

# KEYNOTE ADDRESS

## SYNCHRONIZATION OF CLOCKS

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### Abstract

Time metrology has moved from milliseconds to picoseconds in the last four decades, and frequency metrology from nine significant digits to sixteen. The ability to synchronize remote clocks has improved dramatically as well. With implementation of GPS (Global Positioning System,) the full long-term frequency stability as well as the frequency accuracy of the best atomic clocks can now be transferred to remote sites. GPS's selective availability, an intentional degradation of system performance, will adversely affect the accuracy and stability of GPS time and frequency for the average civilian user.

In this paper we define terms of reference, discuss various alternatives for clock synchronization and syntonization, and make some comparisons between various techniques used in synchronizing and syntonizing clocks. In the process we review the concepts of time stability and accuracy, frequency stability and accuracy.

## INTRODUCTION

The synchronization of clocks is a subject which has been widely treated throughout the years. With the development of very accurate means for satellite time transfer, the subject has gained substantially in importance. This paper provides a discussion of the relevant issues surrounding clock comparisons and of the various means of comparing them when they are a significant distance apart.

Time transfer systems (or clock synchronization systems) are often characterized by a single number, designating a precision or an accuracy of some number of microseconds or nanoseconds. This is often ambiguous and it is the intent of this work to clarify the characterization of clock synchronization or comparison systems. We will apply these techniques to some current comparison systems for clocks located some distance apart, and project some of our future opportunities — given these techniques, constraints and guidelines.

## BACKGROUND

We are not here generally concerned with measurement noise, that is divider or counter noise — though this can be problematic in some instances. As clocks continue to improve, more attention must be

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paid to the characterization of measurement systems, that is the systems which read the output of clocks. This is especially true if the clocks are remotely located from each other. Characterizing the measurement system is essential if a remote (slave) clock is intended to be optimally synchronized or syntonized to a master clock. In this latter situation, optimum design of the servo system, locking the slave to the master clock, requires a characterization of all of the contributing elements.

A free-running clock can almost always be characterized better than one whose output is servo controlled to another clock. Hence, a computed output or an external micro-phase stepper is useful in providing a synchronized or syntonized output which does not perturb the free-running clock<sup>[1]</sup>. A local set of clocks can be better characterized if there are at least three of them of about the same quality<sup>[2]</sup>. Once a set of clocks is available, then algorithms can be employed to intelligently combine their readings so that the algorithm-computed time and/or frequency can be more stable than that of the best clock in the set. In addition, algorithms can be designed to test for abnormal clock behavior and to desensitize the computed time to any abnormal behavior as well as to failures<sup>[3]</sup>.

If the clocks, as well as the comparison system, are well characterized, then an ensemble of clocks, can be constructed from a set of remotely located clocks. With full characterization of all components, the system of clocks and its associated comparison can be optimized for overall performance. As far as I know, while often applied to local ensembles, this concept has not yet been applied to clock ensembles whose member clocks are in different locations. There are some long-term plans to do this for GPS. We feel that there are potentially significant gains available in the proper application of this concept.

Figure 1 illustrates a straightforward comparison system which measures the time and frequency differences between Clock 1 and Clock 2. Our concern is the characterization of the full noise in the comparison including measurement noise, clock noise and noise introduced in the comparison path and system. In figure 2 we illustrate an additional concern which arises in designing a servo-loop to slave a remote clock to a master clock. The data from the comparison may not be available immediately; hence, in the feedback loop, the measurement noise, path deviations, the delay in acquiring the comparison data will effect the servo design very fundamentally. Practical delays in acquiring comparison data range from milliseconds to times longer than a month. For example, the delay time (data acquisition time) for servo controlling Coordinated Universal Time at NIST (UTC(NIST)) to the international UTC scale is more than a month. Though we will not go into the servo-design theory in this paper, we want to stress that the measurement noise and path noise characteristics and the delay in acquiring comparison data play very important roles in servo design.

Appendix A gives some relevant definitions of words (precision, accuracy, stability) that will be used in this paper. In characterizing systems for comparing clocks which are remotely located to each other, it is important to consider concepts such as: time accuracy, time stability, time prediction error, frequency accuracy, and frequency stability. Each of these has a unique interpretation.

Conceptually, time accuracy is the time difference between the readings of two clocks at some time in a given reference frame. We often define one of the clocks as perfect so that we are assessing the accuracy of a clock relative to some “ideal” clock. One can imagine the transport of a perfect portable clock to accomplish this time difference measurement. Time accuracy is often limited by systematic errors in the comparison system, such as uncertainties in cable delays, and propagation-path-length uncertainties, and is often very hard to measure or assess. In addition, systematic differences between the clocks will contribute to the time inaccuracy. The time accuracy can never be better than time stability and is often much worse.

One of the best ways to observe the time stability is to plot the time residuals, often denoted  $x(t)$ ,

between two clocks after the systematics have been subtracted. Time stability is, often affected by environmental variations (which affect clock and comparison system performance), in addition to the usual kinds of random variations. People commonly measure time stability as the rms deviation of the time residuals from a linear regression to the time deviations. This practice, which can be very misleading, will be discussed in some detail in the body of the paper. If there are periodic terms affecting a time comparison system, then measuring the spectral density of the time or the phase fluctuations may be a very good measure. One may also measure the effect of these periodic terms using  $\sigma_y(\tau)$  (see ref. 2). We will show that for time stability there is often a  $\tau$  (averaging-time) dependence. This is an important consideration which will be discussed later. We also show that  $\tau \star \text{mod}\sigma_y(\tau)$  is a useful measure of the time stability of a comparison system.

The quantity  $K\tau\sigma_y(\tau)$  is a useful measure for estimating the time prediction error in a comparison. We often have a particular power-law spectral density process which is the dominate model for the signal variations from the clocks and/or the comparison system. The value of  $K$  is  $1/\sqrt{3}$  for white-noise PM, 1 for white-noise FM and for random-walk FM, and 1.2 for flicker-noise FM under the assumption of optimum prediction. Sometimes white noise phase modulation is the predominant noise model, in which case the quantity  $\tau \star \text{mod}\sigma_y(\tau)/\sqrt{3}$  is the optimum rms time prediction error for an average over  $\tau$  of  $x(t)$  measurements.

Frequency accuracy for a given primary standard is not a function of integration time and is properly stated as a single number. But the ability of a comparison system, to determine absolute frequency difference between two standards is often a function of the sampling or integration time,  $\tau$ . We will show that the frequency accuracy of a comparison system is also a function of the data processing method. This leads to the idea that there is an optimum method for estimating the absolute frequency difference between two remote clocks or for controlling the frequency of a remote clock.

Frequency stability, similar to time stability, is observed by looking at a plot of the fractional frequency offset,  $y(t)$ , where  $y(t) = \nu(t) - \nu_0)/\nu_0$  with  $\nu(t)$  being the time varying frequency output of a clock and  $\nu_0$  is the clock's nominal frequency. In practice, measured values of  $y(t)$  are observed over some averaging time,  $\tau$ . It is often very useful to observe a  $y(t)$  plot at different averaging times. The frequency stability of a comparison system can be quantified in the same way clocks are characterized, using a  $\sigma_y(\tau)$  or  $\text{mod}\sigma_y(\tau)$  plot. It is sometimes useful to measure the spectral density of the frequency fluctuations to supplement the above time-domain methods, in order to ascertain the presence of different kinds of noise. The kind of noise observed in comparisons between two clocks, and that which may be added by the comparison system, will determine how to optimize estimates of characterization parameters (both systematic and noise) for the clocks and the comparison system. One important example of a characterization parameter is the frequency drift between two clocks.

There are of course important relationships among time accuracy, time stability, time prediction error, frequency accuracy, and frequency stability. These will be discussed later.

## CHARACTERIZATION OF COMPARISONS SYSTEMS

Figure 3 shows the improvement in the U.S. primary frequency standard since the advent of cesium beam technology. The trend line shows an improvement of about a factor of 10 every seven years. We expect to see further improvement, but extrapolations from data such as this are dangerous. There are now good indications that standards based on trapped and cooled ions will yield dramatic improvements. The ultimate potential for these devices is an accuracy of about one part in  $10^{18}$ , but

practical considerations will make this limit difficult to achieve.

In the past, the accuracy of operational comparisons between primary standards fell behind the accuracy of the standards. Further improvements in primary standard accuracy were thus of limited use. However, during the last decade the development and application of two-way satellite and GPS time transfer dramatically changed the picture. With the excellent comparison accuracy available with GPS common-view technique, comparison accuracy is now ahead of clock accuracy. This was a major breakthrough for international time and frequency comparisons, and the GPS technique became the de-facto international standard for comparisons<sup>[6]</sup>. A decision by GPS system operators to intentionally degrade performance as observed by civilian users, the so-called process of selective availability, raises questions which are important in time transfer applications. These will be discussed shortly.

Time transfer using the two-way satellite technique now looks to be a very attractive alternate available to primary timing centers. More information is needed on the accuracy and long-term time stability of this comparison technique as early work has not focussed on these<sup>[6],[7]</sup>. Most of the published results are on short-term time stability.

Important factors for all of these comparison systems include cost and simplicity of use and means for accurately assessing comparison accuracy. The ideal comparison system is one which provides the time difference, the frequency difference, and the relative time and frequency stability of the clocks along with the uncertainties associated with the comparison system. If the comparison system is to be widely used, the cost should be low. Of course, there is no single system which now meets this ideal. Figure 4 shows a plot of some of the more common comparison techniques now being used. We have used both  $\sigma_y(\tau)$  and  $\text{mod}\sigma_y(\tau)$  to characterize the frequency stability of these comparison systems, because, in some cases, white-noise phase modulation (PM) is the limiting random process and  $\sigma_y(\tau)$  characterization is ambiguous for that process.

When white-noise phase modulation is the predominant noise in a comparison system, some important equations for optimal estimation of time and frequency between the clocks are:

$$\hat{x}(i) = a_0 + a_1 \cdot i \quad \text{and} \quad (1)$$

$$s_x = \sqrt{\frac{1}{N-2} \sum_{i=1}^N (x(i) - \hat{x}(i))^2} \quad (2)$$

Here the  $\hat{x}(i)$  is the optimal estimate of the time difference between the clocks at the measurement point  $i$ . The “ $a_0$ ” and the “ $a_1$ ” coefficients are determined by minimizing the variance around the linear regression line, so the meaning of optimum is for a minimum variance. The  $x(i)$ ’s are the measured time difference over the  $N$  measurements. The confidence on the estimate of the intercept “ $a_0$ ” is  $s_0$ :

$$s_0 = 2s_x / \sqrt{N} \quad (3)$$

and the confidence of the estimate of the mean value  $(\bar{x} = \frac{1}{N} \sum_{i=1}^N x(i))$  is  $s_{mean}$ :

$$s_{mean} = s_x / \sqrt{N}. \quad (4)$$

The confidence of the estimate of the slope, (“ $a_1$ ” the frequency difference) is  $s_1$ :

$$s_1 = \frac{\sqrt{12}s_x}{\tau_0 N^{3/2}} = 2\text{mod}\sigma_y(\tau) \quad (5)$$

Equation 1 is the classical equation for a linear regression, which is often computed as a fit to the time residuals. The application of this equation is optimal only for white noise processes. We assume there are  $N$  values each  $\tau_0$  apart. In this case, the standard deviation (given by Equation 2) is a measure of the time stability at the data sampling rate — sometimes called the time of the time difference measurements. The  $N - 2$  expression in the denominator shows that two degrees of freedom have been removed with the estimation of the  $a_0, a_1$  terms of Equation 1. The mean value confidence interval in Equation 4 is half that of the intercept, and is the optimum estimate of the time difference between the clocks at the mid-point time. The solution to equation 1 at the midpoint is equal to the mean value. Equation 5 shows the value of using  $\text{mod}\sigma_y(\tau)$  to determine the confidence of the estimate of the frequency difference,  $a_1$ . If the residuals are not white, then the  $\tau$  dependence will not be  $\tau^{-3/2}$ , and the linear regression will not give the optimum estimate of the time and frequency difference of the clocks. If the residuals are white, the value of  $\text{mod}\sigma_y(\tau)$  gives the proper value of the confidence for any averaging time,  $\tau$ . The rapid improvement ( $\tau^{-3/2}$ ) gained in estimating the absolute frequency difference by increasing the averaging time is clearly illustrated by the use of  $\text{mod}\sigma_y(\tau)$ .

Linear regression analysis is often used to model processes which do not have a white spectrum for the residuals. In this case, the linear regression coefficients and their confidences can often be very misleading. A  $\text{mod}\sigma_y(\tau)$  diagram will indicate if one is or is not legitimate in using linear regression analysis, and if not then it gives a measure of the effects of the degradation caused by the actual random processes on the estimate of the frequency difference between the two remote clocks.

Figure 5 is a plot of the rms time prediction error seen in currently available clocks and oscillators. The data has been used in an optimum fashion to predict into the future over an interval,  $\tau_p$ . The rms time deviation can be defined in many ways. This is one useful approach. The next four Figures, 6, 7, 8, and 9, are plotted with exactly the same ordinate and abscissa as Figure 5. They can then be overlaid to see the effects of various systematic effects, either in the clocks or in the comparison system. Figure 6 has the ordinate labeled with both the white PM level (usually arising from the comparison system) and the time accuracy. The time accuracy number provides a hard limit in comparing the time difference between two clocks. In contrast, the white PM level is a function of integration time, and if other processes are not limiting, knowledge of the time difference improves as the square root of the number of measurements averaged — consistent with equation 4. If the residuals are white PM, one may also write from the concept of time averaging of measurements the following equation:

$$s_{rms}(t_0) = \frac{\tau^{3/2}}{\sqrt{3}t_0} \text{mod}\sigma_y(\tau), \quad (6)$$

where "s" denotes the classical standard deviation of the  $x(i)$  taken  $t_0$  apart ( $\tau = n\tau_0$ ) as in Equation (2). Since the numerator in Equation 6 is constant for white PM, the improvement in  $s_{rms}(t_0)$  is proportional to  $t_0^{-1/2}$ . This is not surprising since  $t_0$  is the window over which the phase (or the time) has been averaged. If  $t_0$  becomes the full data length, then, as expected, Equation 6 is the standard deviation of the mean. Here again, a  $\text{mod}\sigma_y(\tau)$  diagram provides a good visualization of the estimate of the time difference estimate uncertainty and of the time stability (as limited by the clocks and/or the comparison system).

Figure 7, 8 and 9 are included for the readers convenience. Figure 7 shows the accumulated time

difference as a function of time for two clocks whose frequencies differ by various fixed amounts. In this case the abscissa could also be the prediction interval. Figure 8 shows the rms time deviation as a function of the prediction interval as caused by flicker noise frequency modulation (FM) (a common noise in clocks). Notice that the slope is the same as for frequency offset. The factor 1.2 is the  $K$  factor for flicker noise where optimum prediction has been assumed. Figure 9 shows the large time deviation error that results from frequency drift. The labels for the different lines are fractional frequency drift per day expressed as powers of 10. The quadratic nature of the time deviation resulting from frequency drift often causes this kind of error to be the predominate long-term systematic error.

Figure 10 is a plot of  $\tau \text{mod} \sigma_y(\tau)$  as a function of  $\tau$ . With  $\tau = n\tau_0$ , this shows whether or not one benefits from averaging  $n$  values of the  $x(i)$  time-difference measurements. One of the advantages of this new approach is that it illustrates the benefit of averaging the time difference measurements, whether or not the instabilities are in the comparison system or in the clocks. If the measurement noise residuals are a white PM process, then the time stability will improve as the square root of  $\tau$ . If it is a flicker PM process there will be no improvement with averaging. If the plot degrades with increasing  $\tau$  (slope greater than 0), then there are probably non-stationary processes perturbing the comparison system. In the case of Loran-C, we see a double hump at one-half day and at one-half year caused by diurnal variations and annual variations. We have longer-term common-view data using GPS than are plotted, and know that the time stability does not continue to improve as square root of  $\tau$ . In this case, the nonstationary processes are probably related to ionospheric modeling errors and errors in the Kalman estimates of the satellites' ephemerides. Multipath distortion effects at the antenna can sometimes cause several nanoseconds of bias in the time inaccuracy, but do not change the slope in this type of plot, if the bias is constant.

For two-way-satellite time transfer, the noise limit does not continue decreasing as indicated by the short-term results in Figure 10. Daily deviations of the order of a few nanoseconds have been observed, but these will likely be reduced as the systems are improved and better characterized. This characterization of the two-way satellite time transfer technique will be very important for the future — especially for averaging of one day and longer. A determination of the time accuracy of this technique will be very important as well. Theoretically, both the time stability and the time accuracy of two-way time transfer should provide an excellent means for comparing widely separated clocks. The primary drawback to this technique is the need for broadcasting from each station, a requirement which adds cost and involves licensing with government agencies.

## THE FUTURE OF COMPARISON SYSTEMS

It is clear that the best means for comparing widely separated clocks involves satellite techniques. For clocks in close proximity (that is, within a modest number of kilometers) perhaps optical fibers will provide the best comparisons<sup>[9]</sup>. As we develop higher accuracy and more stable clocks, we will need to use higher frequencies to achieve better phase resolution for the comparisons.

It appears that the GPS system could be pushed to a time accuracy approaching a few nanoseconds. For short-baseline comparisons, studies suggest that one might achieve accuracies as low as 0.1 ns<sup>[9]</sup>. Time stabilities for GPS common-view comparisons yield  $\tau \text{mod} \sigma_y(\tau)$  of about 1 nanosecond times  $\tau^{-1/2}$ , where  $\tau$  is in days. At  $\tau = 1$  day, this product actually ranges from 0.8 to 8 ns for the many international time stability measurements which use the GPS common view method. With ionospheric calibrators and more-exact, a post-ephemeris data for the satellites, the GPS common-view technique could yield a comparison limit for frequency accuracy approaching  $10^{-17}$ . This would require about

three months of integration under the assumption of ideal white-noise phase modulation. Codeless ionospheric calibrators, which measure the real ionospheric delay, are now becoming available for GPS. There is also the promise that precise post-measurement ephemerides will be made available to the civilian sector (the non-PPS user). With these advances the GPS common-view method for time and frequency transfer could be even better than it has been, but the price for this would be additional processing along with a significant delay in access to data needed to calculate all errors. The following table summarizes the anticipated compensation for using GPS in the common-view mode.

TABLE					
GPS COMMON-VIEW TIME-TRANSFER ERROR SOURCES					
(WITH SELECTIVE AVAILABILITY ON)					
SOURCE	COMMENTS	RMS TIME ACCURACY (ns)			
CLOCK DITHER	CANCELS IN C-V MODE				--
EPSILON	DEPENDS ON THE BASE-LINE	30	to	50	
IONOSPHERE (BDCST)	DEPENDS ON TOD AND COORD.	5	to	40	
TROPOSPHERE	DEPENDS ON ELEV. AND WEATHER	2	to	5	
MULTIPATH	DEPENDS ON GROUND PLANE AND REFLECTION	4	to	8	
RECEIVER	DEPENDS ON THE MAKE AND MODEL	1	to	100	
C-V TIME TRANSFER ERRORS (NO COMPENSATION)			31	to	120
(WITH SELECTIVE AVAILABILITY ON AND WITH COMPENSATION)					
SOURCE	COMMENTS	RMS TIME ACCURACY (ns)			
CLOCK DITHER	CANCELS IN C-V MODE				--
EPSILON	COMPUTED EPHEMERIS (Some Days After)	3	to	5	
IONOSPHERE	WITH IONOSPHERIC CALIBRATOR	2	to	3	
TROPOSPHERE	DEPENDS ON ELEV. AND WEATHER	2	to	5	
MULTIPATH	WITH CHOKE-RING ANTENNA GND. PLANE	2	to	4	
RECEIVER	DEPENDS ON MAKE AND MODEL	1	to	100	
C-V TIME TRANSFER ERRORS (WITH COMPENSATION)			5	to	100

The right column lists rms estimates for each of the time accuracy error elements with the sum at the end of each column being the square root of the sum of the squares. EPSILON is the intentional insertion of errors in the broadcast ephemeris. The meanings of other terms in the table are:

C-V	- GPS common-view mode	Elev.	- Elevation
TOD	- Time of Day	Refl.	- Reflections
GND	- Ground	BDCST	- As Broadcast

How well the systematics of the two way satellite timing technique can be understood is yet to be determined. From a theoretical point of view this technique should be better, in both time stability

and time accuracy, than the GPS common-view technique. The method could provide about an order of magnitude of improvement.

An often overlooked experiment which could lead to time transfer improvement is the Scout Rocket Experiment which involved flight of a hydrogen maser<sup>[10],[11]</sup>. This experiment used a microwave Doppler cancellation method and an ionospheric calibration system. From the published data it is estimated that time stability,  $r \text{mod} \sigma_y(r)$  over several hours was about ten picoseconds. With this level of stability available from a satellite-born hydrogen maser, cycle ambiguity of the clocks microwave signal could be resolved from pass to pass or from day to day. This could yield frequency comparisons over 24 hours of  $10^{-16}$ . If the residuals for the comparison process were white PM from day to day, it would take only a few weeks to measure frequency difference at the  $10^{-18}$  level. At this level, relativity considerations become very important, and they well be very difficult to calculate. But, with bigger and better computers coming in the future, perhaps the relativity issues would be solvable.

## CONCLUSION

In order to synchronize (or syntonize) a system of clocks in an optimum way, it is necessary to know both the stability characteristics of the clocks as well as those of the comparison system. The characterization the random variations in clocks is pretty well understood, but that of comparison systems is not. It is often the case that the standard deviation of the time residuals is non-convergent for both clocks and comparison systems, in which case it is not a useful measure. In this paper we have presented some reasonable ways to describe and to characterize comparison systems. These allow us to better specify time and frequency comparisons. This issue is becoming more important as system synchronization and syntonization requirements become more stringent.

We have explained how time accuracy, time stability, time predictability, frequency accuracy and frequency stability are separate and distinct concepts. Important relationships between these concepts were presented. These have implications for accurate time comparisons. For example, knowing the kinds of random instabilities in the clocks and in the comparison system allows one to optimally estimate the absolute time and frequency differences between widely separated clocks. As we anticipate more accurate frequency standards, very careful design as well as characterization of comparison systems will be required to take advantage of the improved standards. Even at current time comparison levels, there is a need for better specification of the performance of comparison systems. We have presented one reasonable approach with the hope that this will stimulate discussion and even adoption of a standard method for characterizing the accuracy and stability of the comparison process.

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# APPENDIX

## DEFINITIONS

- **ACCURACY**

The degree of conformity of a measured or calculated value to its definition (see Uncertainty).

- **PRECISION**

The degree of mutual agreement among a series of individual measurements; often, but not necessarily, expressed by the standard deviation.

- **UNCERTAINTY**

The limits of the confidence interval of a measured or calculated quantity.

- **FREQUENCY INSTABILITY**

The spontaneous and/or environmentally caused frequency change within a given time interval.

- **REPRODUCIBILITY**

A) With respect to a set of independent devices of the same design, the ability of these devices to produce the same value.

B) With respect to a single device, put into operation repeatedly without adjustments, the ability to produce the same value.

- **ERROR**

The difference of a value from its assumed correct value.

- **DRIFT**

The systematic change in frequency of an oscillator with time.

- **AGING**

The systematic change in frequency with time caused by internal changes in the oscillator.

## REFERENCES

- [1] D.W. Allan, M.A. Weiss, and T.K. Peppler, "In Search of the Best Clock," IEEE Transactions on Instrumentation and Measurement, IM-38, 624-630 (1989).
- [2] D.W. Allan, "Time and Frequency (Time-Domain) Characterization, Estimation, and Prediction, of Precision Clocks and Oscillators", IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control, UFCC-34, 647-654 (1987).
- [3] M.A. Weiss and T. Weissert, "A New Time Scale; AT1 Plus Covariance," submitted to Proc. of 20th Annual Precise Time and Time Interval Planning Meeting (PTTI), Redondo Beach, CA, Nov. 30 - Dec. 1, 1989.
- [4] D.J. Wineland, J. Bergquist, J. Bollinger, W. M. Itano, D. Heinzen, S. Gilbert, C. Manney, and C. Weimer, "Progress at NIST Toward Absolute Frequency Standards Using Stored Ions," Proc. of 43rd Annual Symposium on Frequency Control, Denver, CO, May 31 - June 2, 1989, pp. 143-150.
- [5] D. W. Allan, et. al., "Accuracy of International Time and Frequency Comparisons via Global Positioning System Satellites in Common-View," IEEE Transactions on Instrumentation and Measurement, IM-34, 118-125 (1985).

- [6] D. Hanson, "Fundamentals of Two-Way Time Transfers by Satellite," Proc. of 43rd Annual Symposium on Frequency Control, Denver, CO, May 31 – June 2, 1989, pp. 174–178; D.A. Howe, et. al., "NIST-USNO Time Comparisons Using Two-Way Satellite Time Transfers," 43rd Annual Symposium on Frequency Control, Denver, CO, May 31 – June 2, 1989, pp. 193–198.
- [7] M. Imae, H. Okazawa, T. Sato, M. Urazuka, K. Yoshimura, and Y. Yasuda, "Time Comparison Experiments with Small K-Band Antennas and SSRA Equipments via a Domestic Geostationary Satellite," IEEE Transactions on Instrumentation and Measurement, IM-32, 199–203 (1983).
- [8] G. Lutes and M. Calhoun, "Simultaneous Transmission of a Frequency Reference and a Time Code Over a Single Optical Fiber", submitted to Proc. of 20th Annual Precise Time and Time Interval Planning Meeting (PTTI), Redondo Beach, CA, Nov. 30 – Dec. 1, 1989.
- [9] P.F. MacDoran, D.J. Spitzmesser, L.A. Buennagel, "SERIES: Satellite Emission Range Inferred Earth Surveying," Proc. of the Third International Geodetic Symposium on Satellite Positioning, Las Cruces, N. Mexico, February 1982.
- [10] R.F.C. Vessot and M.W. Levine, Gen. Relativ. Gravit. 10 (1979) 181; R.F.C. Vessot et. al., Phys. Rev. Lett. 45 (1980) 2081.
- [11] D.W. Allan, C.O. Alley, N. Ashby, R. Decher, R.F.C. Vessot and G.M.R. Winkler, Ultra-Accurate International Time and Frequency Comparison Via an Orbiting Hydrogen-Maser Clock, Journal De Physique, Colloque C8, 395–413, 1981.

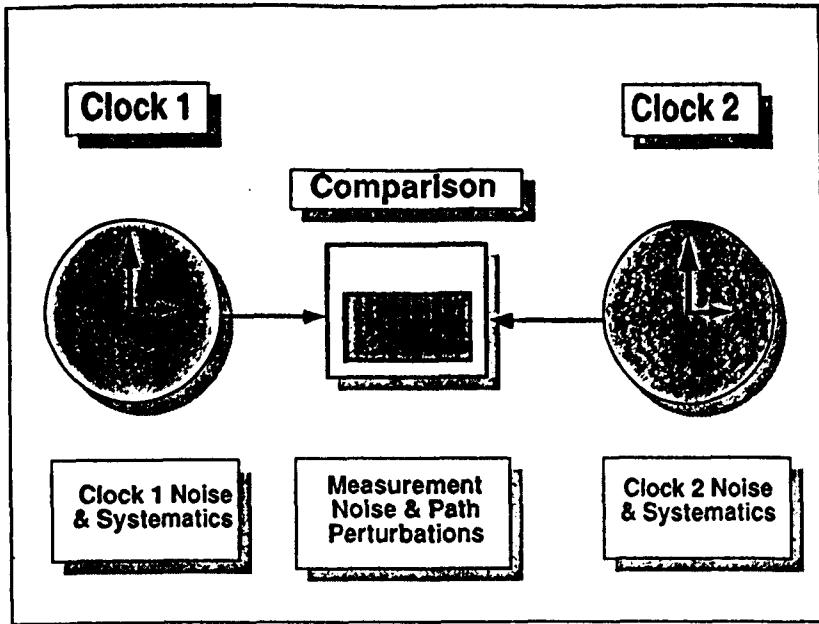


Figure 1. This figure shows two clocks, some arbitrary distance apart, being compared by some generic comparison system. In principle, the comparison system can be co-located with either or both of the clocks or with neither of the clocks. In general, the measured values coming from the comparison system will have variability due to the clocks noise, delay variations in the connecting links, and variations in the comparison system itself. Characterizing the performance of the links and the comparison system is important. Otherwise, understanding what variations come from the clocks and what comes from the comparison system and the links would be impossible.

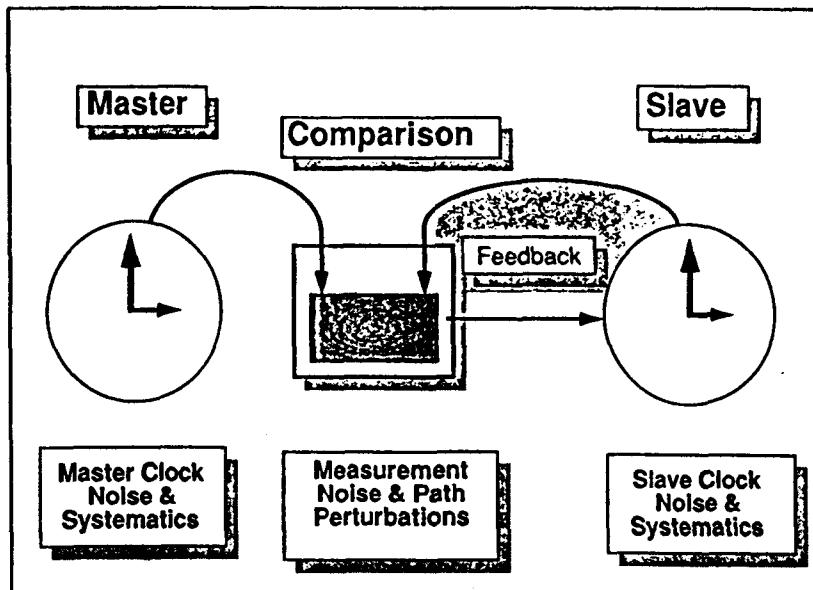


Figure 2. This figure is similar to Figure 1. Again, we are measuring the time and frequency difference between two clocks located some distance apart. In this case we wish to servo control the time and/or frequency of the slave to the master. A proper characterization of the links between the clocks in combination with the comparison system is essential for the proper design of a feedback system to control the slave clock. Another important parameter for the feedback design is the delay associated with the comparison system.

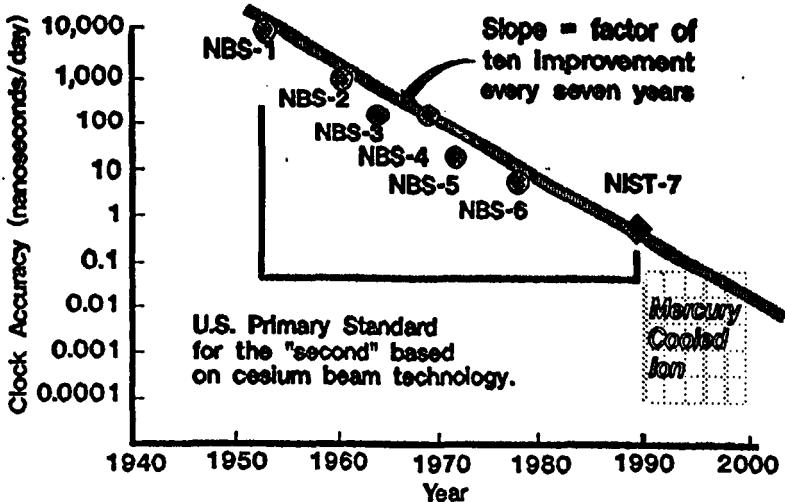


Figure 3. Depicted here is the continual improvement in atomic frequency standards of the U.S. The overall trend is a factor of 10 improvement every seven years. If this trend line continues, and there is good indication that it may, then more careful attention is needed both on the design as well as on the proper characterization of comparison systems for these standards. Note: one nanosecond per day corresponds to a fractional frequency of about a part in  $10^{14}$ .

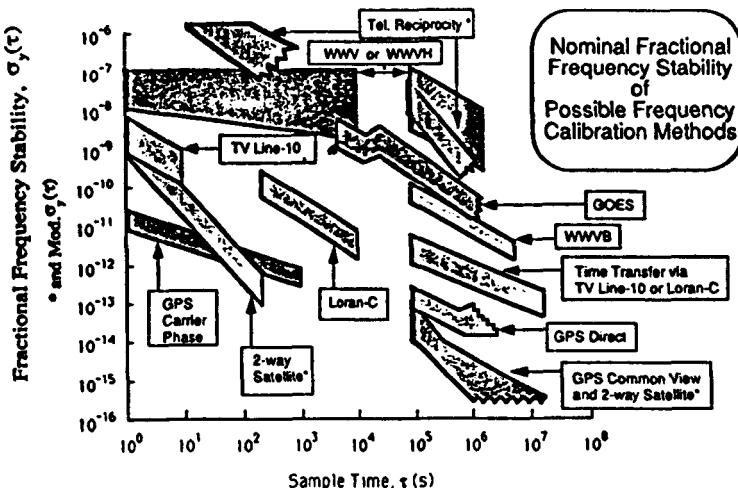
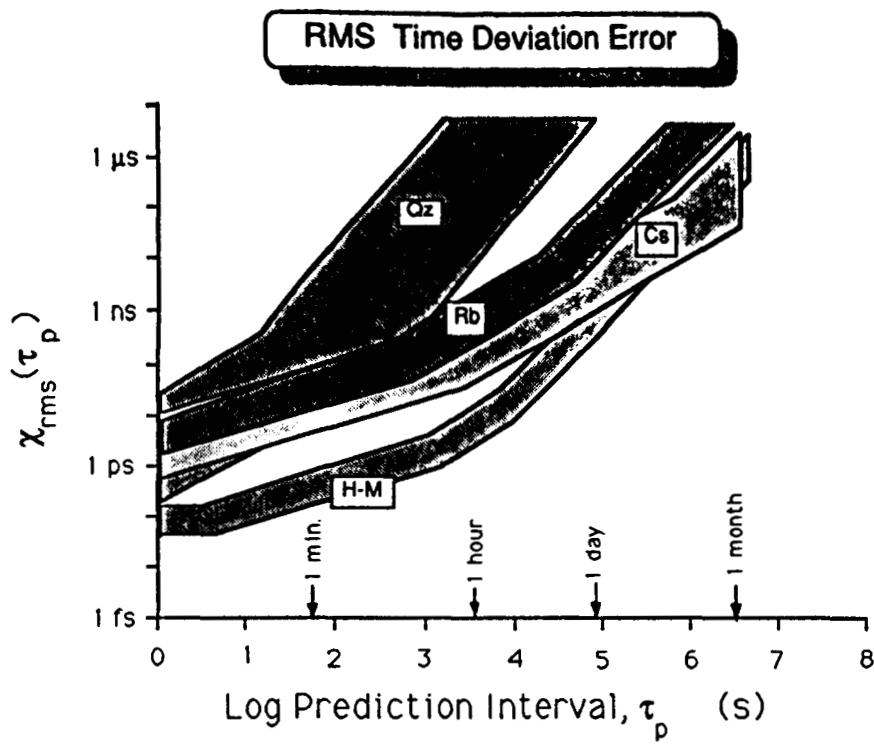
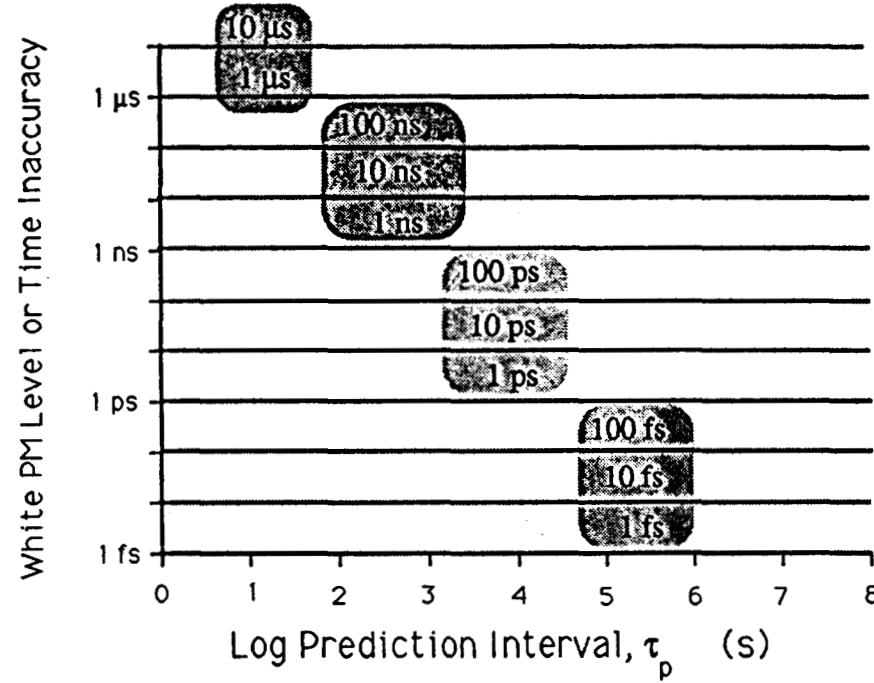


Figure 4. This plot gives nominal frequency stability of several important comparison systems. The stabilities are characterized using  $\sigma_y(\tau)$  except where indicated by an \*\*. Mod.  $\sigma_y(\tau)$  was used in those cases where white noise PM was predominant for some range of sample times  $\tau$ , and an asterisk \*\* denotes those. The "Tel. Reciprocity" data were analyzed under the assumption of reciprocity of the path (measure the round trip time and divide by two to calibrate the path delay). The short-term data were measured locally and the long-term data were measured between Colorado and Hawaii via communication satellite. We often found that telephone modems contributed more noise than the path. What is plotted is the composite. The WWV and WWVH time-and-frequency transmissions at 2.5, 5, 10 and 15 MHz (WWV also broadcasts at 20 MHz) are limited in their stabilities by sky-wave-path variations. GOES East and GOES West are NOAA weather satellites broadcasting UTC(NIST) on two slightly different frequencies near 468 MHz. Here, the stability is limited by the knowledge of the satellites' ephemerides. WWVB is NIST's 60 kHz time-and-frequency broadcast service; in this case the propagation path stability is limited by the fluctuations in the earth-ionosphere waveguide. The TV Line-10 method involves line of sight transmissions in the TV band. It can operate with an atomic clock at the transmitter or with two clock sites receiving the TV Line-10 arrival times concurrently and subtracting one set of numbers from the other. Stability limitations here are often caused by the receiving equipment. Loran-C is a ground-wave navigation signal (at 100 kHz) operated by the U.S. Coast Guard. The time is monitored and controlled with respect to UTC(USNO). The stability is limited by propagation path variations. Two-way satellite time transfer uses spread-spectrum modems operating with different up-link and down-link carrier frequencies in one of several different bands (C, Ku, and K). The short-term stability for two-way satellite time transfer is basically limited by signal to noise and bandwidth considerations. Currently, the long-term performance seems to be limited by equipment instabilities. One can only extract frequency information from the "GPS Carrier Phase" measurements, and the stability seems to be limited by the GPS on board clocks. Time and frequency stability of directly received GPS signals is limited mainly by variations in the GPS Kalman state estimates for the system. If one is using an L1 GPS timing receiver only, then the ionospheric modeling errors can contribute additional instabilities. In some cases, signal multipath errors and/or receiver instabilities can also contribute significant instabilities. Using GPS in the common-view mode cancels out the clock instabilities of the GPS satellites and cancels some of the broadcast satellite-ephemeris instabilities. The stability limits for the common-view mode arise from the same mechanisms as for GPS direct measurements except that some of the mechanisms are reduced by common-mode cancellation.



**Figure 5** This is a plot of the time prediction error,  $\chi_{rms}(\tau_p)$ , as a function of the prediction interval for commercially available precision clocks. Qz denotes quartz-crystal oscillator clock; Rb denotes rubidium gas-cell frequency-standard clock; Cs denotes cesium-beam frequency-standard clock; and H-M denotes active hydrogen maser clock. This prediction error is calculated from  $K\sigma_y(r)$  with  $K$  being chosen for an optimum prediction estimate. The value of  $K$  depends on the type of noise.



**Figure 6**

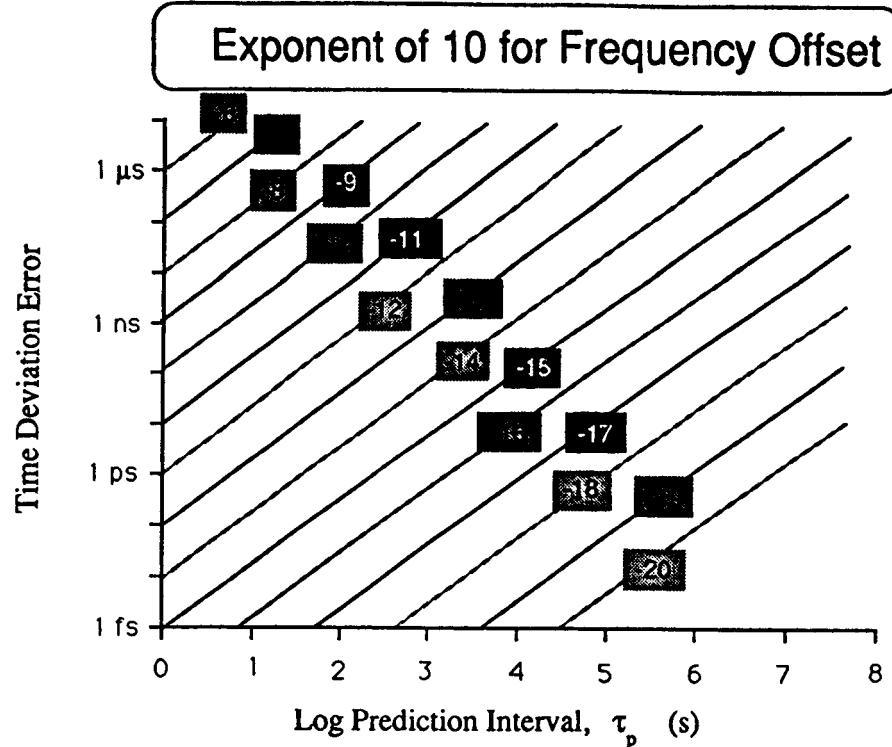


Figure 7

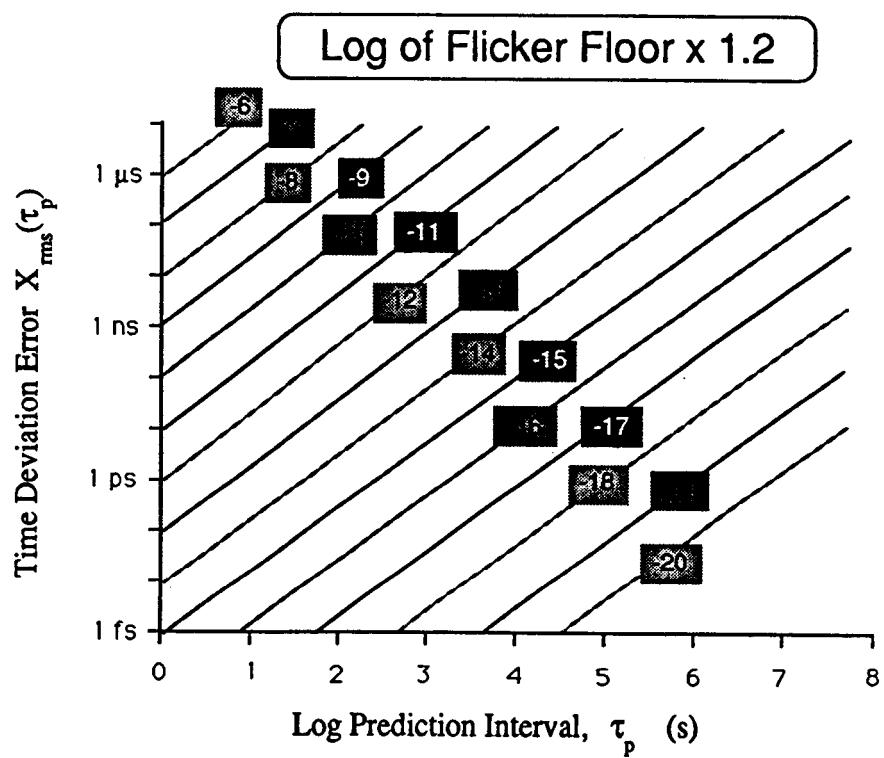


Figure 8

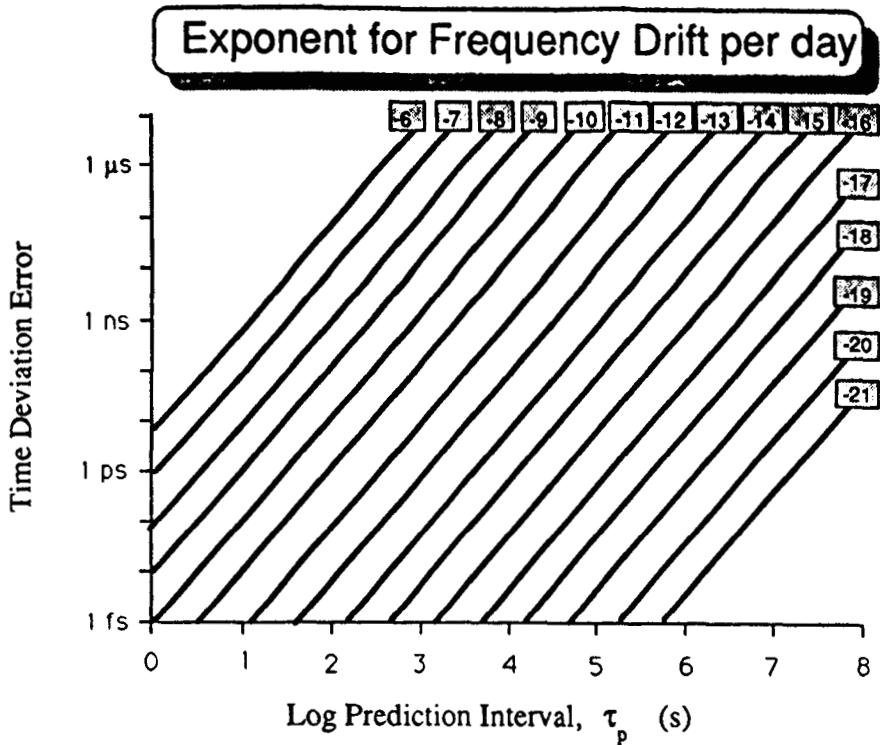


Figure 9

Figures 6, 7, 8 and 9 The ordinates and abscissas of these four plots are the same as those for Figure 5. Figure 6 can represent either the time accuracy or the white noise PM level. The time accuracy is often limited by systematic effects and averaging values does not improve it. The white noise PM is well represented by the standard deviation of the measurements, and, if this noise is the limiting noise, then averaging values will improve the knowledge of the time as the square root of the number of values averaged. Figure 7 is the time accumulation over some interval,  $\tau_p$ , due to a systematic frequency difference (or offset) between the two clocks being compared. Figure 8 is the rms time deviation resulting from a random flicker FM process — often observed in long-term clock comparisons. The  $1.2 (1/\sqrt{\ln 2})$  factor is the  $K$  factor for flicker noise FM. "Flicker Floor" means the value of  $\sigma_y(r)$  where there is a  $r^0$  dependence, that is, where there is no further improvement in stability with increasing  $r$ . The curves in Figures 7 and 8 have the same slope (+1) even though they arise from different mechanisms. Figure 9 demonstrates the long-term significance of time deviation errors resulting from a linear frequency drift in a clock. The plus-two (+2) slope corresponds to the quadratic departure of the time of the drifting clock. If frequency drift exists in a clock, this error along with environmental perturbations is often the main cause of long-term time deviations.

## RMS Time Estimate due to Random Noise

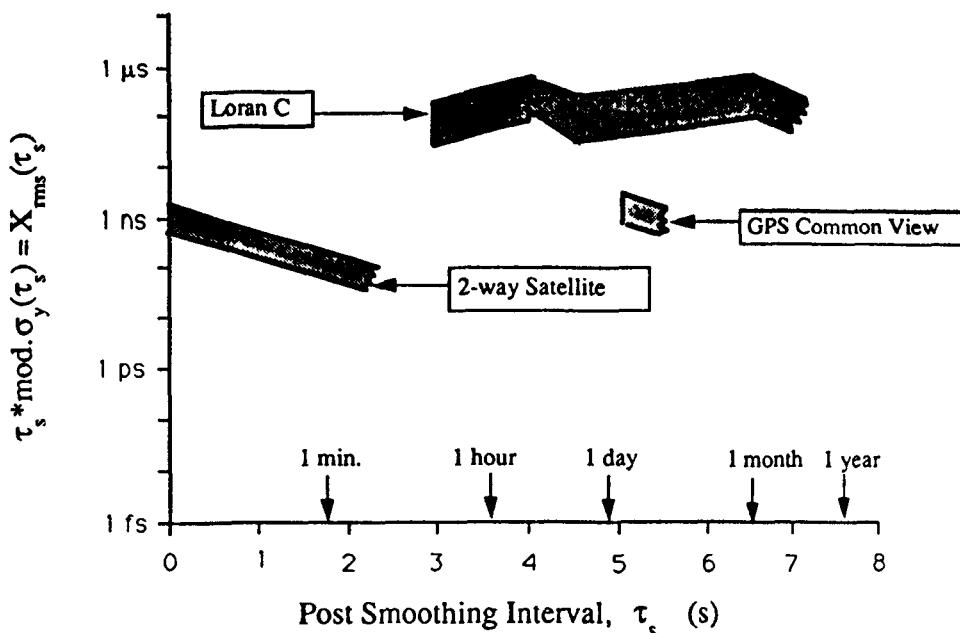


Figure 10. This type of plot can be used to determine whether or not smoothing or averaging the data is beneficial. We have here defined the time stability as the product  $\tau_s \text{mod.} \sigma_y(\tau_s)$ . For flicker noise PM, white noise FM, flicker noise FM and random-walk noise FM the standard deviation of the time residuals grow without bound as the data length increases. Hence, it is not a good measure. The above product is a good measure, is convergent and is data-length independent. This measure can also show the effects of systematic effects, of environmental perturbations as well as the different kinds of noise processes that may be driving the instabilities in the comparison system and/or in the clocks. The different comparison methods plotted are explained in Figure 4.