7]R. D. Deslattes, H. P. Layer, and W. G. Schweitzer, Jr., to be published.

[8]We are indebted to R. K. Kirby of the National Bureau of Standards for this measurement.

[9]R. D. Deslattes, Rev. Sci. Instrum. 38, 815 (1967).

[10]I. Henins and J. A. Bearden, Phys. Rev. 135, A890 (1964).

[11]J. A. Bearden, in *Precision Measurement and Fundamental Constants,* edited by D. N. Langenberg and B. N. Taylor, National Bureau of Standards Special Publication No. 343 (U.S. GPO, Washington, D.C., 1971), p. 251.

[12]J. S. Thomsen and A. F. Burr, Amer. J. Phys. 36, 803 (1968); J. A. Bearden, Phys. Rev. 137, B181 (1965).

[13]J. A. Bearden, Phys. Rev. 137, B455 (1965).

[14]J. A. Bearden, A. Henins, J. G. Marzolf, W. C. Sauder, and J. S. Thomsen, Phys. Rev. 135, A899 (1964).

[15]A. Henins, in *Precision Measurement and Fundamental Constants,* edited by D. N. Langenberg and B. N. Taylor, National Bureau of Standards Special Publication No. 343 (U.S. GPO, Washington, D. C., 1971), p. 255.

[16]E. R. Cohen and B. N. Taylor, to be published. We are indebted to these authors for prepublication communications.

[17]J. A. Bearden, Phys. Rev. 137, B181 (1965).

Observation of the $g-2$ Resonance of a Stored Electron Gas Using a Bolometric Technique*†

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A bolometric technique is used to determine the g-factor anomaly a of the free electron from measurements made on 78°K electrons contained in a Penning-style quadrupole ion trap. The value obtained is $a=0.001\,159\,667(24)$, where the electron g factor is given by $g=2(1+a)$.

Precision measurements of the g-factor anomaly a [where $g=2(1+a)$] of the free electron have provided the most sensitive tests of QED up to the present time. In this paper a new method of measuring a is described which differs in every detail from the method used to obtain the most precise, currently accepted experimental value of a.[1]

We have applied a bolometric technique described earlier[2,3] to the measurement of the cyclotron and the $g-2$ resonances of an electron gas stored in a quadrupole ion trap. The value of a for the free electron is then determined using corrections derived from direct measurements of the perturbations caused by the trapping fields and the stored electrons.

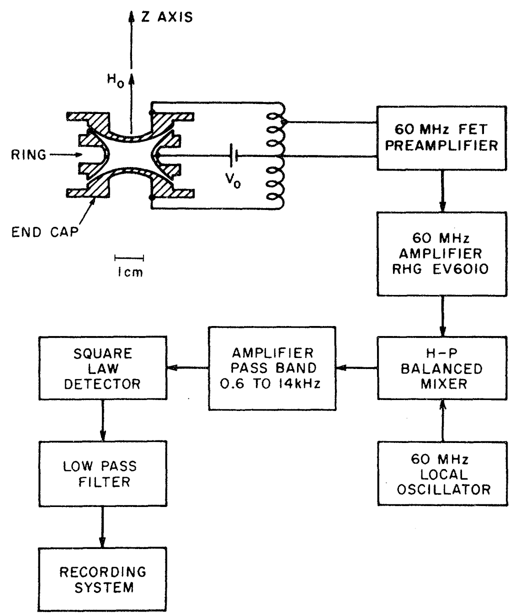

FIG. 1. Schematic representation of the cylindrically symmetric trap electrodes and the bolometric detection circuit.

A block diagram of the detection apparatus and the trap electrodes is shown in Fig. 1. The trap's molybdenum electrodes are hyperbolas of revolution (inside diameter of ring $\equiv 2R_0$; axial separation of the end caps $\equiv 2Z_0$). An electrostatic potential

$$V = V_0(r^2 - 2z^2)/(R_0^2 + 2Z_0^2), \tag{1}$$

where $r^2 = x^2 + y^2$, results when the ring electrode is held at the positive dc potential V_0 with respect to the two end caps. This confines electrons along the z axis. Confinement in the x-y plane is achieved by applying a static, homogeneous axial magnetic field $\vec{B} = B_0\hat{k}$, where $\hat{k}$ is a unit vector along the z axis. Electrons are injected through a hole in one end cap and ionize residual gas atoms, producing slow electrons which are trapped. Three separate types of motion are combined in the observed motion of the electrons: (1) harmonic oscillation parallel to the z axis with frequency $\omega_{z0} = [4eV_0/(mR_0^2 + 2mZ_0^2)]^{1/2}$, where m is the electron's mass; (2) cyclotron motion with frequency $\omega_c = eB_0/mc$ (this frequency is shifted slightly to $\omega_c' = \omega_c - \omega_m$ by the radial electric fields; ω_m is defined below); (3) magnetron motion consisting of a slow drift of the center of the cyclotron orbits about the z axis at the frequency ω_m, related to ω_c and ω_{z0} by $2\omega_m^2 - 2\omega_m\omega_c + \omega_{z0}^2 = 0$. The harmonic axial motion of the electrons at frequency ω_{z0} induces a current in the tank circuit formed by the capacitance between the end caps and the coil connecting them, shown in Fig. 1. In the absence of external heating, the Joule heating of the loss resistance of this circuit causes the temperature of the electrons to approach exponentially that of the tank circuit with a time constant of $2\tau_{zt} = 0.06$ sec.

Pressures in the trap[4] are maintained below 5×10^{-11} Torr. The extrapolated electron lifetime at this pressure is several weeks. The electron temperature is deduced from the noise voltage across the tank circuit which is amplified, square-law detected, and filtered to yield a dc voltage proportional to the noise temperature T_t of the tank circuit. When the electrons are externally heated or cooled, the tank-circuit noise voltage —and hence the dc level—is changed accordingly. The sensitivity η of this technique for detecting changes of the electron-gas temperature is given by[2,3]

$$\eta \equiv \frac{T_t - T_0}{T_e - T_0} = \frac{n/n_c}{1 + n/n_c}, \tag{2}$$

where the "critical number" $n_c = n\gamma/(1-\gamma)$, T_0 is the equilibrium temperature of the tank circuit, T_e the temperature of the electron gas, n the number of electrons, and γ the ratio of detected noise level with electrons to the noise level without electrons.

A small departure of the electrostatic potential from that given by Eq. (1) is caused by the space charge of the electron gas. By including the effect of space charge on the classical equations of the electrons' motion one finds[5]

$$\omega_m(R_m, n) = \frac{\frac{1}{2}\omega_z^2(R_m, 0)}{\omega_c - \omega_m} + \frac{c}{B}\frac{E_r^e}{R_m}, \tag{3}$$

$$\omega_c'(R_m, n) = \omega_c - \frac{\frac{1}{2}\omega_z^2(R_m, 0)}{\omega_c - \omega_m} - \frac{c}{B}\frac{dE_r^e}{dr}, \tag{4}$$

$$\omega_z^2(R_m, n) = \omega_z^2(R_m, 0) - (e/m)dE_z^e/dz, \tag{5}$$

where $\omega(R_m, n)$ is the oscillation frequency as a function of radial distance from the center and electron number; E_r^e and E_z^e are the radial and axial space-charge electric fields, respectively; and R_m is the radius of the ω_m motion. (In the remainder of the paper, an ω without an indicated dependence on position or electron number is to be taken as an ensemble average over the trap.)

The shifted $g-2$ transition at frequency

$$\omega_{g-2}' \equiv \omega_s - \omega_c' = \omega_s - \omega_c + \frac{\frac{1}{2}\omega_z^2}{\omega_c - \omega_m} + \frac{c}{B}\frac{dE_r^e}{dr} \qquad (6)$$

is measured by alternately applying a transverse magnetic field at the spin precession frequency ω_s and a transverse rf magnetic field gradient (whose frequency is swept through the value ω_{g-2}').[3] At resonance this two-step cycle ideally has the following effect on the cyclotron and spin systems. The transverse magnetic field at ω_s heats the spin system to a very high temperature, i.e., equalizes the spin populations. The transverse magnetic field gradient at ω_{g-2}' thermally couples the spin and cyclotron systems so that the spin temperature approaches that of the cyclotron motion. The extra energy given up by the spin system heats the cyclotron motion and the tank circuit via electron-electron collisions and the radiative coupling. The maximum size of the expected temperature increase of the tank circuit is $\Delta T/T_0 = (\hbar\omega_c/2kT_0)^2(n/n_c)\tau_{zt}/t_c$, where t_c is the period of the ω_s-ω_{g-2}' excitation cycle.[3] For $n/n_c = 2$, $T_0 = 80°K$, $B = 7.7$ kG, $\tau_{zt} = 0.03$ sec, and $t_c = 0.0008$ sec, this amounts to a change in the tank-circuit temperature of 3 parts in 10^3.

Figure 2(a) shows the results of a shifted $g-2$ run with $n/n_c = 2$, $T_0 = 78°K$, and $B = 7.7$ kG. The data were taken using a voltage-to-frequency converter and a multichannel analyzer. The spin frequency was continually frequency modulated over a range of 40 kHz at approximately a 3-kHz rate. In the ω_s-ω_{g-2}' cycle, ω_s was on 380 μsec, followed by 20 μsec of dead time. ω_{g-2}' was on 360 μsec, followed by 40 μsec of dead time. ω_s was on 380 μsec, etc. The large sidebands on either side of the center peak are due to the square-wave amplitude modulation of ω_{g-2}' at 1.25 kHz. The small sidebands at approximately 500 Hz on either side of the large peaks are evidently due to a beat between the rate at which the spin is frequency modulated and the rate at which it is amplitude modulated. Phase information of this type is not included in the simple description of the electron gas in terms of its translational temperature presented above. However, the peaks should be symmetric about the center peak.[3] The fact that the signal is less than 3 parts in 10^3 indicates that the electrons are not fully disoriented and then thermalized in the applied cycle.

The raw data points were smoothed by taking the incremental count value of a given point and adding one half of the value of its two immediate neighbors and dividing by 2. This is justified since the signal covers several channels. In the absence of a complete theoretical description of the expected signal, the data points were connected by smooth lines. The error bar on the central peak indicates the approximate statistical deviation associated with each datum point, from bandwidth considerations. The error in determining the line center is approximately 8 ppm and is indicated by the horizontal bar on Fig. 2(a).

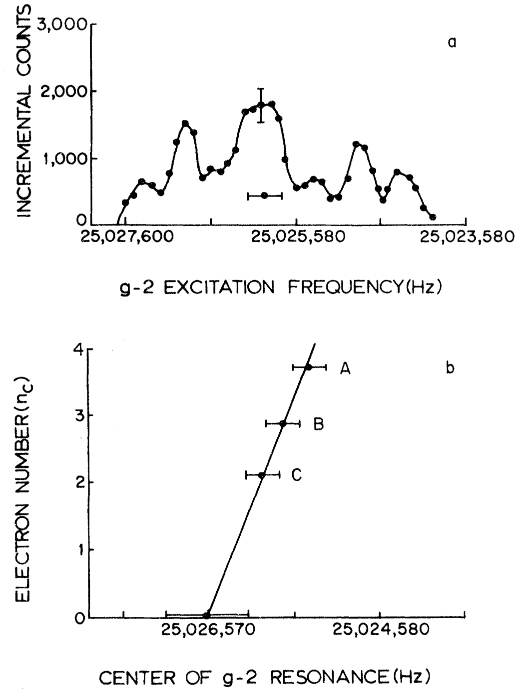

FIG. 2. (a) Shifted $g-2$ resonance with $2.1n_c$ electrons at a magnetic field of 7.7 kG and translational temperature 78°K. Baseline, $\sim 2.5\times10^6$ counts. (b) Measurement of the center of shifted $g-2$ resonance versus measured initial number for three different bunches of electrons.

Figure 2(b) shows a plot of the position of the center of the shifted $g-2$ resonance versus electron number in units of n_c. Each point represents the results of approximately 4.5 h of data on a separate batch of electrons. Data for points A, B, and C were taken with all other parameters held constant, except for twice as much power in the $g-2$ excitation in the case of point C. The change in applied $g-2$ power for point C had a negligible effect on the position of the $g-2$ resonance. (See paragraph on the measurement of ω_z below.) The value of a for free electrons is found by applying Eq. (6) to the definition of a.

Thus,

$$a = (\omega_s - \omega_c)/\omega_c$$
$$= \left(\omega_{g-2}' - \frac{\frac{1}{2}\omega_z^2}{\omega_c - \omega_m} - \frac{c}{B}\frac{dE_r^e}{dr}\right)\omega_c^{-1}. \qquad (7)$$

Figure 2(b) was used to correct for the space-charge effects included in the term $(c/B)dE_r^e/dr$. One would expect this term to vary linearly with electron number, assuming that the dimensions of the space-charge cloud remain constant. The nearly infinite electron lifetimes indicate that radial diffusion in the trap is very small. The z extent is fixed by the well depth and the electron temperature. The shifts due to the applied trapping fields are then calculated from direct measurements of ω_z, ω_c, and ω_m.

The extrapolation approach to eliminating the effect of space charge was made necessary by the lack of information on the absolute number of electrons and exact spatial distribution of the electrons. Estimates of the absolute value of n_c vary from 500 to 5000. Calculations based on the width of ω_m and ω_z yield shifts of ω_c' of the same order of magnitude as the shifts in the value of $\omega_{g-2}'/2\pi$ displayed in Fig. 2(b).

A typical cyclotron spectrum is shown in Fig. 3(a). The sideband separation is equal to ω_m.[3] One of the sidebands corresponds to the true cyclotron frequency and is stationary under changes of ω_m. The value of $\omega_c/2\pi$ for the $g-2$ runs was 21 516 437 000 ± 10 000 Hz. The frequency ω_z is measured by noting the frequency at which the electrons maximally load the Q of the tank circuit. The value obtained for the above $g-2$ runs was $\omega_z/2\pi = 56.685(2)$ MHz. The applied trap voltage was 12.446(3) V. No shift of ω_c or ω_z with electron number was observed beyond the quoted uncertainties.

Direct measurements of ω_m [as defined in Eq. (3)] have been made by slightly modulating the potential of the ring electrode of the trap at a frequency near $\omega_z^2/2(\omega_c - \omega_m)$. Figure 3(b) shows a number of instances of ω_m heating of the electron cloud. Traces 1 and 3 were taken with $1.5n_c$ and $3.75n_c$ electrons, respectively, newly injected into the trap. Trace 2 was taken after about 3 h of data runs. These data indicate that the value of the term $\omega_z^2/4\pi(\omega_c - \omega_m)$ is constant to within 50 Hz over the active volume of the trap and equal to 74 666(50) Hz.

The measurements of ω_m and ω_c were made with both ω_{g-2}' and ω_s off. Measurements of ω_z were made with ω_{g-2}' and ω_s on and modulated,

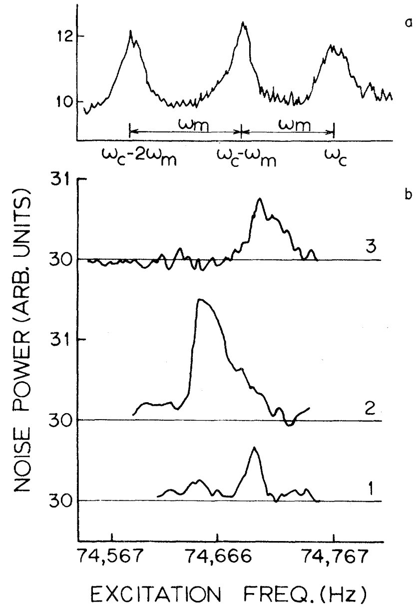

FIG. 3. (a) Cyclotron spectrum obtained from the molybdenum trap with the excitation power adjusted to yield approximately equal heights for all three components. Normally $\omega_c - 2\omega_m$ is 25% and ω_c is 10% of the height of the central $\omega_c - \omega_m$ peak. $\omega_c/2\pi = 21\,516\,437\,000$ (10 000) Hz. $\omega_m/2\pi = 74\,666(50)$ Hz. (b) Heating of the electron cloud due to modulating the potential of the ring electrode at a frequency near $\omega_m/2\pi$. The three traces illustrate the effect of changing electron numbers and residence time in the trap (see text).

as was normally done during the $g-2$ data runs, and also with them off. A shift of less than 1 part in 2000 in the voltage necessary to maximize the tank-circuit loading was noted. This would imply a shift of less than 2 ppm in a due to ω_{g-2}' and ω_s if the $g-2$ measurement weighted the electron distribution in the same way as the ω_z measurement. Relativistic effects are of order kT_e/mc^2, or less than 0.1 ppm of a and are thus neglected.

The value of a derived from this work is $a = 1159\,667 \pm 24) \times 10^{-9}$ (uncertainty ~ 21 ppm). All except 1 ppm of the error assigned to a is due to uncertainties in the extrapolation of Fig. 2(b) to eliminate space-charge effects. This value agrees well with the currently accepted theoretical value,[6,7] $(1159\,652.9 \pm 2.4) \times 10^{-9}$ (uncertainty ~ 2.1 ppm) and also with the value of Rich and Wesley,[1,8] $(1159\,656.7 \pm 3.5) \times 10^{-9}$ (uncertainty ~ 3.0 ppm). Further work is under way to reduce the size of the space-charge effect.

The authors are grateful to H. G. Dehmelt for suggesting this research topic and for many illuminating and vigorous discussions, to J. Jonson for glass work, and to D. Russell for machining the trap electrodes.

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[1] A. Rich and J. C. Wesley, Rev. Mod. Phys. 44, 250 (1972).

[2] H. G. Dehmelt and F. L. Walls, Phys. Rev. Lett. 21, 127 (1968).

[3] F. L. Walls, Ph.D. thesis, University of Washington, 1970 (University Microfilms, Ann Arbor, Michigan, 1970).

[4] Pressure was monitored with a General Electric Triggered Discharge Gauge.

[5] S. Liebes and P. Franken, Phys. Rev. 116, 633 (1959).

[6] M. J. Levine and J. Wright, Phys. Rev. Lett. 26, 1351 (1971).

[7] T. Kinoshita and P. Cvitanovic, Phys. Rev. Lett. 29, 1534 (1972).

[8] S. Granger and G. W. Ford, Phys. Rev. Lett. 28, 1479 (1972).

Shock-Wave Compression of Liquid Deuterium to 0.9 Mbar*

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Liquid-deuterium samples initially at 21°K and 1 atm were shock compressed to pressures of 200 and 900 kbar and temperatures of 4500 and 7000°K, respectively. The 900-kbar points were obtained by reflecting the 200-kbar shock wave from a brass cover. A theoretical model which uses an effective intermolecular pair potential gives reasonable agreement with the data. Using this potential we obtain a lower limit for a metallic phase transition at 1.7 cm^3/mole and at 4.2 and 4.8 Mbar for H_2 and D_2, respectively.

Our interest in the equation of state of the hydrogen isotopes stems in part from their importance in determining the structure of the outer planets, Jupiter, Saturn, and Uranus. These are largely composed of hydrogen, and its presence in insulating and metallic states will have a profound influence on the observed planetary properties. It is generally believed that the outer, low-pressure regions are molecular and the interiors are metallic. This metallic transition boundary has previously been estimated[1] to occur at 0.52 and 0.80 times the radius in Jupiter and Saturn, respectively. Uncertainties in the equation of state of the molecular phase can significantly alter this estimate.[2] Recently, experimental workers[3] have reported a metallic transition at 2.8 Mbar, which would indicate that the above estimates are in error by about 10%.

Recent work in controlled fusion has led to the possibility of using high-powered lasers to compress solid D_2 to the high temperatures and densities required for the fusion reaction to take place.[4,5] An important input into the proper design of experiments will necessarily involve an accurate equation of state of D_2. Since existing technology now allows us to make measurements to nearly one million atmospheres, an improved understanding of planetary structure and controlled fusion is possible. In this report we present the results of such measurements.

Shock compression of a number of liquid-deuterium samples with projectiles from a large two-stage light-gas gun[6] is reported here. In this gun, a piston driven by a charge of propellant compresses a charge of hydrogen gas into a truncated conical volume at the end of a long cylindrical barrel. The gas is allowed to expand into a second stage where it accelerates a projec-