

**FORTY-FOURTH ANNUAL SYMPOSIUM ON FREQUENCY CONTROL**  
**NEW INEXPENSIVE FREQUENCY CALIBRATION SERVICE FROM NIST**

by

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Abstract

A new inexpensive frequency calibration service from NIST is now available. This service takes advantage of the operation of the NIST Automated Computer Time System (ACTS), which was begun in 1988. Software to access the service from several types of computers was released at the same time. Time and frequency dissemination by this modest-accuracy service depends on the reciprocity of the telephone system. The round-trip delay is measured by the NIST equipment. The advance of an on-time marker is adjusted so as to arrive at the user's site on time. A frequency calibration method taking advantage of this service has been designed and preliminary tests conducted. A computer is not required to access this service. All that is required is a telephone modem, a simple peripheral circuit to generate an on time marker and standard time and frequency measurement and data processing equipment.

Introduction, Review and Relevant Statistical Measures

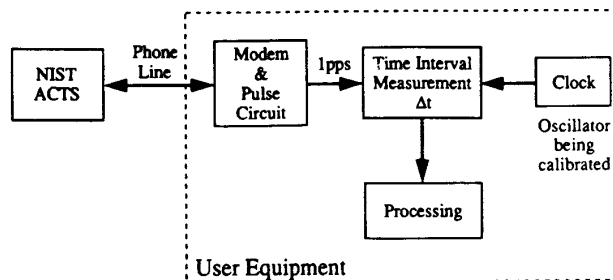
The intent of this paper is to demonstrate an extremely cost-effective, modest-accuracy method of remotely obtaining time and frequency via a telephone modem. The time and frequency so obtained are traceable to the National Institute of Standards and Technology's (NIST) time scale, UTC(NIST). We will demonstrate the accuracy and stability of both time and frequency available with this technique. [1]

The Automated Computer Time Service (ACTS) system has been described previously [1-3]. The accuracy

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and stability of ACTS is based on the assumption that the telephone path for the sending and receiving signal is reciprocal. Under this assumption, the round-trip time is measured by an automatic system at NIST, and a time marker in the time-code is then advanced in order to arrive at the user's site on time. The basic equipment needed is a telephone line that can call (303) 494-4774, a modem and a simple circuit outlined in reference 3 to generate an on-time one pulse-per-second (1 pps) or one pulse-per-two-seconds. It was evident from the preliminary measurements that the modems contributed substantially to the uncertainties of the measurements.

A block diagram of this method is as follows:



Proper processing is essential for the best calibration accuracy. The accuracy is a function of the calling rate, the modem used, and the quality of the oscillator employed. The time accuracy limit for NIST ACTS is about a millisecond. A good choice of modems gives a time stability of NIST ACTS of better than 0.1 ms.

In order to test the reciprocity of the telephone transmission system, we used the same brand of modems at

each end. We tested at both 300 bits/s and 1200 bits/s on a local telephone switching network. Finding the 300 bits/s to be better, we then used it over a long distance (from WWVH, Kauai, Hawaii to NIST, Boulder, Colorado) where both land-links and a satellite-link were involved.

We use a basic statistical theorem to evaluate the time and frequency stability and accuracy. If a time series has a white spectrum (the sequential values of the time series are random and uncorrelated), then the optimum estimate of the mean is the simple mean.

The confidence on the estimate of the mean is the standard deviation of the mean, which is the standard deviation divided by the square root of the number of values in the series. If the series does not have a white spectrum, then the standard deviation of the mean will not be an unbiased estimate. In some cases, such as a 1/f spectral density for the random deviations, the standard deviation and the standard deviation of the mean are not convergent, and hence are not useful measures.

We define the following measure, which is equal to the standard deviation for  $\tau$  equal to the data spacing and to the standard deviation of the mean for  $\tau$  equal to the data length [4,5]

$$\sigma_x(\tau) = \tau * \text{mod}\sigma_y(\tau) / \sqrt{3} \quad (1)$$

The  $\text{mod}\sigma_y(\tau)$  denotes the square root of the "modified Allan variance." [4,5] This quantity in equation 1 has the following properties:

- 1) it is equal to the classical standard deviation of the time residuals for  $\tau = \tau_0$ , the data spacing interval, if the process has a white spectrum;
- 2) it equals the standard deviation of the mean of the time residuals for  $\tau = N\tau_0$ , the data length (N is the number of data points), if the process has a white spectrum;
- 3) it is convergent and well behaved for most of the random processes encountered in time and frequency metrology;
- 4) the  $\tau$  dependance indicates the power-law spectral density model appropriate for the data;

5) and the amplitude of  $\sigma_x(\tau)$  at a particular value of  $\tau$  provides an estimate of the spectral density coefficient for any one of the common kinds of power-law spectra.

We have the following relationships

$$\begin{aligned} S_x(f) &\sim f^\beta, \\ \sigma_x^2(\tau) &\sim \tau^\eta; \\ \beta &= -\eta - 1, \\ -2 &< \eta < 4, \end{aligned} \quad (2)$$

where  $S_x(f)$  is the spectral density of the time residuals,  $x$ , and  $\beta$  denotes the kind of power-law spectrum. Given a  $\log \sigma_x(\tau)$  versus  $\log \tau$  plot, the slope of the values will nominally follow a slope equal to  $\eta/2$  for power-law spectra in the range indicated. The spectral levels also can be calculated using information supplied in NIST Technical Note 1337 [4].

If a set of time or phase residuals has a white noise spectrum ( $\beta=0$ ,  $\eta=-1$ ), a linear regression to the residuals gives the optimum estimate for the mean time (the mid-point to the fit) and to the frequency (the slope of the fit). If the linear regression equation for the  $n$ th value is given by

$$\hat{x}(n) = a_0 + a_1 n, \quad (3)$$

for a set of N measurements with standard deviation  $\sigma$ , standard deviation of the mean  $\sigma_m$  and a data spacing  $\tau_0$ , then the mean value is

$$\bar{x} = a_0 + \frac{a_1 N}{2}. \quad (4)$$

The confidence of its estimate is  $\sigma_m$ ,

$$\begin{aligned} \sigma_m &= \frac{\sigma}{\sqrt{N}} \\ &= \sigma_x(N\tau_0) \end{aligned} \quad (5)$$

The confidence,  $s_1$ , on the frequency estimate,  $a_1$ , is given by :

$$\begin{aligned} s_1 &= \sqrt{12} \sigma N^{-\frac{3}{2}} / \tau_0, \\ &= 2 \text{ mod}\sigma_y(N\tau_0). \end{aligned} \quad (6)$$

If the residuals do not have a white spectrum, then  $\sigma_x(N\tau_0)$  and  $2\text{mod}\sigma_y(N\tau_s)$  will take on different values than the first equation listed in (5) and (6), respectively. These different values will be in a direction to accommodate the improvement or the degradation in the confidence intervals appropriate to the particular power-law spectra noise process.

### Time and Frequency Stability and Accuracy of ACTS

#### Measurements Using a Local Switching Network

For the modems we used, the 1200 bits/s rate had an instability which was about two to three times that obtained using 300 bits/s modems. Figure 1 is a plot of the frequency stability using a pair of 300 bits/s modems of the same model. As stated in the previous section, the frequency accuracy obtainable is given by  $2\text{mod}\sigma_y(\tau)$ . We repeated the experiment for sets of 7 000 points while still connected to the same line to see how far we could push the long term stability. Each sequential set gave a frequency stability plot nearly identical to that shown in Figure 1.

Notice that the data exhibit a slightly steeper slope than  $\tau^{-3/2}$ . The  $\tau^{-3/2}$  model corresponds to white phase or time modulation (PM). That the slope is steeper indicates the presence of more high frequency noise than white noise should have. This is probably due to a combination of processes such as the digitization granularity and the asynchronous nature of modems, which are built for communications rather than for timing. This suggests that less short term noise could be obtained if a modem were designed and built for accurate and stable timing. In any case, we see that the data are averaged at least as quickly as white noise, and that accepting a white noise model is conservative.

We found that the approximate  $\tau^{-3/2}$  behavior continued to beyond  $10^4$  s. We measured  $\text{mod}\sigma_y(\tau = 7\ 3/4\ \text{hours}) = 2.1 \times 10^{-11}$ . At this integration time and longer the spectrum was no longer white. This measure implies a frequency calibration accuracy of better than one part in  $10^{10}$  with an integration time of  $\tau \geq 10^4$  s.

The time stability,  $\sigma_x(\tau = 2\ \text{s})$ , as shown in Figure 2, was about 55  $\mu\text{s}$  for the local network, which would yield  $\sigma_x(\tau = 2 \times 10^4\ \text{s}) = 550\ \text{ns}$ . This turned out to be statistically significant as long as we maintained the same connection. We measured  $\sigma_x(\tau = 14\ 000\ \text{s}) = 230\ \text{ns}$ —a smaller number than predicted with the assumption of white noise PM. As before we assume this smaller number

is due partially to the digitization and asynchronous noise in the modems.

One application of time stability is the maintenance of synchronization. In principle it seems that one could maintain synchronization to less than 1  $\mu\text{s}$  once a path (particular connection) was calibrated. If connection is lost and a good enough clock exists to fly-wheel time to 1  $\mu\text{s}$  over the next  $10^4$  s while recalibrating, then the 1  $\mu\text{s}$  synchronization accuracy could be maintained. This method may be convenient in a local calling area.

If connection is lost and no fly-wheel method for recalibrating exists, then the time stability degrades markedly from the white PM model. We will show the amount in the long-link analysis in the next section.

#### Measurements via long-link

We chose the path between Boulder, Colorado (NIST) and Kauai, Hawaii (WWVH) because of known reference clocks on each end and because the link involved both land and satellite paths. We made measurements more than once a day on some days as well as nominally once per day over a few weeks. As is the case with white noise the variance appeared to be interval independent. The variance was much larger from call to call than during a call.

Figures 3 and 4 for this long-link measurement using ACTS correspond to Figures 1 and 2 for the local switching network. In Figure 3 we measured  $\text{mod}\sigma_y(\tau=512\text{s}) = 1 \times 10^{-8}$  with the  $\tau^{-3/2}$  behavior (white PM) being a reasonable model. Some departure from this power-law spectra is observed, and again we assume this departure to be driven in part by data quantization and the asynchronous nature of communication modems. If the trend in Figure 3 continued, we should achieve a stability of  $10^{-10}$  at  $\tau = 10^4$  s.

Figure 4 is representative of several such curves we plotted. We obtain the following results for time stability and frequency accuracy calibrations. The values of  $\sigma_x(\tau=2\text{s})$  ranged from 65  $\mu\text{s}$  to about 90  $\mu\text{s}$ , and reached values less than 10  $\mu\text{s}$  in almost all cases at  $\tau = 100$  s. As with the local network, the standard deviation of the mean was only statistically significant as long as connection was maintained.

Figure 5 is a plot of one set of the time difference measurements. Each diamond symbol represents one measurement of the NIST ACTS time transfer, taken every

2.5 seconds. The symbols are simply plotted at the relative time of the measurement. The digitization and asynchronization effects are obvious in the apparent curves that the eye resolves, though these curves do not represent consecutive measurements.

Figure 6 is a plot of  $\sigma_x(\tau)$  using as input the mean values from several sets of data, the connection being broken after each measurement. We measured a white noise level of  $\sigma_x(\tau=1.9d) = 270 \mu s$ , a degradation of about 3 or 4 from the 2 s measurements. The mean value of 15 measurements was 219  $\mu s$  with a standard deviation of the mean of 70  $\mu s$ , indicating that there are some systematic errors in the ACTS system. Thus time transfer accuracy, or synchronization, at the 1 ms level is quite reasonable with ACTS, but systematics appear to limit the time accuracy at the few hundred  $\mu s$  level.

Figure 7 shows a  $\text{mod}\sigma_y(\tau)$  diagram for the average values. It is apparent that, with about a week's worth of measurements taken a few seconds per day to assure statistical veracity, we can obtain NIST traceability of frequency accuracy at better than 1 part in  $10^9$ . This could be done automatically using low rates of calling, that is, a rate such that the telephone charge would be less than \$2 for the whole time.

The confidence on the frequency calibration can also be written as follows:

$$s_1 = \frac{\sqrt{12} \sigma_x(\tau_0)}{\sqrt{N} T}, \quad (7)$$

where  $T = N\tau_0$ , is the data length. Equation (7) holds if the spectrum of the time deviation is white, which appears to be the case from Figure 7. The confidence improves as the data length and as the square root of the number of measurements. Various options can therefore be among calling cost, desired accuracy and the amount of time to obtain the calibration. The fastest way to obtain a frequency calibration is to stay connected to take advantage of the better short-term stability. If a user wishes to maintain connection, please call (303) 497-3294 for arrangements for a special line; otherwise, an automatic disconnect occurs after 55 s. It appears that an accuracy of better than 1 part in  $10^9$  could be reached in less than  $10^4$  s for the long link. Time accuracy, on the other hand, cannot be improved much by averaging because of the apparent biases present.

## Conclusions

A comparison of the time accuracy, the time stability, and the frequency calibration accuracy is plotted in Figure 8 for several current time and frequency dissemination systems. We see that the accuracies and stabilities for ACTS are quite competitive with some of the other traditional ways of obtaining traceability to NIST. In addition, Figure 9 indicates the cost effectiveness, which makes the ACTS calibration approach extremely attractive for the range of accuracies and stabilities it provides.

Figure 10 is a plot of the fractional frequency stability of the ACTS calibration approach compared with a wide variety of other techniques. The figure shows that a calibration accuracy of better than 1 part in  $10^9$  is readily and inexpensively obtainable either from a single long call or from a sequence of short calls averaged over a few days. With 300 bits/s modems of the same model, time accuracies better than 1 ms are available either by land and/or satellite links. Time stabilities pulse-to-pulse are better than 0.1 ms, but from call to call degrade to about 0.3 ms.

The ACTS telephone number is (303) 494-4774. Example user software can be obtained for \$35 by calling (301) 975-6776 (refer to RM 8101, software for Automated Computer Time Service) [2,3]. The broadcast time code includes the Modified Julian Date, the year, month and day, the hour, minute and second, advanced alerts for daylight savings time and for leap seconds, the time difference between UTC and UT1 (earth time), and the amount by which the time code is advanced in order to arrive at the user's site on time.

The system will work at 1200 or 300 bits/s. Using a variety of modems, we observed much larger inaccuracies and instabilities than reported above - amounting to a few milliseconds. We found that 300 bits/s modems from a single supplier were the best combination.

## Acknowledgements

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## References

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- [4] "Characterization of Clocks and Oscillators," NIST Technical Note 1337, Ed. by D. B. Sullivan, D. W. Allan, D. A. Howe, and F. L. Walls.
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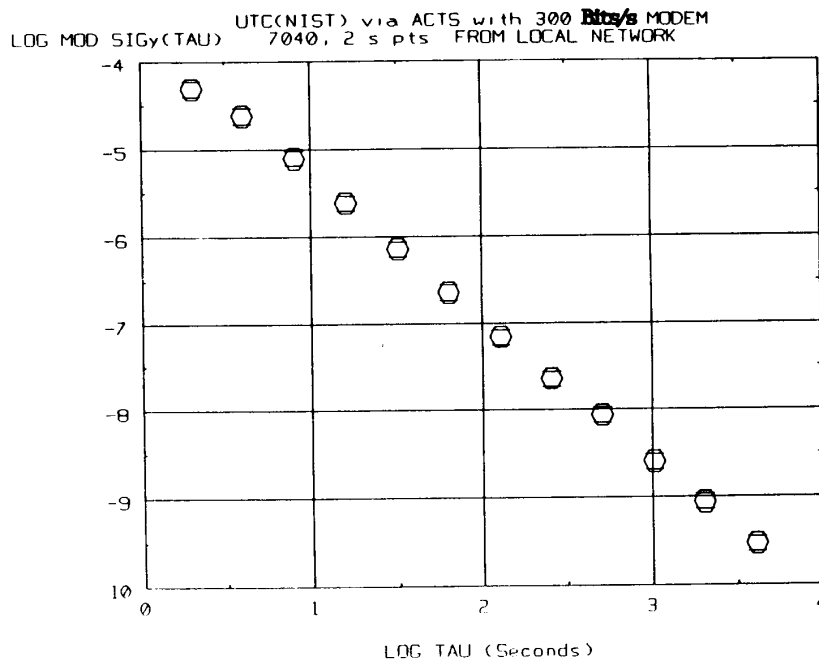


Figure 1. A plot of the fractional frequency stability,  $\text{mod}\sigma_y(\tau)$ , for UTC(NIST) as accessed over a local telephone switching network via the NIST ACTS system. The confidence on a frequency calibration is given by  $2\text{mod}\sigma_y(\tau)$ . The stability was analyzed for  $\tau$  values longer than those shown, and the same nominal white PM ( $\tau^{-3/2}$ ) model continued down to a level of about  $3 \times 10^{-11}$ .

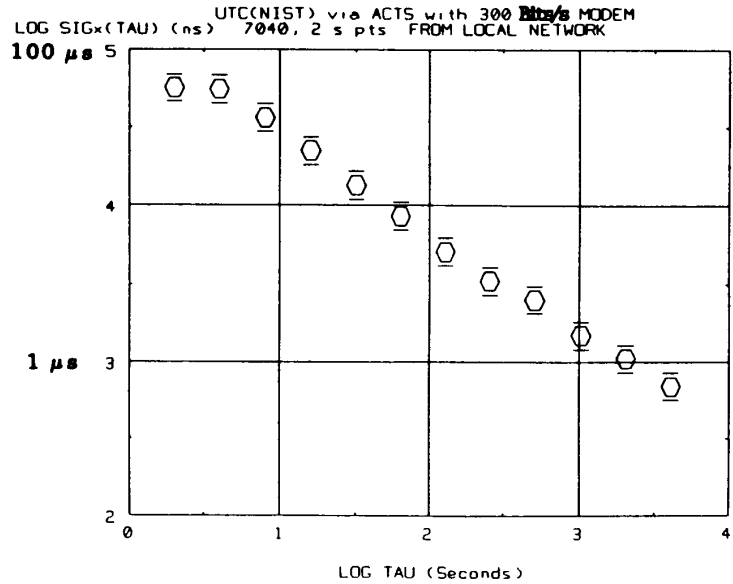


Figure 2. A plot of the time stability,  $\sigma_x(\tau)$ , of UTC(NIST) as accessed through the NIST ACTS system over a local telephone switching network. The stability improves to better than  $1 \mu\text{s}$  for long enough integration times. This stability is lost when connection is broken. If, for example, a quartz oscillator could fly-wheel and maintain a microsecond accuracy while the system was being recalibrated via ACTS, then a system accurate to less than  $1 \mu\text{s}$  could be maintained in the long term. This idea might be useful in a local calling network where there would be little expense for a dedicated line.

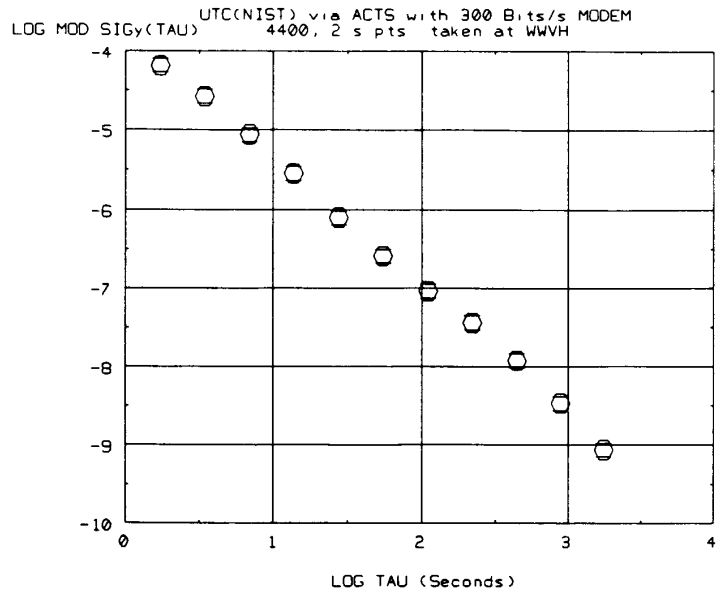


Figure 3. A plot of the fractional frequency stability,  $\text{mod} \sigma_y(\tau)$ , for UTC(NIST) as accessed over a long-link telephone network using the NIST ACTS system. The long-link involved both land-links and a satellite link between NIST Boulder, CO and WWVH Kauai, HI. The confidence for a frequency calibration is given by  $2\text{mod} \sigma_y(\tau)$ . The data in this figure would indicate that a frequency calibration accuracy of less than 1 part in  $10^9$  is available from a continuous long-link connection with an averaging time greater than about an hour.

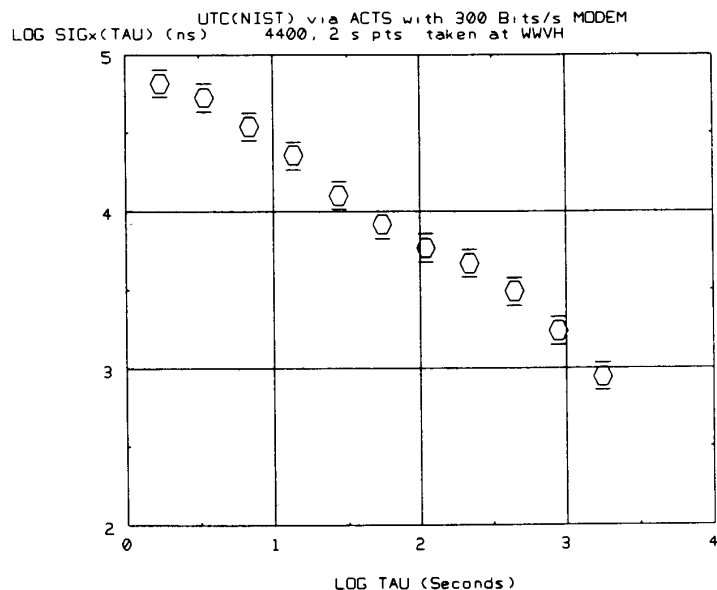


Figure 4. A plot of the time stability,  $\sigma_x(\tau)$ , of UTC(NIST) as accessed using the NIST ACTS system over a long-link telephone network using the NIST ACTS system. The long-link involved both land-links and a satellite link between NIST Boulder, CO and WWVH Kauai, HI. The stability improves to less than  $1 \mu\text{s}$  for long enough integration times. This stability is lost when connection is broken. A single measurement has less instability than the noise introduced in repeated calls.

WWVH - UTC(NIST) via ACTS: 1000, 2 s SEQUENTIAL READINGS

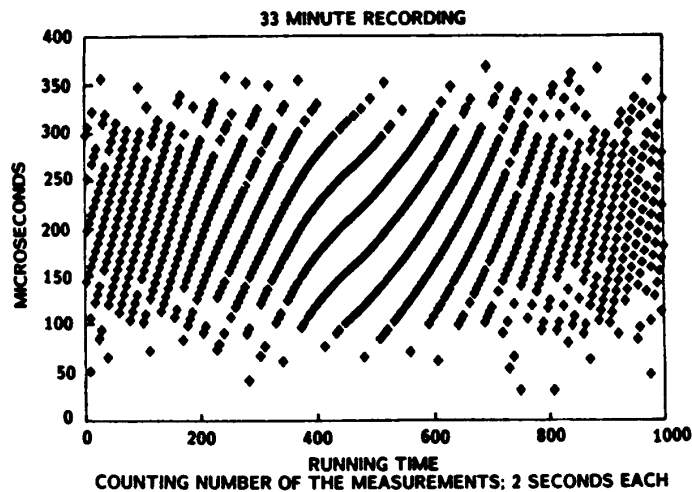


Figure 5. A plot of the individual ACTS measurements, each taken with two seconds spacing and with 300 bits/s modems of the same model at NIST Boulder and at WWVH Kauai. Each diamond symbol represents one measurement of the NIST ACTS time transfer. The symbols are simply plotted at the relative time of the measurement. The digitization and asynchronization effects are obvious in the apparent curves that the eye resolves, though these curves do not represent consecutive measurements. The mean value is  $209 \mu\text{s}$  and  $\sigma_x(N\tau_0) < 10 \mu\text{s}$ . Hence, biases in ACTS limit its accuracy. The accuracy appears to be well under a millisecond.

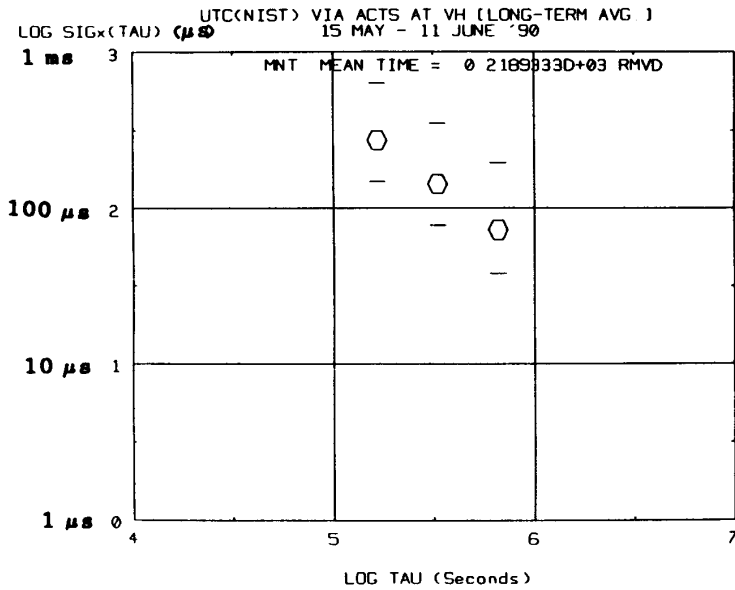


Figure 6. A plot of the time stability of the average values,  $\sigma_x(\tau)$ . Each call accesses UTC(NIST) via ACTS through a 300 bits/s modem at WWVH Kauai. The benefit of averaging the individual call's average value is illustrated by this plot since the values have a white spectrum ( $\tau^{-1/2}$ ). This plot shows that a repeated set of short calls, reasonably spaced, is more economical for a frequency calibration; see equation (7).

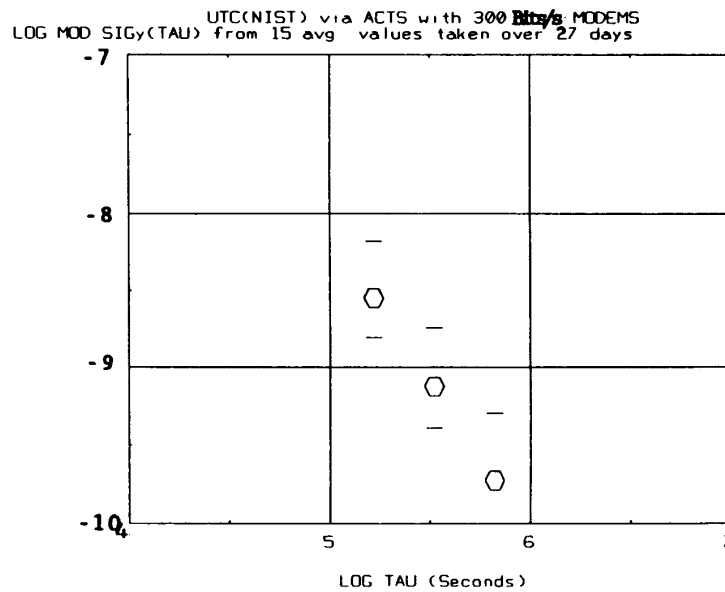


Figure 7. A plot of the fractional frequency stability,  $\text{mod } \sigma_y(\tau)$ , obtained from the average values of a series of calls with average spacing 1.9 days. Since the spectrum appears to be nominally white ( $\tau^{-3/2}$ ) and the calibration accuracy is  $2\text{mod } \sigma_y(\tau)$ , we see that calling once each night (when calls are less expensive) for about a week yields a frequency calibration of better than  $1 \times 10^{-9}$ .



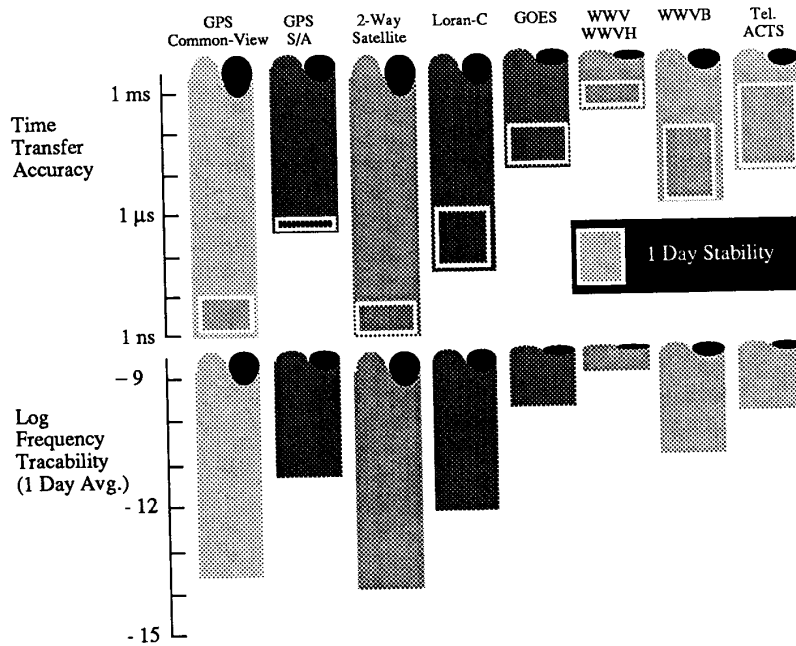


Figure 8. This bar chart compares the time accuracy, the time stability from day to day and the frequency calibration accuracy of several of our current dissemination systems.

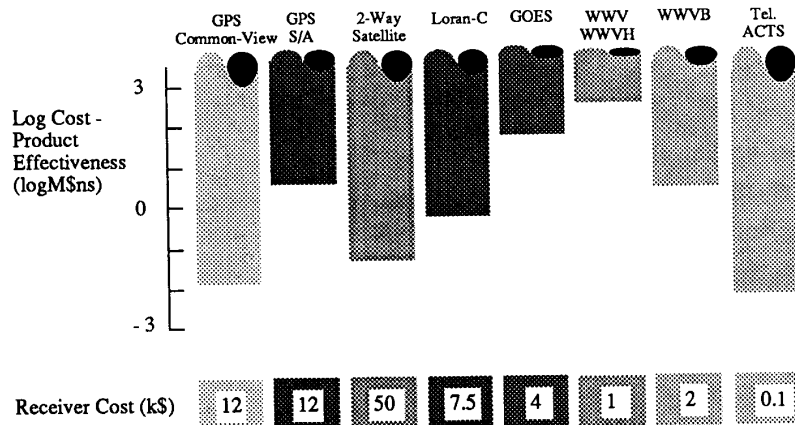


Figure 9. This chart illustrates the cost times the day-to-day time stability as a measure of the cost effectiveness of various time and frequency dissemination systems that are currently available. The units are mega-dollar nanoseconds. The further the bar goes down - the more cost effective the system.

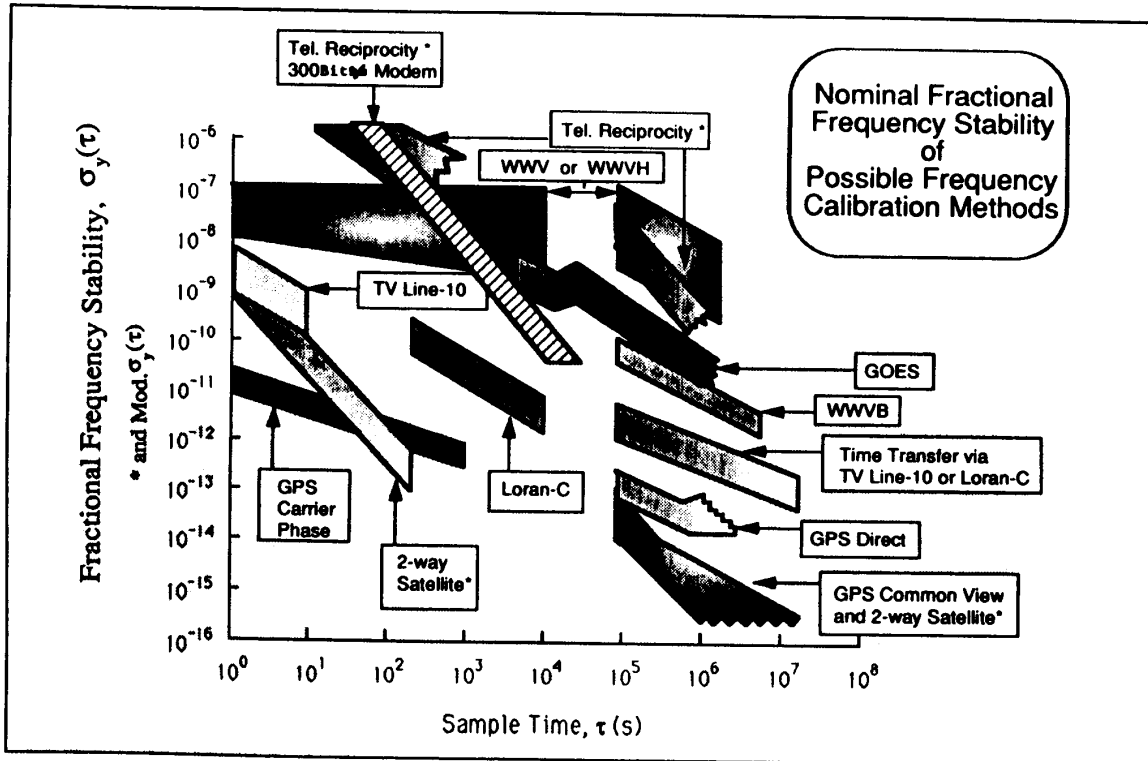


Figure 10. This fractional frequency stability plot compares most of the currently available time and frequency dissemination systems. The "\*" indicates where  $\text{mod. } \sigma_y(\tau)$  has been used in order to distinguish if the deviations have a white PM spectrum. Otherwise, the conventional  $\sigma_y(\tau)$  is used as the stability measure. The stability of the reciprocity of the telephone lines clearly makes this service a competitive one for the low accuracy user.