

Rydberg constant and fundamental atomic physics

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(Received 25 April 1988)

A detailed report on the current status of measurements of the Rydberg constant is given. Our recently reported value of $R_\infty = 109\,737.315\,73(3) \text{ cm}^{-1}$ has been confirmed by three other laboratories within experimental error. An additional check on the iodine cell, the heart of our wavelength and frequency reference, confirms a negligible pressure shift. The possible role of the Rydberg constant in fundamental atomic physics lies in tests of quantum electrodynamics and in improvement of the realization of the meter. We propose that the hydrogen spectrum be used to realize the meter in the optical domain, as an alternative to the current frequency chains. For the realization to be useful, improvement of the current precision of the Rydberg constant by a factor of 2 or more is required.

I. INTRODUCTION

The Rydberg constant R_∞ is one of the most accurately measured, fundamental physical constants:^{1,2}

$$R_\infty = 109\,737.315\,73(3) \text{ cm}^{-1} \quad (\text{error of 3 parts in } 10^{10}). \quad (1)$$

R_∞ ties together several areas of fundamental physics (see Fig. 1). R_∞ enters into the least-squares adjustments of other fundamental constants, in which it is an “auxiliary” constant, as an essentially exact input.

For example, consider the measurement of a magnetic field B by nuclear magnetic resonance. The resonant frequency ν is given by $\nu = \gamma'_p B$, where γ'_p can be determined by the equation

$$\gamma'_p = \frac{\alpha^2 c (\mu'_p / \mu_B) (2e/h)}{4R_\infty} \quad (2)$$

given by Cohen and Taylor.² As they point out the quantity γ'_p can be determined more accurately indirectly by substituting the quantities on the right, than by direct measurement of γ'_p . We list these quantities, with the corresponding errors in parts in 10^6 , as listed by Cohen and Taylor:²

$$\alpha = 7.297\,353\,08 \times 10^{-3} \quad (0.045),$$

$$\frac{\mu'_p}{\mu_B} = 1.520\,993\,129 \times 10^{-3} \quad (0.011),$$

$$\left[\frac{2e}{h} \right] = 4.835\,976\,7 \times 10^{14} \text{ A/J} \quad (0.30),$$

$$c = 299\,792\,458 \text{ m/s} \quad (0.0),$$

$$R_\infty = 109\,737\,315.73 \text{ m}^{-1} \quad (0.001).$$

The result, from Eq. (2), is $\gamma'_p = 267\,515\,255 \text{ s}^{-1} \text{ T}^{-1}$, with an error of 0.31 parts in 10^6 , which is, of course, the value obtained by Cohen and Taylor.²

For a discussion of this example and for the details of their computation see Cohen and Taylor.² The subsequent change in R_∞ of a few parts in 10^9 changes γ'_p by only a hundredth of its quoted error.

In turn, other fundamental constants, such as the proton-electron mass ratio and fine-structure constant,² are needed to interpret experimental results used to find R_∞ . Presently, the uncertainties in these constants have a negligible effect on the determination of R_∞ .

R_∞ is basic to all calculations of atomic and molecular energy levels, which often are expressed in atomic units of 27.2 eV, twice the value of the Rydberg constant. Comparison of theoretical energy levels and experimental wavelength measurements are made through the Rydberg constant. In turn, determination of R_∞ depends on simple systems where the theory is trustworthy.

Theoretical calculations usually involve three levels of analyses: nonrelativistic wave functions and energies, relativistic and quantum-electrodynamical (QED) corrections (Fig. 1). R_∞ is measured by means of spectra of elementary atomic systems, such as hydrogen, where it enters the nonrelativistic formula for the wavelength of atomic hydrogen transitions:

$$\frac{1}{\lambda} = R_\infty \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]. \quad (3)$$

This formula is subject to corrections due to fine-structure, hyperfine-structure, reduced-mass, and quantum-electrodynamic corrections. In turn, the interpretation of these spectra often depends on a precise value of R_∞ . For example, measurements of the Lamb shift of the ground state of H or of the 1S-2S interval in

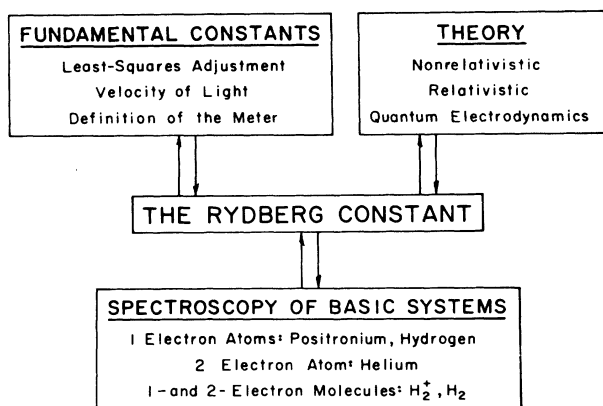


FIG. 1. The Rydberg constant ties together three areas of fundamental physics: atomic theory, spectroscopy of basic systems, and the least-squares adjustment of the fundamental constants.

positronium currently depend on R_{∞} . With continued improvement in theory, the helium atom may become a new source for the determination of R_{∞} .¹

The first determination of $R_{\infty} = 109\,675\text{ cm}^{-1}$ by Rydberg in 1890 was accurate to better than 1 part per 10^3 .³ By the early 1970s optical spectroscopy had brought the experimental accuracy to a limit of better than 0.1 part per 10^6 .³ Laser spectroscopy, introduced to the hydrogen atom by Hänsch and Schawlow,³ has further improved the accuracy by more than three orders of magnitude to the present value of 3 parts in 10^{10} . As this paper will show, the measurement of the Rydberg constant by laser spectroscopy has approached a limit in accuracy which will require new approaches to achieve further improvement.

II. METHOD

A. General plan of the experiment

The latest experiment is a modification of previous efforts by Amin *et al.*⁴ and by us.⁵ In the two previous experiments, the Rydberg constant was determined by a set of wavelength measurements of the Balmer- α line in hydrogen and deuterium. To achieve better precision, the wavelength of the $2S\text{-}4P$ Balmer- β transition is measured. Briefly, these experiments are optical analogs of the classical Lamb-Retherford measurement of the Lamb shift. Instead of microwaves, a cw dye-laser beam is used to quench the metastable atomic beam through either the Balmer- α or the Balmer- β transition. A $^{127}\text{I}_2$ -stabilized He-Ne laser provides a wavelength and frequency standard. By using the Balmer- β line, we are able to improve our previous accuracy of 0.7 parts in 10^9 to 0.3 parts in 10^9 . This improvement over the Balmer- α measurement is partly due to the higher transition frequency ($6.17 \times 10^8\text{ MHz}$) and narrower linewidth (12.9 MHz) of the Balmer- β line.

An independent value of R_{∞} is calculated from each of the four components $2S_{1/2} \rightarrow 4P_{1/2}$ and $2S_{1/2} \rightarrow 4P_{3/2}$ in both hydrogen and deuterium. The consistency of these is a measure of the reliability of our new value for the Rydberg constant.

B. Experimental technique

Figure 2 shows the basic scheme of this experiment. The apparatus consists of five parts: the metastable atomic beam chamber, the dye laser, the standard laser, the measuring étalon, and the data recording system.

In the atomic beam chamber, a tungsten oven is used

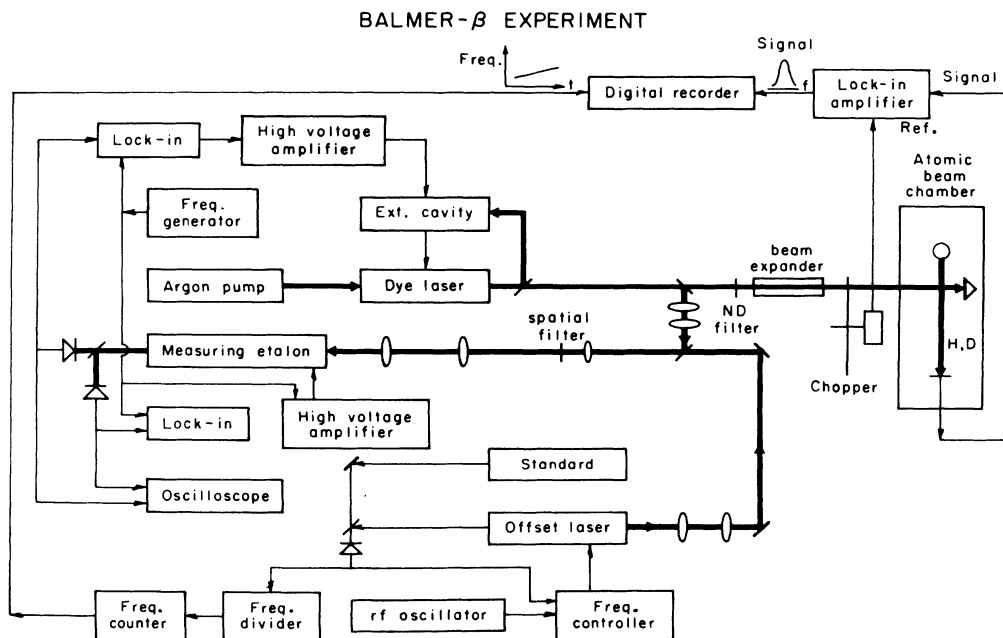


FIG. 2. Overall plan of the experiment to measure the Balmer- β wavelength with a direct comparison to the standard wavelength (He-Ne: I_2^{127}).

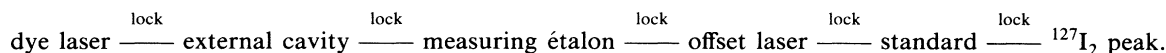
to dissociate molecular hydrogen (or deuterium). A beam of H atoms emerges from a slit in the oven to be bombarded and excited by electrons. All low-lying excited states return to the ground state in $\sim 10^{-8}$ sec, with the exception of the metastable $2S_{1/2}$ state, which has a lifetime $\sim \frac{1}{7}$ sec and lives long enough to reach a secondary electron emission detector. (See Fig. 2.)

In place of the microwaves in the Lamb-Retherford experiment, a chopped, cw dye-laser beam crosses the atomic beam at right angles and excites the $2S_{1/2}$ state to the $4P_{1/2}$ or $4P_{3/2}$ states. As most of the atoms in the $4P$ states drop down to the ground state, there is a quenching of the metastable atoms. The first-order Doppler shift is eliminated by crossing the laser beam and its retroreflection with the atomic beam at an angle of 90° . A lock-in amplifier, referred to the chopper, detects the quenched part of the atomic beam when the dye laser is

tuned to a Balmer- β transition. The dye laser is stabilized to an external cavity. The wavelength of the dye laser is measured in a piezoelectrically (PZT) scanned étalon, by comparison with a standard laser. Both laser beams are mode matched into the étalon.

To avoid errors caused by scanning nonlinearities, a frequency-locked chain is used. The standard laser (^3He - ^{20}Ne , $^{127}\text{I}_2$) is constructed identically to the standard laser used to establish the new definition of the meter⁶ ($c = 299\,792\,458$ m/s), to an error of 1.6 parts in 10^{10} . Another He-Ne laser is offset-locked from the standard laser with a best frequency equal to that of a scannable rf oscillator. Two feedback loops, one fast and one slow, lock the dye laser to an external cavity. A set of slower loops lock the dye laser to the measuring étalon and the measuring étalon to the offset laser.

Thus the frequency-locked chain is formed as follows:



This chain precisely sweeps the dye-laser frequency across the Balmer- β transition by tuning the rf oscillator. A two-channel digital recorder makes a simultaneous plot of the offset frequency and the signal from the quenched atomic beam (see Fig. 2). More detailed figures and descriptions of the experiment are given elsewhere.¹

C. Apparatus

This section describes the apparatus, many of the aspects of which are described in previous publications.^{4,5}

(1) *Metastable atomic beam machine.* The metastable atomic beam machine produces and detects a metastable beam ($2S_{1/2}$) of atomic hydrogen or deuterium and contains a region in which this beam interacts with a crossed laser beam. The apparatus is similar to that used by Amin *et al.*⁴

(2) *Source.* As before,^{4,5} atomic hydrogen or deuterium is produced by thermal dissociation in a tungsten oven at 2850 K. The metastable atoms in the $2S_{1/2}$ state are produced by an electron bombardment current of 2 mA at 15 V. The electron bombarder is improved over earlier models by lengthening the anode to 2 cm length. Accidental quenching by electric fields is avoided by keeping the anode near ground potential. About 3 out of 10^5 atoms are excited to the metastable $2S_{1/2}$ state, 99.8% polarized in the substate α ($m_s = +\frac{1}{2}$).

Contamination of the secondary electron emission detector is reduced by continuous cold trapping of the detector chamber with liquid nitrogen, by use of a low back-streaming diffusion pump and by gold plating the target.

(3) *Laser-atomic beam interaction region.* To eliminate Zeeman shifts in the interaction region, a box made of high permeability alloy is placed about the interaction region inside the vacuum chamber. This shields the interaction region from stray magnetic fields from, for example, the electron-gun magnet and from the earth. The magnetic field inside the box is measured to be smaller

than 20 mG (2×10^{-6} T). To eliminate possible Stark shifts, a piece of copper waveguide, 2×5 cm in cross section and 25 cm in length, is put inside the magnetic box to shield against stray electric fields. The electric field is estimated to be of the order of 1 mV/cm or less.⁷

(4) *Dye-laser system.* The dye laser used in this experiment was built at the National Institute of Standards and Technology (NIST). It is in a ring configuration, uses Coumarin 480 dye, and is stabilized to an external reference cavity. The pump is an argon-ion laser operated with an all-line uv output power of 2.5 W. The cw output power of the dye laser at the Balmer- β transition wavelength (486.2 nm) starts at 120 mW with fresh dye. However, it is typically 60 mW or less during the 20–40 h lifetime of the dye. A hydrogen optogalvanic cell is used for the preliminary tuning of the dye-laser frequency to near coincidence with the Balmer- β transition. Tuning can be done within the Doppler width of approximately 5 GHz in a room-temperature discharge.

The locking technique of Hänsch and Couillaud⁸ is used with a 10-cm-long confocal reference cavity. In their scheme, the polarization of the light reflected from the cavity is analyzed and provides dispersion shaped resonances which give the error signal for two servo loops (> 100 Hz and dc to 100 Hz). When the dye laser is stabilized to the external reference cavity, the rms frequency jitter is less than 0.4 MHz. Control of the laser frequency is done by adjusting the reference cavity length with its PZT. This gives a continuous, stable, and smooth tuning over a range of 300 MHz.

(5) *$^{127}\text{I}_2$, ^3He - ^{20}Ne standard laser.* Possible pressure shift in frequency. The standard laser used in the experiment is provided by NIST. It is a helium-neon (He-Ne) laser with an internal iodine cell operating near 633 nm. The He-Ne laser is third-derivative locked to one of seven hyperfine components of the $R(127)$ 11-5 line in $^{127}\text{I}_2$ that coincide with the He-Ne laser gain curve. The third-derivative signal is obtained by modulating the laser at 2.5 kHz with a modulation width of 6 MHz.

The Bureau International des Poids et Mesures (BIPM) assigns an error of 3.4 parts in 10^{10} to this standard laser.⁹ However, the one used in this experiment is constructed identically to the one used in the experiments of Jennings *et al.*⁶ in which they measured the frequencies of several iodine hyperfine components to an error of 1.6×10^{-10} . The absolute frequencies of the *g* and *i* components were measured to be

$$473\,612\,340.492(74) \text{ MHz for } g,$$

$$473\,612\,214.789(74) \text{ MHz for } i.$$

Since the standard iodine-stabilized He-Ne laser is mechanically, electrically, and optically identical to that used by Jennings *et al.*,⁶ we assume the same values and uncertainty for the frequency of our reference laser when it is locked to the *g* or *i* hyperfine components. There could be corrections for differences in temperature of the iodine cell (which causes changes in the iodine pressure), for the depth of frequency modulation and for the laser power. The values for these parameters were close to those used by Jennings *et al.*,⁶ so that the corrections were at most a few parts in 10^{11} and thus were negligible.

However, one possibility is that the standard cell could have leaked or outgassed since the original calibration, thereby adding some foreign gas to the original iodine filling. The order of magnitude of frequency shift due to foreign gases can be estimated from the known value for iodine, which is approximately 80 kHz for a pressure of 130 Pa (about 1 mm Hg), or 2 parts in 10^{10} . Although it is a simple matter to check the external operating parameters of laser power, modulation, and temperature, this is not the case for the interior pressure of a sealed-off cell.

Marx and Rowley developed an ingenious method for testing standard iodine cells for foreign gases.¹⁰ They used the Hanle effect, which is a magnetic depolarization of resonance radiation. In this effect, the intensity of resonance radiation changes as a magnetic field is applied to the atoms or molecules. This intensity change follows a Lorentzian dependence on the magnetic field. The measured half-width is a measure of the pressure broadening and therefore of the total pressure. This method is used in our experiment to check for the presence of foreign gases.

Figure 3 shows the experimental arrangement. An argon-ion laser beam at 501.7 nm and power of 120 mW has its vertical plane of polarization rotated by 90° by a half-wave plate to a horizontal orientation convenient for fluorescence detection, passes through a beam expander and then through the iodine cell. The iodine molecules in the cell are excited by the 501.7-nm resonance radiation and give off fluorescence, which passes through a polarizer and an orange filter, and then is detected by a photocell. The detector lies on an axis parallel to the original polarization and normal to both the laser beam and magnetic field directions. All three directions are mutually orthogonal. The magnetic moment of the excited state precesses in a nonzero field, thereby increasing the light intensity passed through the polarizer onto the photocell. The beam expander reduces the laser intensity to 30 mW/cm², which reduces the power broadening to a

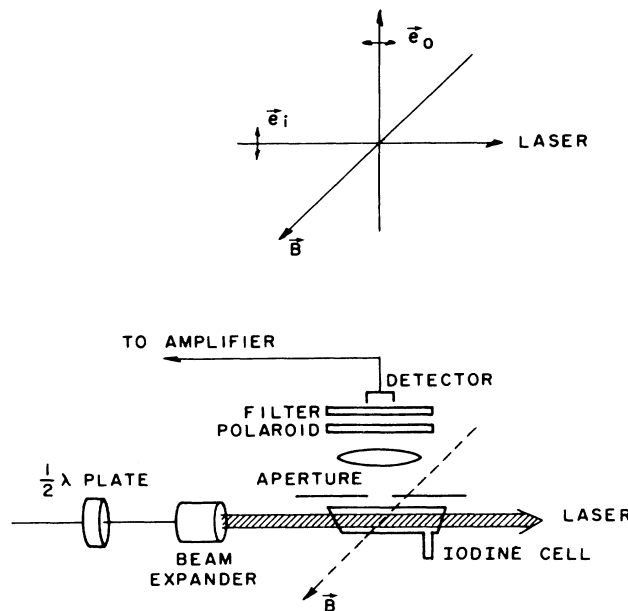


FIG. 3. Experimental arrangement to use the Hanle effect to detect foreign gases in a closed iodine cell used for a standard wavelength. The view is from above the apparatus.

negligible value. In order to know the iodine pressure (which produces a pressure shift of its own), the iodine cell temperature is measured with a probe attached to the cell. The experiment differs in one detail from that of Marx and Rowley.¹⁰ Instead of a ratiometer, the laser light-intensity control keeps the power stable enough to make satisfactory intensity measurement while changing the magnetic field strength.

Marx and Rowley point out that a misalignment of the apparatus would produce an asymmetry between positive and reversed magnetic fields.¹⁰ As a check on the alignment of our apparatus, we measure the linewidth for both field directions and take the average of the two results. We position a magnetron magnet at different distances from the cell to provide magnetic fields up to 0.12 T (1200 G). The magnetic field strength is measured with a Hall probe gaussmeter which is calibrated by nuclear magnetic resonance. The intensity of the fluorescence varies by about 8% from zero field to infinity. To show the Lorentzian function graphically, the normalized difference at each field from the infinite field value is plotted in Fig. 4. Also shown is a curve of the function of the form

$$Y = A_1 + A_2 / (1 + A_3 H^2),$$

which is fit to the data by the program GRIDLS, taken from Bevington.¹¹ The parameter A_3 gives the half-intensity field. The error ϵ in A_3 is found from the condition

$$\chi^2(A_3 + \epsilon) = \chi^2(A_3) + 1, \quad (4)$$

given by Bevington.¹¹ The measured half-width for the cell is 0.0239(4) T and 0.0253(6) T with the field reversed, with an insignificant asymmetry. The average value is

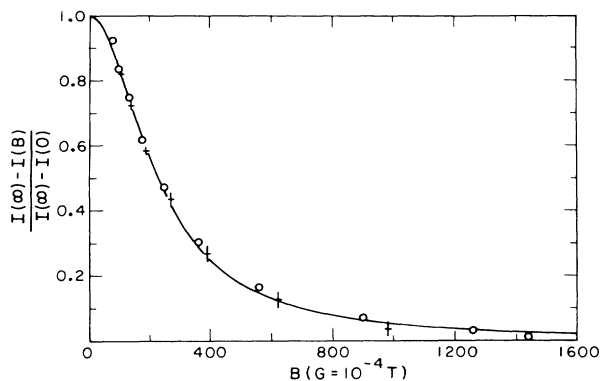


FIG. 4. Plot of the parameter $[I(\infty) - I(B)]/[I(\infty) - I(0)]$, a normalized Lorentzian function derived from the data, which is used to evaluate the effect of foreign gases in the standard laser.

0.0246(4) T, which corresponds to a shift of 0.0007(4) T from the value of 0.0239 T, which is inferred for iodine at the equilibrium vapor pressure of 26.1(3) Pa at 293.0(2) K, the ambient temperature.¹² This would correspond to a frequency shift of 5(3) kHz if the background gas were I_2 . This is a fractional shift of $1.0(6) \times 10^{-11}$, less than the quoted error by more than an order of magnitude. We have thus determined that foreign gas in the cell made by one of us (H.P.L.) at NIST Gaithersburg has a negligible contribution to our error budget.

(6) *Frequency chain.* To provide a tunable but precisely known and controlled frequency source, a He-Ne laser is heterodyned and offset locked to the standard laser. This lock is obtained by detecting the beat frequency between the standard and offset laser on a photodiode and by comparing this beat note to the frequency from an rf oscillator controlled by a stepping motor. The voltage error derived from this comparison is used to control a piezoelectrically driven mirror on the offset laser in such a way as to lock the beat and oscillator frequencies together. The control loop forces the beat frequency to track the rf oscillator; the offset laser is precisely scanned by tuning the oscillator frequency. A frequency counter measures the offset frequency, over a range of 7–100 MHz to a fraction of 1 kHz. This apparatus, along with the range of available I_2 lines, provides a scannable standard laser which can be precisely frequency offset locked to the iodine-stabilized reference laser at any value over a frequency range of almost 400 MHz. A second servo-control loop locks a cavity mode of the measuring étalon to resonance with the offset-locked He-Ne laser. A third servo-control loop locks the frequency of the dye laser to resonance with another cavity mode of the measuring étalon. Thus the frequency of the dye laser is known and can be precisely scanned by scanning the frequency of the rf oscillator.

(7) *Measuring étalon.* The measuring étalon consists of a pair of mirrors, one flat and one curved. The surfaces of the measuring étalon mirrors are good to $\lambda/200$ at 589 nm. The radius of the curved mirror has been measured by one of us to be $r = 284.3 \pm 0.1$ cm.¹³ The mirrors have

a reflectivity $R = 94\%$ at both 486 and 633 nm, which gives a cavity finesse of about 50. The spacers are hollow cervit blocks (coefficient of thermal expansion 2×10^{-8} K⁻¹). One mirror is movable by a piezoelectric element.

The length of the étalon is chosen by the following consideration: All low-order transverse modes are set as far as possible from the fundamental modes, by judicious choice of cavity length, so that the resonant frequency for the planoconcave étalon avoids overlapping modes caused by imperfect mode matching.¹³

The mirror phase shift is canceled by the method of virtual mirrors, which involves subtracting the order numbers for two different interferometer spacings. The two étalon spacings are measured by gauge blocks to be

$$L_1 = 12.34890 \text{ cm}, \quad L_2 = 82.03979 \text{ cm}.$$

The virtual mirror spacing is estimated by the difference of spacer lengths, which is

$$L = L_2 - L_1 = 69.69089(5) \text{ cm}.$$

To better mode match the dye-laser beam into the étalon, cylindrical lenses are used to convert the elliptical dye-laser beam to a circular shape, which is then focused onto a 25- μ m spatial filter. The beam transmitted by the filter is then nearly matched into the lowest-order transverse mode of the étalon. The standard and dye-laser transmission fringes are separated by blue and red interference filters. The length of the étalon is modulated, which produces a corresponding modulation of the transmitted power at the cavity resonance. The resulting modulated signal is amplified and sent into lock-in amplifiers for the locking chain.

III. DATA ANALYSIS AND CORRECTIONS

A. Line-shape analysis

The natural linewidth of the H and D Balmer- β transition is 12.9 MHz.¹⁴ It is broadened by the first-order Doppler effect caused by the atomic beam divergence (17 MHz for H and 12 MHz for D), by laser power broadening (typically about 2.5 MHz), and by hyperfine structure (hfs) (about 2.5 MHz or less). The broadened linewidth is about 28 MHz for H and 23 MHz for D. Figure 5 shows the experimental points of the H resonances fitted with a theoretical Voigt profile of 28 MHz width.¹⁵ The only variable parameters are the percentage quenching at the line center and the line-center position. The fit is excellent. The calculated linewidths agree with experiment for both H and D. These preliminary experiments act as a check of our understanding of the experimental conditions.

Typically the laser power corresponds to 50% quenching of the $4P_{3/2}$ component, with a laser intensity of about 2–3 mW/cm². The signal-to-noise ratio is about 1000:1 with a 3-s time constant.

B. Corrections and errors

Corrections for each transition with error estimates for the Rydberg constant are listed in Table I.

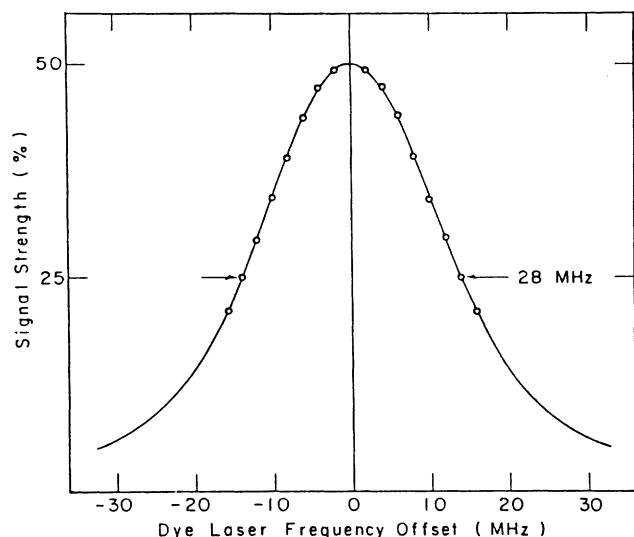


FIG. 5. Comparison of experimental line shape (points) with theoretical Voigt profile for hydrogen, Balmer- β transition. The only adjustable parameters are the maximum percentage quenching and the position of the line center.

1. $2S_{1/2}$ state hfs

Since the motional electric field in the electron gun almost completely quenches the $F=I-\frac{1}{2}$ levels, only the upper level $F=I+\frac{1}{2}$ survives after the beam leaves the electron gun. Therefore the $2S$ hfs is completely resolved. The measured values of this hfs cause negligible errors (about 10^{-13}) in R_∞ .¹⁶ The shifts due to the incomplete quenching of the levels $F=I-\frac{1}{2}$ are negligible.

2. $4P_{1/2}$ and $4P_{3/2}$ hfs

Since the $4P$ hfs is not resolved, corrections must be estimated. The transition into the magnetically shielded transition region occurs in approximately 0.5 ms. According to the uncertainty principle, the width of the state is now approximately 0.3 MHz; the Zeeman splitting between the sublevels is approximately 0.03 MHz. Thus the atoms lose quantization and all the sublevels of the $2S_{1/2}$, $F=I+\frac{1}{2}$ state become equally populated. The $4P$ hfs corrections in Table I are calculated under this assumption. The correction for each of the four lines is large in magnitude (3 or 9 parts in 10^{10}), but the net correction is zero. The agreement among the four independent values for R_∞ gives credence to our analysis.

3. Second-order Doppler shift

In order to calculate the second-order Doppler shift, the speed of the metastable hydrogen beam is measured by using the first-order Doppler effect. The laser beam is deliberately misaligned, and the frequency difference between the Doppler-shifted transition peaks caused by counterpropagating optical waves is measured. The result for hydrogen is 9500 m/s, in good agreement with the expected value for a velocity distribution characteristic of an effusive source. For deuterium the speed is smaller by a factor of $\sqrt{2}$ due to the doubled mass.

4. Others

Many other corrections and sources of possible error are discussed in detail by Zhao.¹ Only those of magnitude 10^{-11} or greater are mentioned in Table I.

TABLE I. Corrections, errors, and variance for R (parts in 10^{10}).

Transition $2^2S_{1/2}$ to $n=4$	Corrections					Error	Contribution to variance
	H, $P_{1/2}$	H, $P_{3/2}$	D, $P_{1/2}$	D, $P_{3/2}$	Mean		
Effects ^a							
Standard Laser						1.5	37
Statistical						1.5	33
Stark, light shifts, other minor						1.0	14
Second-order Doppler	5.7	5.7	2.8	2.8	4.3	0.9	12
$4P$ hfs	9.1	−9.1	2.8	−2.8	0.0	0.5	4
Lamb shifts and other QED ^b	16 973.0	16 921.6	16 990.8	16 939.4	16 959.2	0.2	1
Fundamental constants (α , M_p/M_e , M_d/M_p)						0.1	
Photon recoil	−13.6	−13.6	−6.8	−6.8	−10.2		
Refractive index	−0.2	−0.2	−0.2	−0.2	−0.2		
$2S$ hfs	720.0	720.0	221.2	221.2	470.6		
Total ^b	721.0	702.8	219.8	214.2	464.5	2.6	100
Root-mean-square error	1.1	2.6	2.3	1.3			
Rounded to						3.0	

^aA possible foreign gas shift has been measured to be less than 0.10(6) parts in 10^{10} .

^bLamb shifts, other QED corrections, and nuclear size effects are built into the calculations of Ref. 25 as corrected by us for new values for Lamb shifts and fundamental constants and thus do not appear in the totals.

C. Virtual mirror method

Any precise, absolute, interferometric wavelength determination must correct for the wavelength dependence of the mirror phase shift. With the method given by one of us,^{17,18} the phase-shift correction can be estimated with an error of about 2%. Unfortunately, because the present experiments involve wavelengths that are far apart (486 and 633 nm), a double stack coating must be used. The phase shift for such a coating changes by 2π between these two values and causes an unacceptable error of the order of 10^{-8} in the absolute wavelength. For this reason, we use the virtual mirror method, which eliminates the phase-shift problem by using two mirror spacings. The phase shifts are thereby canceled.

The étalon spacings are determined by the method of exact orders.¹⁸ This method requires knowledge of the integral order number, which requires a measurement of the mirror spacing to 3×10^{-5} cm. Initially, the étalon spacers are measured in the machine shop. Although this is known to 0.00005 cm, because of possible lack of parallelism of the étalon and spacer axes, the uncertainty in the effective spacing is 0.003 cm. By means of isotope shifts between H and D, the virtual mirror spacing is determined to a precision of 1.4×10^{-4} cm, which gives the spacing to within a few orders. Finally, only one possible integral order is consistent with all four H,D: $4P_{1/2,3/2}$ lines. A detailed description of this procedure is given by Zhao.¹

The frequency ν for a measured hydrogen or deuterium line is given by the expression

$$\nu = \frac{N + \phi}{N_s + \phi - (2L_1/c)(\nu_{s_2} - \nu_{s_1})} \nu_{s_2}, \quad (5)$$

where N is the integral order number difference between the two mirror spacings for the dye laser tuned to the atomic line, N_s is the integral order number difference between the two mirror spacings for the standard laser, and ϕ is the difference in diffractive phase shifts $\phi_2 - \phi_1$ between the two mirror spacings, where

$$\phi_i = \frac{1}{\pi} \cos^{-1} \sqrt{1 - L_i/r}, \quad i = 1, 2. \quad (6)$$

L_i is the étalon length, r is the radius of curvature of the spherical mirror,¹³ c is the speed of light (299 792 458 m/s), and ν_{s_i} is the frequency of the standard plus offset used for each measurement. (For convenience, different hyperfine components of the 633-nm $^{127}\text{I}_2$ line are used.)

IV. RESULTS

The corrections and estimated errors are listed in Table I. The major sources of error (70% of the contribution to the variance) are statistical (1.5 parts in 10^{10}) and the standard laser (1.6 parts in 10^{10}). The root-mean-square error is 2.6 parts in 10^{10} , which we round to 3 parts in 10^{10} .

Table II shows the measured frequencies of the four transitions observed in our work. The $4P$ fine structure and the isotope shifts can be calculated trivially by subtraction of the entries in Table II. The agreement between theory and experiment is excellent.¹

TABLE II. Measured frequencies and calculations of the Rydberg constant.

Atom transition $2S_{1/2}$	H $4P_{1/2}$	$4P_{3/2}$	D $4P_{1/2}$	$4P_{3/2}$
Frequency ($\nu-6.16\times10^8$) (MHz)	519 973.57(7)	521 345.54(16)	687 756.82(14)	689 128.94(10)
Fractional correction ($\Delta\nu/\nu$) (10^{-10})	721.0	702.8	219.8	214.2
Corrected frequency	520 018.02(7)	521 388.87(16)	687 770.38(14)	689 142.15(10)
Calculated ^a frequency	520 016.169	521 387.299	687 768.960	689 140.464
Rydberg constant ^b $R-109\,737$ (cm^{-1})	0.315 770(12)	0.315 720(29)	0.315 692(25)	0.315 741(14)
Combined by atoms ^b $R-109\,737$ (cm^{-1})	0.315 745(25)		0.315 717(25)	
Grand average	$R=109\,737.315\,731(16)\text{ cm}^{-1}$			

^aBased on Ref. 25 and $R_\infty = 109\,737.315\,44 \text{ cm}^{-1}$, $1/\alpha = 137.035\,989\,5$, $2S$ Lamb shift for H = 1057.8514(19) MHz, for D = 1059.235(27) MHz, $m_p/m_e = 1836.152\,701(37)$.

^bErrors shown are statistical only.

V. DISCUSSION

A. Comparison with other results: Sources of error

Table III and Fig. 6 compare the present measurement of R with some recent results. Boshier *et al.*²¹ quote the same error as our result. The other groups have larger errors. All now agree within the quoted errors. These Rydberg-constant determinations come from measurements of wavelengths of a wide variety of transitions: Balmer- α (2S-3P) by Zhao *et al.*,⁵ two-photon, high Balmer series members (2S-8D, 10D) by Biraben and co-workers,¹⁹ and two-photon Lyman- α (1S-2S) by Beausoleil *et al.*²⁰ and Boshier *et al.*²¹ All previous discrepancies are now resolved.^{5,20}

Tables I and III are helpful in comparing the major sources of variance in this paper, in other current measurements, and in planning for future experiments.

The previous experiment by Zhao *et al.*⁵ suffered from a statistical error of 5 parts in 10^{10} due to the large linewidth of the Balmer- α line. The current experiment uses the Balmer- β line, with its three times higher Q value (ratio of frequency to linewidth) to achieve the current, quoted error of 3 parts in 10^{10} . The other experiments all use two-photon transitions to achieve much higher Q values.¹⁹⁻²¹ However, the larger error (6 parts in 10^{10}) of Beausoleil *et al.* arises principally from a secondary wavelength standard with an error of 4 parts in 10^{10} . The error by Biraben and co-workers (6 parts in 10^{10}) arose mainly from relative wavelength measurements.

One notable conclusion follows from an examination of these tables. In almost every case, the weakest link in the chain of errors is the wavelength and frequency measurement, which involves a standard and measuring the wavelength ratio or the frequency interval to a hydrogen line.

At the present time, the uncertainty in the second-order Doppler effect is not significant. In the present experiment, with a beam at ~ 2850 K, the second-order Doppler effect introduces a small fraction (12%—see Table I) of the estimated variance. Merely cooling the

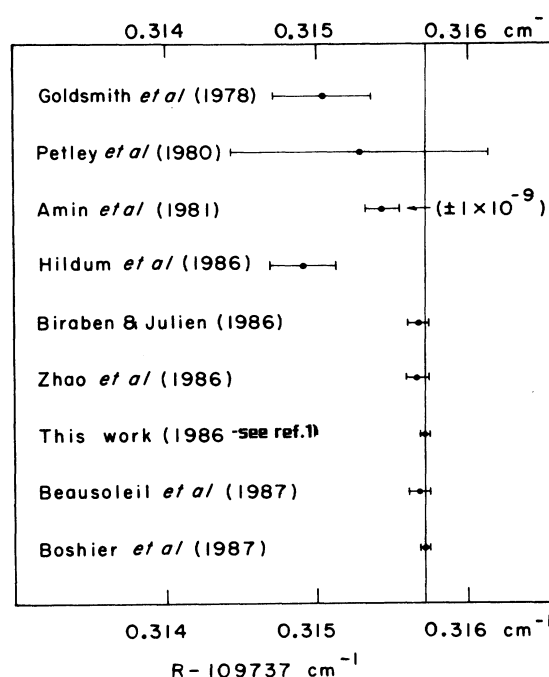


FIG. 6. Comparison of recent measurements of the Rydberg constant.

beam to room temperature would be adequate until precision of the order of 1 part in 10^{11} is required; cooling it to liquid-nitrogen temperatures would be adequate until the precision reaches a few parts in 10^{12} .

B. Need and possibility of higher precision in the Rydberg constant

One test of fundamental theory is the measurement of the Lamb shift of the ground state of hydrogen by subtracting the relativistically corrected, theoretical value of the 1S-2S wavelength (based on the Rydberg constant) from the experimentally measured 1S-2S value. Present-

TABLE III. Comparison of measurements of R , errors, and variances.

Group	Transition	$R - 10973\,731.50\text{ m}^{-1}$	Error	Standard	Variances (parts in 10^{20})			
					Relative wavelength or frequency measurement	Statistical	Other	Total
Zhao <i>et al.</i> ^a	2S-3P	0.069	0.007	3	10	25	5	42
Beausoleil <i>et al.</i> ^b	1S-2S	0.071	0.007	16	18	3		38
Biraben, Garreau, and Julien ^c	2S-8D, 10D	0.069	0.006	4	15	1	7	27
Zhao <i>et al.</i> ^d	2S-4P	0.073	0.003	3		2	2	7
Boshier <i>et al.</i> ^e	1S-2S	0.073	0.003	7	1		1	9

^aReference 5.

^bReference 20. "Other" sources of error are included in the statistical column.

^cReference 19. See note added at end of the present paper for a new result.

^dReference 1, present experiment.

^eReference 21.

ly, within the experimental error of 1 part in 10^4 in the Lamb shift, there is agreement with theory.²¹ The theoretical uncertainty is a factor of 10 smaller than the experimental error. A critical test of QED by this method of measuring the wavelength or frequency of the $1S-2S$ transition directly requires at least an order-of-magnitude improvement of the precision to which R is given.

The question arises as to how such an improvement could be accomplished. In all current investigations,^{1,5,19-21} the Rydberg constant is determined by a comparison of the wavelength or frequency of an atomic hydrogen line with a standard line, such as I_2 or Te_2 . The wavelength of these standards depends on the current definition of the meter, which is defined to be the distance in vacuum that light travels in $1/299\,792\,458$ of a second, where the second is defined to be exactly $9\,192\,631\,770$ periods of the ground-state hyperfine transition in ^{133}Cs . Thus frequency and wavelength are equivalent by the equation $\lambda\nu=c$. Secondary standards of wavelength in a particular frequency domain can be defined now by what is called "realization of the meter," which consists of measuring the frequency of a particular atomic or molecular line. Table IV shows the realization of the meter in various regions of the electromagnetic spectrum, that is, some points at which absolute frequencies and wavelengths have been measured.

The realization of the meter by a long frequency chain reaching from the Cs clock into the visible range^{22,23} by means of the He-Ne laser stabilized to the i th component of the $R(127)11-5$ line in $^{127}I_2$ to 1.6 parts in 10^{10} is the major limiting factor in the current precision in the Rydberg constant. The last link in this long chain reaches from the methane-stabilized He-Ne laser in the infrared at $3.39\,\mu m$ to the iodine-stabilized He-Ne laser at $633\,nm$ in the red. For this last link, two results can be compared: for the wavelength ratio,¹³

$$5.359\,049\,260\,6(11),$$

and for the frequency ratio,⁶

$$5.359\,049\,258\,9(9).$$

This realization of the meter in the optical domain by direct frequency techniques⁶ has not significantly improved the results over the wavelength comparison.¹³ We do not know of any major current efforts to improve these results. Therefore it seems likely that, in the near future, improvement in precision in the Rydberg constant will lie in infrared and/or microwave standards, where the realization of the meter is more precise (see Table IV).

C. Realization of the meter through the Rydberg constant: standards

If progress in the measurement of R_∞ continues, we anticipate a precision of 1 part in 10^{10} or better;¹ then R_∞ will be more precise than the realization of the meter in the optical domain. At that time, an additional method for realizing the definition of the meter in the optical region could be recommended.^{1,19} The wavelengths in vacuum of atomic hydrogen lines, as calculated by quantum electrodynamics and the Rydberg constant, can be used to realize the definition of the meter. This definition would allow various transitions in the hydrogen atom to complement the frequency chains, which in the past have linked the cesium clock with secondary wavelength standards, such as $^{127}I_2$ in the red or Te_2 in the blue.

The procedure might be carried out as follows.

(1) The Rydberg constant would be measured in the infrared or microwave region by direct multiplication of cesium to a hydrogen frequency or by use of secondary standards which exist at longer wavelengths (see Table IV).

(2) Secondary wavelength standards at other parts of the spectrum (such as the visible and ultraviolet) would be established by comparison of convenient lines of other atoms or molecules with atomic hydrogen frequencies.

(3) Comparison of these secondary standards with more than one atomic hydrogen line would act as a test of quantum-electrodynamic calculations. An example of this are the two experiments, one by Zhao *et al.*¹ and one by Boshier *et al.*²¹ and Barr *et al.*²¹ which form the following chain of comparison. Balmer- β at $486\,nm$ —Zhao

TABLE IV. Realization of the meter.

Standard transition	Frequency (MHz) Wavelength (μm)	Reliability (1 standard deviation)
^{133}Cs hfs ^a	$f = 9192.631\,770$	10^{-13}
H maser ^b	$f = 1420.405\,751\,7667$	0.6×10^{-12}
$CO_2:O_3O_4$ ^c	$f = 29\,054\,057.446\,66$	3.4×10^{-12}
He-Ne:CH ₄ ^d	$f = 88\,376\,181.608$ $\lambda = 3.392\,231\,397\,0$	0.44×10^{-10}
He-Ne: $^{127}I_2$ i^e	$f = 473\,612\,214.789$ $\lambda = 0.632\,991\,398\,1$	1.6×10^{-10}

^aReference 22.

^bReference 23.

^cReference 24.

^dReference 9.

^eReference 6.

et al.—He-Ne I_2 i at 633 nm—Barr *et al.*—Te₂ standard at 486 nm—Boshier *et al.*—frequency doubling to 243 nm and two-photon 1s-2s transition in H. The overall precision of this comparison was to 3 parts in 10^{10} , which just equals the absolute accuracy quoted by both groups.

In conclusion, the shortage of sufficiently accurate, absolute frequency standards in the optical region is an inconvenience to fundamental research, but can be overcome by a variety of wavelength and frequency intercomparisons. The Rydberg constant currently plays, and can continue to play, an important role in this area of basic research.

D. Comments upon future experiments

One might ask, what are the sources of errors in future experiments which aim to increase the precision of determinations of R to 1 part in 10^{10} or better? We suppose that the major errors in this experiment, frequency standard and statistical, can be reduced by a factor of 3 or

more. On the other hand, errors associated with the static, dynamic, and blackbody radiation-induced Stark effect will not go away easily. They promise to be the most troublesome sources of uncertainty as the principal quantum number increases.

Note added. After the completion of this article, Biraben and co-workers completed a remeasurement of the Rydberg constant by improvement of their technique. The error due to all sources, other than the standard, is 5 parts in 10^{11} . Their final result is

$$R_{\infty} = 109\,737.315\,714(19) \text{ cm}^{-1},$$

in excellent agreement with all recent measurements.²⁶

ACKNOWLEDGMENTS

This research was aided by National Science Foundation Grant No. NSF-PHY-8419105 and by support from the U.S. National Bureau of Standards. We thank Dr. Barry Taylor of NBS for helpful suggestions about our discussion of the fundamental constants.

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¹These results were first presented by Ping Zhao, W. Lichten, Zhi-Xiang Zhou, H. P. Layer, and J. C. Bergquist in *Advances in Laser Science—II* (Seattle, WA, 1986), Proceedings of the Second International Laser Science Conference, AIP Conf. Proc. No. 160, edited by W. C. Stwalley, M. Lapp, and G. A. Kenney-Wallace (AIP, New York, 1986), p. 402; see also P. Zhao, Ph.D. thesis, Yale University, 1986 (unpublished). This paper is an expanded version of Phys. Rev. Lett. **58**, 1293 (1987); also see Ping Zhao, W. Lichten, Zhi-Xiang Zhou, H. P. Layer, and J. C. Bergquist, in Proceedings of the Eighth International Conference on Laser Spectroscopy, Åre, Sweden, 1987 [in *Laser Spectroscopy VIII*, edited by W. Persson and S. Svanberg (Springer-Verlag, New York, 1988)].

²E. R. Cohen and B. N. Taylor, Committee on Data for Science and Technology Bulletin No. 63, 1986, Tables 1, 6, and 7. Certain constants, such as c , μ_0 , and ϵ_0 are exact by definition. Others, such as $\mu_e/\mu_B = 1 + \alpha_e$, $1 + m_e/m_p$, etc. achieve higher precision by adding 1 to a small measured quantity. If these redefined quantities are not classified as measured constants, then the Rydberg constant is the most accurately measured fundamental physical constant.

³For a more detailed historical summary see Ping Zhao, W. Lichten, Zhi-Xiang Zhou, H. P. Layer, and J. C. Bergquist, in Proceedings of the Eighth International Conference on Laser Spectroscopy, Åre, Sweden, 1987 [in *Laser Spectroscopy VIII*, edited by W. Persson and S. Svanberg (Springer-Verlag, New York, 1988)].

⁴S. R. Amin, C. D. Caldwell, and W. Lichten, Phys. Rev. Lett. **47**, 1234 (1981).

⁵P. Zhao, W. Lichten, H. P. Layer, and J. C. Bergquist, Phys. Rev. A **34**, 5138 (1986). This is an improved version of an earlier experiment (Ref. 4).

⁶D. A. Jennings, C. R. Pollock, F. R. Petersen, R. E. Drullinger, K. M. Evenson, J. S. Wells, J. L. Hall, and H. P. Layer, Opt. Lett. **8**, 136 (1983). For comparison with Ref. 13, the offsets

published in Ref. 9 were used.

⁷Michael Littman (private communication).

⁸T. W. Hänsch and B. Couillaud, Opt. Commun. **35**, 441 (1980).

⁹See the column on the new definition of the meter, Metrologia **19**, 163 (1984).

¹⁰B. R. Marx and W. R. C. Rowley, National Physical Laboratory Report No. MOM 51, 1981 (unpublished); W. R. C. Rowley and B. R. Marx, Metrologia **17**, 65 (1981).

¹¹P. R. Bevington, *Data Reduction and Error Analysis for the Physical Sciences* (McGraw-Hill, New York, 1969).

¹²M. W. Chase *et al.*, Joint Army Navy Air Force Thermochemical Tables, 3rd ed. [J. Phys. Chem. Ref. Data **14**, Suppl. **1**, 1358 (1985)].

¹³H. P. Layer, R. D. Deslattes, and W. G. Schweitzer, Jr., Appl. Opt. **15**, 734 (1976). This particular measurement was for the k peak, which is readily corrected to the i peak, or other hyperfine components. See also Ref. 9 and G. Bönsch, Appl. Opt. **22**, 3414 (1983) for confirmation and reference to several other confirming measurements.

¹⁴E. U. Condon and G. H. Shortley, *The Theory of Atomic Spectra* (Cambridge University Press, Cambridge, England, 1967), p. 136, Table 5⁵.

¹⁵For calculation of Voigt profiles and linewidths, see J. T. Davies and J. M. Vaughan, Astrophys. J. **137**, 1302 (1963). See A. Corney, *Atomic and Laser Spectroscopy* (Oxford University Press, New York, 1977), Chap. 8 for further discussion and references.

¹⁶J. Heberle, H. A. Reich, and P. Kusch, Phys. Rev. **101**, 612 (1956); **104**, 1585 (1956).

¹⁷W. Lichten, J. Opt. Soc. Am. A **2**, 1869 (1985).

¹⁸W. Lichten, J. Opt. Soc. Am. A **3**, 909 (1986).

¹⁹F. Biraben, J. C. Garreau, and L. Julien, Europhys. Lett. **2**, 925 (1986); F. Biraben and L. Julien, Opt. Commun. **53**, 319 (1985). For a similar proposal to the present one on the realization of the meter, see L. Julien, F. Biraben, and B. Cagnac, BNM Bulletin No. 66, 1986 (unpublished), p. 31. For an additional discussion of this proposal, see D. H. McIntyre and

- T. Hänsch, *Metrologia* **25**, 61 (1988).
- ²⁰R. G. Beausoleil, D. H. McIntyre, C. J. Foot, B. Couillaud, E. A. Hildum, and T. W. Hänsch, *Phys. Rev. A* **35**, 4878 (1987). All the following measurements by this group were significantly lower than the measurements in Refs. 1 and 5: T. W. Hänsch, M. H. Nayfeh, S. A. Lee, S. M. Curry, and I. S. Shahin, *Phys. Rev. Lett.* **32**, 1336 (1974); J. E. M. Goldsmith, E. W. Weber, and T. W. Hänsch, *ibid.* **41**, 1525 (1978); E. A. Hildum, U. Boesl, D. H. McIntyre, R. B. Beausoleil, and T. W. Hänsch, *ibid.* **56**, 576 (1986). Beausoleil *et al.* attributed the differences between their work and earlier measurements by Hänsch and co-workers to "chirp" caused by pulsed lasers. However, the work by Goldsmith *et al.* was done with a cw laser.
- ²¹M. Boshier *et al.*, in *Proceedings of the Eighth International Conference on Laser Spectroscopy*, Åre, Sweden, 1987 [in *Laser Spectroscopy VIII* (Springer-Verlag, New York, 1988)]. The quoted error, presented in that paper, of 5 parts in 10^{10} has been reevaluated to be 3 parts in 10^{10} in M. G. Boshier *et al.*, *Nature (London)* **330**, 463 (1987). For calibration of the wavelength standard used by this group, see J. R. M. Barr, J. M. Girkin, A. I. Ferguson, C. P. Barwood, P. Gill, W. R. C. Rowley, and R. C. Thompson, *Opt. Commun.* **54**, 217 (1985).
- ²²*Proceedings of the Sessions of the 13th General Conference of Weights and Measures, Paris, 1968* (Gauthier-Villars, Paris, 1968), p. 103; *Proceedings of the Sessions of the 14th General Conference of Weights and Measures, Paris, 1971* (BIPM, Sevres, 1972), pp. 77 and 78; both reprinted in *Time and Frequency: Theory and Fundamentals*, Natl. Bur. Stand. (U.S.) Monograph No. 140, edited by Byron E. Blair U.S. GPO, Washington, D.C., 1974).
- ²³H. Hellwig *et al.*, *IEEE Trans. Instrum. Meas.* **IM-19**, 200 (1970); L. Essen *et al.*, *Nature (London)* **229**, 110 (1971).
- ²⁴A. Clairon, B. Dahmani, A. Filimon, J. Rutman, *IEEE Trans. Instrum. Meas.* **IM-34**, 265 (1985); A. Clairon and B. Dahmani, *Bull. Bur. Natl. Metrologie (Paris)* **16**, 33 (1985).
- ²⁵G. W. Erickson, *J. Phys. Chem. Ref. Data* **6**, 831 (1977).
- ²⁶M. Allegrini, F. Biraben, B. Cagnac, J. C. Garreau, and L. Julien, in *Proceedings of the Conference on the Hydrogen Atom, Pisa, 1988* (Springer-Verlag, New York, in press); (private communication).