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IMPACT OF ATMOSPHERIC NON-RECIPROcity ON SATELLITE TWO-WAY  
TIME TRANSFERS

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Abstract

In the two-way time transfer method no knowledge is necessary concerning the locations of the slave and the master clock stations relative to the satellite nor does one need any knowledge of the atmospheric delay. In this method, signals are exchanged back and forth, via a satellite transponder, between the master and slave stations. Because of the two-way signal exchanges the path delay cancels out in the relevant calculations leaving only the difference in the clock readings at the master and slave stations. (This statement assumes that all cable delays and other associated equipment delays are known and accounted for in the calculations.) However, the cancelling of path delay is critically dependant on the assumption that the signal path delay from master to satellite to slave is equal to the delay from slave to satellite to master--that is, the paths are reciprocal.

In actual practice this is hardly ever the case. Generally speaking the uplink signal frequencies to the satellite from both master and slave to the satellite differ from the downlink frequencies from the satellite to the master and slave stations so that the reciprocity assumption does not hold over the ionospheric portion of the path. Thus one must make calculations to see if the lack of reciprocity is significant in comparison to the degree of timing accuracy required. It is possible, under some circumstances to measure the degree of non-reciprocity directly. This is because the signals relayed by the satellite are generally available at both the slave and master stations simultaneously.

Introduction

The continuing improvement in frequency standards, and consequently clocks, creates a parallel need to perfect the means for comparing these devices at numerous locations throughout the world. In the past, as now, electromagnetic signals are the mainstay for these comparisons.

In the first few decades of this century, clock stability was not better than path stability--the variation in arrival time--of radio signals, so that, for all practical purposes, remotely located clocks could be compared with little or no loss due to signal path delay instability. However, with the introduction of atomic frequency standards in the 1950's, radio signals propagated over long distances through the ionosphere no longer had sufficient path stability to permit unimpaired clock comparisons. To some extent these problems were reduced by averaging out path instabilities--the averaging was usually achieved by integrating a signal for long periods of time--often weeks and even months. Loran-C and VLF navigation signals used in timing were and still are, treated in this way.

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Another, less used scheme, entails averaging several signals, referenced to the same clock and broadcast simultaneously from the same location. Thus one could, for example, average WWV time signals on several different carrier frequencies at the same time. However, even in the best case, these schemes do not produce immediate results and often require considerable bookkeeping. Ultimately, path delay variations with periods of a year or more make the averaging approach impractical.

Fortunately, the introduction of atomic frequency standards in the 50's was accompanied by the communications revolution--arriving with the launch of the first artificial earth satellite. With communication satellites one could relay information reliably at high rates over intercontinental distances. The high rates were a consequence of the satellites' broadband communication channels and the reliability was due both to the technical excellence of satellite technology and to the frequencies at which the satellites operated. Whereas normal, long-distance-terrestrial-communication systems operate at frequencies which are low enough to reflect radio signals between the earth and the ionosphere, communication satellites operate at frequencies high enough for their signals to easily penetrate the ionosphere. That is, satellite signals pass through the ionosphere little affected by the ionospheric irregularities that afflict signals propagated around the world by the ionosphere.

The nearly vacuum line-of-sight path stability coupled with the large channel bandwidths was just what the timing community needed to greatly improve clock comparisons over great distances. The latter supported the transmission of very narrow pulses (or more recently their sophisticated cousins--spread spectrum signals) thus insuring that time-of-arrival measurements could be made with great precision and the former insured that path delay variations did not mask the phase variations inherent in the clocks--the phenomena we want to study.

The first satellite clock comparisons were carried out in the early 1960's with the Telstar communication satellite followed by numerous comparisons in the ensuing years. However due to the expense and the complexity, both political and technical, of carrying out these time transfers no routine comparisons were established. It is only today that we are beginning to see the development of routine comparisons over communication satellites as a result of the availability of both more economical ground equipment and modest channel costs. With this greater availability two-way satellite time comparisons are now reaching a mature stage where many factors, once essentially ignored, are now being scrutinized at a level not deemed necessary in the past. In this paper I shall focus on one of these factors--the reciprocity, or non-reciprocity--of atmospheric path delay through the atmosphere.

### Background

Most communication satellites are in orbits well above the ionosphere so a signal from earth to satellite and return travels through the bulk of the ionized medium surrounding the earth as well as, of course, the lower neutral atmosphere lying between the surface of the earth and the bottom of the ionosphere, near 60 kilometers.

Several things impact signals in both the neutral and ionized regions. First, there are large scale structures which lead to (1) changes in the propagation velocity and (2) rotation of the plane of polarization of the radio signal. Second, the signal may be partially or completely absorbed by various mechanisms in the intervening media, and finally the signals may be scattered by small scale turbulent structures in the media leading to what is usually termed scintillation. In this paper I shall center on the first effect.

### Phase Delay

In the general case, a signal propagating through the atmosphere will be both absorbed and delayed. As an illustration, consider a plane wave represented by the equation

$$\vec{E}(z, t) = \vec{E}_0 e^{j(kz - 2\pi\nu t)}. \quad (1)$$

Here  $k$  is the propagation constant  $2\pi\nu/c$  in free space,  $c$  is the speed of light,  $\nu$  the signal frequency, and  $E_0$  the field amplitude.  $n$  is the complex refractive index equal to  $n_R + jn_I$ . When the imaginary part of the refractive index is negative, the medium is dissipative and the wave decays exponentially. It is usual to define the power absorption coefficient as

$$\alpha = \frac{4\pi\nu}{c} n_I.$$

In the atmosphere the propagation constant is

$$kn_R = \frac{2\pi\nu}{c} = \frac{2\pi\nu}{v_p},$$

where  $n = n_R$  and  $v_p$  is the phase velocity,  $c/n_R$ . As a rule of the thumb the phase velocity is about 0.03 percent less than  $c$  in the lower neutral atmosphere.

Here we are concerned primarily with how much extra time  $\Delta t$ --more accurately the extra phase path time, as we shall discuss in more detail later--it takes the signal defined by Equation (1) to travel through a medium with refractive index  $n$  as compared to the time it takes to travel through free space. In other words we want to evaluate the quantity

$$\Delta t = \frac{1}{c} \int_{\text{path}} (n_R - 1) dz.$$

### Reciprocity in the Lower Neutral Atmosphere

Since  $(n - 1)$  in the neutral atmosphere is a very small number it is common to introduce the refractivity  $N$  defined by

$$N = 10^6 (n_R - 1).$$

With this definition we may write the excess path length,  $L$ , as

$$L = 10^{-6} \int_{\text{path}} N(z) dz.$$

Thus the problem of evaluating non-reciprocity in the neutral atmosphere reduces to knowing--experimentally, theoretically or some combination thereof--appropriate expressions for  $n$  (or equivalently for  $N$ , the refractivity).

An often used empirical expression for  $N$  in the neutral atmosphere at radio frequencies is

$$N = 7790 \frac{P_o}{T} - 6480 \frac{P_v}{T} + 377 \times 10^5 \frac{P_v}{T^2}, \quad (2)$$

which is sometimes referred to as the Smith-Weintraub Equation [1]. Here  $T$  is the temperature in Kelvins,  $P_o$  the partial pressure of dry air, and  $P_v$  the partial pressure of water vapor in pascals. (1 millibar = 100 newtons per square meter = 100 pascals; 1 atmosphere = 1013 millibars).

The first two terms are related to the displacement polarizations (due to the electric field of the penetrating signal) of the gaseous constituents in the air and the third term is due to the permanent dipole moment of water vapor. Equation (2) is accurate to less than 1% at frequencies below 100 GHz.

What concerns us here is the form of Eq. (2.) First we notice that it is dependent on the partial pressures of water vapor and the gaseous constituents of air and the temperature. More important we notice that there is no dependence on the frequency.

In general, ref [2], the up and down frequencies in two-way time transfers are not the same and therefore a frequency dependent term in the Smith-Weintraub would introduce non-reciprocity. That is, the excess path,  $L$ , the length in the two directions would not be the same and thus not cancel out. Happily, this is not the case.

Before we leave the subject of reciprocity in the neutral atmosphere, it is useful to consider the situation a little more carefully since, strictly speaking,  $N$  does depend, if ever so slightly, on frequency.

At radio frequencies the refractivity due to water vapor in the neutral atmosphere is about 20 times greater than it is in the near infrared or optical regions--certainly a frequency-dependent effect. At first glance this result may seem strange until we consider the underlying mechanisms leading to the Smith-Weintraub Equation.

If we look at the resonant spectrum of the gases making up the neutral atmosphere we find that there are a great many lines at frequencies well above 10 GHz where most satellite two-way time transfers are made today. Figure 1 schematically shows one such line. The figure shows that the tail of the real part of the refractive index approaches 1 as  $\nu$  approaches  $\infty$  but does not quite reach 1 as  $\nu$  approaches 0. In other words all the spectral lines above the range of frequencies involved in the time transfer contribute to the refractive index as illustrated schematically in Fig. 2. Here we see the tails of many lines adding together to produce the refractive index at the frequencies of interest. Thus, strictly speaking  $N$  does have a frequency dependence at radio frequencies.

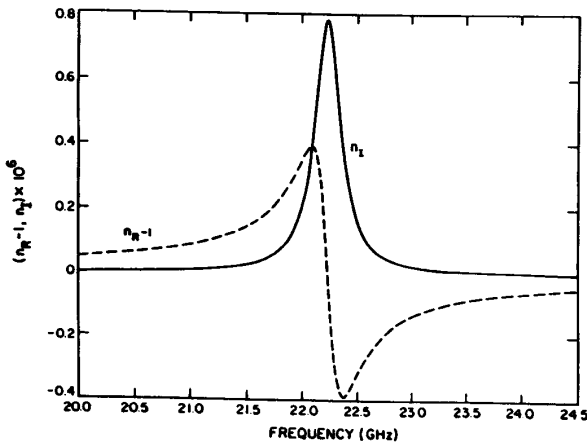


Fig. 1 Refractive index variation of water vapor line with frequency.

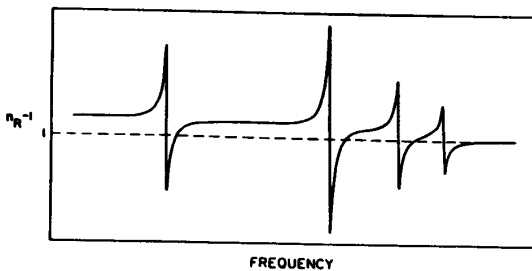


Fig. 2 Low frequency refractive index variation due to summation of tails of higher frequency lines.

How important is this dependence for present-day, two-way time transfers? The answer is that the dependence produces a non-reciprocity that is well below the noise floor, about 0.5 ns, in present two-way exchanges. However, we can imagine circumstances where the effect is not negligible.

Under normal atmosphere conditions a signal propagated at 10 GHz to a satellite directly overhead encounters an extra delay of about 8 ns due to the gaseous part of the atmosphere and about 1 ns more due to water vapor. An optical signal taking the same path undergoes essentially the same 8 ns delay due to the atmospheric gases but only about 0.05 ns due to the water vapor. (This difference is largely due to the fact that there aren't a large number of water vapor line tails adding together to affect the refractive index at optical frequencies.) In other words, assuming that the Smith-Weintraub Equation holds at optical frequencies leads to an error of about 1 nanosecond for propagation in the direction of the zenith. At 5 degrees elevation angle the error increases to about 10 ns.

Thus, two-way time transfers involving both optical and microwave signals could involve non-reciprocities near and above the 0.5 ns noise floors of present-day, two-way exchanges. Although I know of no such recent exchanges or any planned for the future, it is a point to keep in mind.

### Absorption in the Troposphere

For all practical purposes atmospheric absorption of electromagnetic signals in the neutral atmosphere can be ignored unless broadcasting signals in or very near an absorption line--an unlikely situation. However, for the sake of completeness we shall consider such a possibility.

Absorption itself will, of course, not lead to non-reciprocity. However it may lead to the appearance of non-reciprocity.

Consider a pulse whose initial shape, shown in Fig. 3, propagates through some absorbing medium where the absorption is frequency dependent. In this case it will emerge altered in shape as the figure shows. We will assume that the time reference point on the transmitted pulse, before absorption, is at the half power point also shown in the figure. The question is, "After the pulse has been distorted due to absorption, how does one determine the location of the time reference point on the received pulse?"

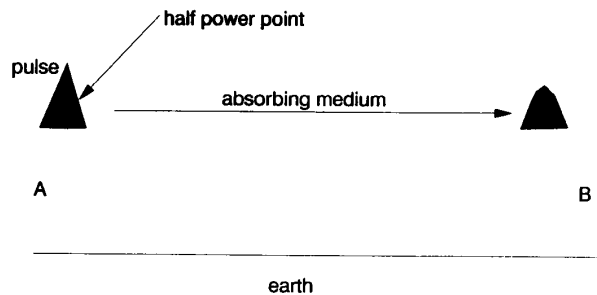


Fig. 3 Pulse distortion due to absorption.

There is no easy answer to this question. However, whatever method is chosen to define the time reference point at one receiver, the same procedure must be used at the other receiver. If different criteria are used then we will have the appearance of non-reciprocity.

In the next section, when we consider radio signals propagated through the ionosphere, I shall pick up this topic again.

### Reciprocity in the Ionosphere

As in the neutral atmosphere, reciprocity, or the lack thereof, in the ionosphere depends on the form of the refractive index for an ionized medium imbedded in a magnetic field.

Before giving the appropriate expression for the refractive index in the ionosphere let's consider the important underlying phenomena for a moment.

As an electromagnetic signal propagates through the ionosphere the electrons in the ionosphere "see" the signal's fluctuating electric field and will thus move in some manner under the signal's influence. In addition, the detailed orbits of the electrons are constrained by the magnitude and configuration of the earth's magnetic field.

That this is indeed the case is demonstrated by the Appleton formula

$$\mu^2 = 1 - \frac{X}{1 - iZ - [Y_T^2/2(1-X-iZ) \pm \sqrt{Y_T^4/4(1-X-iZ) + Y_L^2}]}$$

for the square of the refractive index  $\mu$  in the ionosphere.

The term  $Z$  is proportional to the collision frequency between the electrons and the other particles,  $Y_L$  is the magnitude of the earth's magnetic field in the direction of propagation of the signal, the longitudinal component,  $Y_T$  is the magnitude of the earth's magnetic field transverse to the signal direction, and  $X$  is proportional to the ratio of electron density,  $N_e$ , to the square of the signal frequency,  $\nu$ .

One thing we notice immediately, in contrast to the Smith-Weintraub expression, for the radio wave refractive index in the neutral atmosphere, is that the refractive index in the ionosphere is frequency dependent. In fact it varies with the reciprocal of the frequency squared. Thus, any two-way system operating with different up and downlink frequencies will not have reciprocity. Whether this is important or not depends on the desired accuracy of the time comparison. I shall return to this point later.

The other thing we notice in the refractive index formula is that it contains terms involving the longitudinal and transverse components of the earth's magnetic field. Here is a possible source of non-reciprocity. However, looking more closely, we see that all terms involving the magnetic field are even powers of the longitudinal and transverse components so that a reversal in propagation direction does not alter the value of the refractive index. That is, the earth's magnetic field does not introduce any non-reciprocity.

#### The Group Refractive Index

As we have seen, the refractive index in the ionosphere is frequency dependent. Because of this we must reconsider our notion of signal delay,  $L$ , as we defined it earlier.

Any finite duration signal can be decomposed into a packet of constituent signals with different frequencies--the well known Fourier decomposition. When the refractive index varies with frequency--when we have a dispersive medium--each Fourier component travels with a different phase velocity,  $v_p$ . In a dispersive medium we are interested in the speed with which the envelope of this group of Fourier components travels--not the speed of each individual component. Also, from a physical point of view, the speed of the envelope, the group velocity,  $v_g$ , is the speed with which the signal energy travels.

It is easy to show that the envelope travels with group velocity [3]

$$v_g = 2\pi\nu/dk.$$

Since the group refractive index is

$$\mu' = c/v_g = (c/2\pi)dk/d\nu = \frac{d}{d\nu}(\nu\mu),$$

we can find the group refractive index for the

ionosphere by substituting the Appleton formula in the above expression.

However, before we do this we may simplify the Appleton formula because in the gigahertz frequency range collisions between the electrons and the other atmospheric constituents can be ignored. With this simplification and after some manipulation we obtain for the Appleton group refractive index

$$\mu' = \mu + \frac{2}{2(1-X) - Y_T^2 - Y_L^4 - 4Y_L^2(1-X^2)} \left[ 1 - \mu^2 - X^2 + \frac{(1-\mu^2)(1-X^2)Y_T^2}{Y_T^4 + 4Y_L^2(1-X)^2} \right].$$

As before we see that all terms involving the magnetic field are raised to even powers, so 180° changes in the direction of signal propagation do not effect the group refractive index and thus there are no non-reciprocity effects involving magnetic fields.

In summary, then, in the ionosphere a magnetic field does not introduce a reciprocity problem, but frequency does.

As a final remark in this section recall that we calculated the delay,  $L$ , in the neutral atmosphere using the appropriate expression for the phase refractive index whereas in the ionosphere we need to use the group refractive index. The reason we use the phase refractive index in the neutral atmosphere and not the group refractive index is that in the neutral atmosphere the phase refractive index is not frequency dependent. In other words, the phase refractive index and the group refractive index are identical in the neutral atmosphere.

#### Ionospheric Non-Reciprocity at 14.300 and 12.007 GHz

The NIST-USNO two way satellite time transfer system ref [2] operates at 14.300 GHz on the uplink and 12.007 GHz on the downlink. This section presents some results indicating the magnitude of the ionospheric non-reciprocity we can expect at these frequencies.

In the gigahertz frequency range we can further simplify the Appleton formula to obtain

$$\mu' = 1 - K_1 \frac{I_N}{\nu^2} - \frac{K_2 I_N B \cos \theta}{\nu^3} - [K_3 I_N^2 + K_4 I_N B^2 (1 - \frac{1}{2} \sin^2 \theta)] \frac{1}{\nu^4} + \dots$$

Here  $I_N$  is the total integrated electron density along the propagation path,  $\theta$  is the angle between the signal direction and the earth's magnetic field  $B$ , the  $K$ 's are constants whose values depend on the units employed, and  $\nu$  is signal frequency. As the equation shows the magnetic field contribution scales as the third and higher order powers of the inverse signal frequency so that we may neglect the magnetic field terms for the analysis that follows.

In these calculations we assume that the total integrated electron content in a vertical column is  $1 \times 10^{18}$  electrons per meter squared. This is a

large value, although near sunspot maximum, not an unheard of value.

At angles of propagation other than the zenith direction the path delay through the neutral atmosphere increases approximately as

$$\left[ \csc[(E^2 + 20.3^2)] \right],$$

where E is the elevation angle in degrees.

Using this approximation Fig. 4 shows the residual non-reciprocity for up and downlink frequencies of 14.300 and 12.007 respectively. As the figure shows, at low elevation angles, the non-reciprocity approaches the noise floor of the present two-way exchanges. However, if history is a reliable guide, the noise floor with better techniques will continue to diminish and finally be of the same magnitude or less than the residual non-reciprocity.

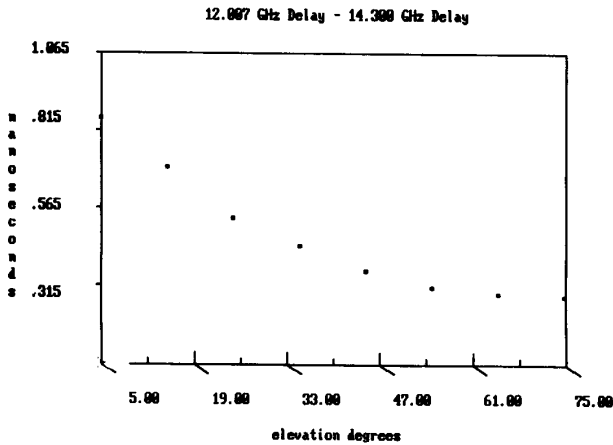


Fig. 4 Path delay non-reciprocity for 12.007-14.300 GHz link corresponding to extreme total integrated electron content.

Of more immediate interest is the fact that many communications satellites operate in the C Band where the uplink frequency is 6 GHz and the downlink frequency 4 GHz. Figure 5 shows the non-reciprocity variation with an elevation angle that we would expect with this pair of frequencies. We have again assumed a total electron content corresponding to daytime maximum sunspot number.

Here we see that the non-reciprocity is well above the measurement noise floor even at high elevation angles. Under more normal circumstances we would expect all the y axis values in this figure to be reduced by about one order of magnitude. But even then we are still above the measurement noise floor at low elevation angles and approach the noise floor as the path direction approaches the zenith.

These results suggest that non-reciprocity should be scrutinized when communications satellites operating at C Band are used in two-way time exchanges. In the next section I suggest a procedure that might be applied to correct for ionospheric delay in circumstances where it cannot be ignored.

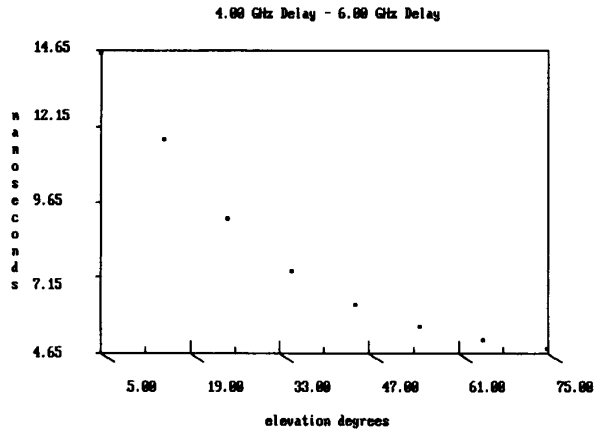


Fig. 5 Path delay non-reciprocity for 4.00-6.00 GHz link corresponding to extreme total integrated electron content.

#### Measuring the Non-Reciprocity

More often than not in two-way time exchanges the up and downlinks frequencies are not under the control of the participants in the exchange but are determined by available satellites. Thus, there may be occasions where non-reciprocity effects are not below the noise floor of the measurement--as the discussion of C Band communication satellites in the previous section shows. In a more extreme case one link might be in the VHF range and the other in the microwave region.

Under some circumstances it may be possible to measure the ionospheric path delay if the signals relayed by the satellite can also be received at the sites initiating the signals. The idea explored here is a variation on the two frequency scheme developed for measuring the ionospheric delay for the Global Positioning Systems (GPS) signals.

Consider the situation shown in Fig. 6. As we have seen the group velocity in the ionosphere is frequency dependent so that the extra delay due to the ionosphere at frequency,  $\nu$ , is of the form

$$T = \frac{KI_N}{\nu^2}$$

where K is a constant depending on the units and  $I_N$ , as before, is the total integrated electron content along the path from A to the satellite.

Consider two signals at  $\nu_1$  and  $\nu_2$  traveling to the satellite from location A and then transponded back to A at frequencies  $\nu_3$  and  $\nu_4$ . If, on the uplink, two pulses are transmitted (simultaneously, one on the carrier at  $\nu_1$  and the other on the carrier at  $\nu_2$ ), from A, they will arrive at the satellite separated in time by an amount,

$$\Delta T_1 = KI_N \left[ \frac{1}{\nu_1^2} - \frac{1}{\nu_2^2} \right]$$

as shown schematically in Fig. 6.

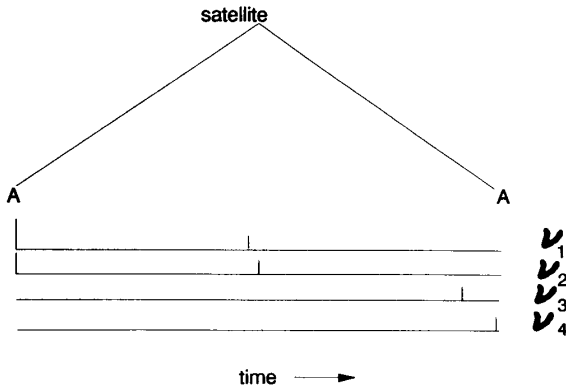


Fig. 6 Schematic representation of pulse delay spreading on transponder up and downlinks to the same location.

These same two pulses transponded back on the downlink signals at frequencies  $\nu_3$  and  $\nu_4$  will arrive back at A separated by an amount.

$$\Delta T_2 = \Delta T_1 + KI_N \left[ \frac{1}{\nu_3} - \frac{1}{\nu_4} \right]$$

which we can rewrite in the form

$$\Delta T_2 = KI_N \left[ \frac{1}{\nu_1} + \frac{1}{\nu_3} - \frac{1}{\nu_2} - \frac{1}{\nu_4} \right]$$

Then solving for the total integrated electron content,  $I_N$ , we have

$$I_N = \frac{\Delta T_2}{K \left[ \frac{1}{\nu_1} + \frac{1}{\nu_3} - \frac{1}{\nu_2} - \frac{1}{\nu_4} \right]}$$

Now, knowing the total electron content we can determine the ionospheric signal path delay at any frequency we desire.

In a practical case the two uplink signals at frequencies  $\nu_1$  and  $\nu_2$  could be at the upper and lower frequency limits of the uplink transponder and similarly  $\nu_3$  and  $\nu_4$  could be at the upper and lower frequency limits of the downlink transponder. Of course, whether this scheme would work in practice depends on a number of factors including the transponder characteristics and the satellite transmitted power.

As a practical illustration of the method discussed here consider a C Band communication satellite operating on 4 and 6 GHz. Typically, the transponder bandwidth on these satellites is 30 MHz so we shall assume this figure for the frequency separation between  $\nu_1$  and  $\nu_2$  on the uplink and  $\nu_3$  and  $\nu_4$  on the downlink. Again, under daytime,

sunspot maximum conditions Fig. 7 shows the total differential delay,  $\Delta T_2$ , we would expect to find for elevation angles ranging from 5 to 45°. These results suggest that, with care, we can measure the total integrated electron content for C-Band transponders in circumstances where it is important.

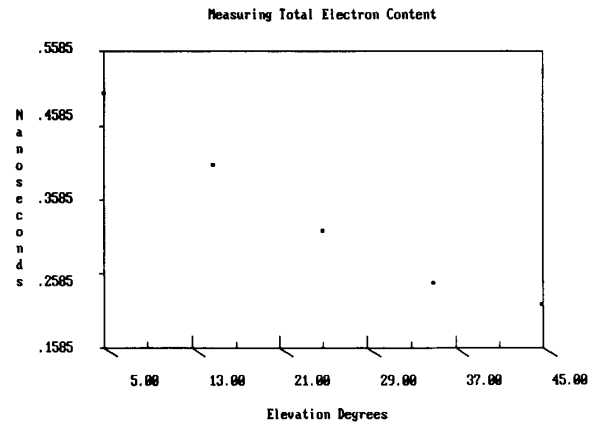


Fig. 7 Variation of pulse delay spreading,  $\Delta T_2$ , with elevation angle of the satellite. The figure corresponds to extreme total integrated electron content for up and downlinks of 6.00 and 4.00 GHz respectively.

#### Absorption in the Ionosphere

As stated earlier, absorption in the ionosphere is produced by free electrons colliding with neutral particles. At gigahertz frequencies we have been considering absorption effects may be ignored. However, for the sake of completeness I shall mention one circumstance under which absorption can be a problem.

The problem arises in connection with two-way exchanges involving low frequency signals propagated over long paths by the ionosphere.

At frequencies of 10 kHz and lower, where the wavelength is 30 km and longer, the actual orbits of the electrons as they collide with neutral particles becomes important in computing absorption. Detailed analysis shows that the electron orbits have a smaller radius in one direction of propagation than they do in the reversed direction. The net effect is that for one direction there is more absorption of the wave than there is in the other. The effect is most pronounced for east-west propagation paths where the absorption is greater for signals propagated to the west than it is to the east.

Here again we can have the kind of problem mentioned earlier in the section on absorption in the troposphere. That is when the pulse shape is altered due to absorption we can no longer be sure where the time reference point is. The problem is compounded for low frequency waves reflected to earth by the ionosphere because now the absorption depends on the direction of propagation. Now not only are the pulse shapes altered by absorption but they are altered in different ways depending on the direction of propagation, thus magnifying the problem of identifying the time reference point on the received wave.

### Conclusions and Summary

For present two-way satellite time transfer systems operating in the 12.00-14.00 GHz frequency range, propagation non-reciprocity is not ordinarily a problem. Sample calculations show that under typical conditions non-reciprocity effects are about an order of magnitude below the 0.5 ns noise floor of current systems. However, at low elevation angles when the integrated electron density is unusually high, daytime sunspot maximum conditions for example, non-reciprocity begins to approach the two-way noise floor.

For two-way satellite time transfers in the 4.00-6.00 GHz region (C-Band), path non-reciprocity cannot be ignored when the electron content is unusually high at any elevation angle and can be a problem at low elevation angles even under normal conditions.

Under certain circumstances it may be possible to correct for ionospheric non-reciprocities using a system analogous to the GPS procedure for correcting for ionospheric delay.

### Acknowledgement

I thank D. W. Hanson and W. J. Klepczynski for several comments and suggestions.

### References

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