THIRD INTERNATIONAL TIME SCALE ALGORITHM SYMPOSIUM*

David Allan

National Institute of Standards and Technology

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*The following are excerpts from the proceedings

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Opening Address

Dr. Bernard Guinot

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One of the roles of the talk to open a session is to demonstrate that what we will do is quite important. Of course you already know the importance of the subject since you are here. Nevertheless a confirmation that we are performing essential duties is always satisfactory, and it helps in warming up the engine.

There has been contrasting opinions concerning the algorithms for time scales.

For instance, some years ago, our colleague from NRC, Costain, was somewhat skeptical. He told me "have a good clock and don't care for algorithms." It might be a good solution, but it is not a natural one. The natural evolution, both for physical and psychological reasons, is that when you have a good clock, you manage to get a second one, then a third one, and you are all faced with the algorithms. On the opposite, Barnes told me that if he can gain 10% by statistics, it saves the cost of one clock over 10 to achieve the same performances, which is by no means negligible.

At the beginning of the era of atomic time, the algorithms for time scales were essentially machines which avoided phase and frequency jumps when clocks failed or when new clocks were entered in the system. Nowadays, to maintain the stability in spite of modifications of the clock ensemble, which includes change in the weighting, remains an essential role of the algorithms. But we have recognized that the result is strongly dependent on the quality of the prediction of the clock frequency.

We have also recognized that there is no unique ideal time scale algorithm. We have to specify first what the criteria of the time scale that have to be fulfilled: frequency accuracy, stability for some given sample time. As I believe that this is an important point, I would like to illustrate by two examples how the criteria may differ.

There are many systems where remote clocks must be maintained in synchronization within stated tolerances. Examples: navigation systems such as GPS, master clocks of time services,... The most convenient method consists in maintaining the synchronism with a master clock of the system. It is evident that the master clock must be more stable than any other clock of the system. More precisely the stability of the master clock must be optimized for a sample time which is of the order of the re-synchronization by absolute means, which are anyway needed. An additional requirement is the reliability of the master clock. This is clearly a domain where we have to rely on an ensemble of clocks and a well adapted algorithm.

I will spend a little more time on the second example which illustrates the need of the very long term stability, which is often loosely referred to as "uniformity", in celestial dynamics.

In this type of research, we observe space coordinates (angles, distances) as a function of time read on a realized time scale. The goal is to fit later a theory. Limitations are the errors of the dimensional measurement and the departure of the realized time scale from ideal. The important point is that the errors of dimensional measurement progressively decrease as a result of improvement of the observational techniques. At the same time the time error increases indefinitely because atomic time is the result of integration of imperfect frequencies. Therefore it will primarily happen that the atomic time error will exceed what should correspond to the dimensional error.

In current solar system dynamics this crowning point appears sufficiently remote, even without progress of the atomic clocks, may be in centuries, so that we feel reasonably comfortable.

But Backer (1982) discovered the first millisecond pulsar and we realized that our crowning point had suddenly jumped to a year, maybe less, after the start of precise pulsar timing. We have to be aware of the importance of this discovery for the time metrology.

Of course, the progress in long term stability needed by pulsar studies is not within the reach of better algorithms: we need better clocks. Nevertheless we cannot change the clock data: but we can use them in an optimum way and the gain is not marginal. This gives life to a new activity: the reprocessing of past data with appropriate algorithms, for which it might be possible to use multi-weight approaches which could use all the clocks in their domain of excellence.

Concerning the algorithms themselves, I would like to paraphrase a famous sentence of Sully, unfortunately I cannot translate it into English "La fourage et pastourage sont les deux mamelles de la France". I will say that prediction and weighting are the two main features of the time scale algorithms. But it appears that with new methods, such as the Kalman filtering, prediction and weights do not appear explicitly anymore. They should nevertheless exist and I believe that it is important to understand what they really are in order to avoid misusing these powerful tools.

I am also convinced that the classical algorithms still require further studies and may offer improved possibilities.

I would also like to mention that the establishment of time scales is not the only domain where stability and accuracy algorithms are realized. Similar problems occur with series of measurements in astronomy and geophysics: for instance, the evaluation of the motion of the terrestrial pole from many observatories. Maybe it was not so crucial than with time, but it may become more important. Our community of time scale algorithms designers is small; it might be useful in the future to share our experience with other researchers in nearby domains.

I personally look forward with the greatest interest the discussions of these two days.

I would like to express to our program chairman David Allan, our thanks for having so timely initiated the 3rd Symp. on time scale algorithms. The IENGF has kindly accepted to host our symposium. I wish to express to them our gratitude to Prof. G.F. Micheletti, Special Commissioner of the Institute for putting such a pleasant meeting place at our disposal and for his welcome address and to Dr. Galliano, local coordinator, and his staff, for the charge of the local organization.

J.S. Boulanger, NRC

Status and Plans for the NRC Primary Cesium and for Synchronizing UTC(NRC)

Notes: Plan to use Hydrogen, Commercial Cs and Primary Cs. over the next year and test algorithm. Overlap for approximately 1 year. UTC(NRC) will continue from Cs V until algorithm version has been well demonstrated to be better. Cs VIB will be an experimental clock. Cs VIC provided input to GPS receiver after July failure of Cs V for two months (2030 UTC, 31 Aug. 1988 Cs V began again to provide the reference).

L.A. Breakiron, USNO

Progress Report on the Time Scale Evaluation Project at USNO

The Time Service Department of the U.S. Naval Observatory has begun a project of research aimed at reevaluating the time–scale algorithm currently in use for the processing of cesium clock data. In particular, our linear algorithm is being compared to higher-order and ARIMA–type algorithms. Also, our unweighted, filtered scheme is being compared to more sophisticated filtering and weighting schemes involving the Allan variance. Increased attention is being paid to environmental factors. A report summarizing the results to data will be presented.

Notes: 40 Cs clocks in six (one with humidity control) vaults. Found annual absolute humidity dependence of either sign. Found temperature coefficient. J45 series tended to be positive relative to option 004. Looked at non equal weighting approach. Used weights at 6 g values and constructed 6 scales plus the regular UT(USNO) algorithm (0,1 weights). H–maser, used in short term and TAI for long term reference: g = 1 hr., 5 hr., 1 d, 5 d, 25 d, 50 d. g = 5 d gave some marginal improvement. Fit a parabola to part of the data and a straight line to the rest. 37% of Δy changes occurred at maintenance periods. 55% of the time something was changed the variance changed. Tested Percival ARIMA algorithm with Robust statistics, and Hampel's Psi function. Two more vaults being built with humidity control.

F. Cordara, V. Pettiti, P. Tavella, IEN

Status Report and Future Trends of the IEN Time Scale

The time scale UTC(IEN) of the Istituto Elettrotecnico Nazionale (IEN) of Torino, Italy, has been realized since 1976 by means of an industrial cesium clock whose rate has been corrected with a phase microstepper to maintain UTC(IEN) in close agreement with UTC.

In recent years, the analysis of the influence of the environmental parameters on the frequency of the atomic clocks maintained at IEN allowed to improve the long term stability of the time scale adopting a compensation of the seasonal rate of changes of some clocks.

Further steps towards a more uniform and reliable time scale generation will be possible adopting suitable algorithms for a small clock ensemble. Some preliminary results obtained using a different weighting criteria and future trends for the IEN time scale realization are analyzed.

Notes: GPS comparison started mid 1985. Last year micro–stepper control was added. Ser. 893 004 Cs with 12 years of life. TA2 is NBS algorithm, target is 20 days. TA1 is simple average. TA1 is better in long-term, TA2 in short term. τ_{min} = approximately 2 weeks, at $\approx 3 \times 10^{-14}$. Temperature control at 0.5° C; plan to control humidity to 5%. Environmental perturbations predominate in long-term. That TA1 is better in long–term than TA2 was not statistically significant.

H. De Boer, PTB

Development of Time Scales at PTB

Notes: Don't need sophisticated algorithms. Cs3 now in test. (69 m/s = $\bar{v}$; $\Delta\nu$ = 44 Hz, $\sigma_y(1s)$ = 6×10^{-12} and 3×10^{-12} for Cs1 and Cs2 respectively. Frequency accuracy primarily due to cavity ϕ-shift of 30×15^{-15} and 15×10^{-15} respectively. $\sigma_y(90\ d) \simeq 6 \times 15^{-15}$ from 10 values. Beam reversal every few weeks causes modulation. Hence after 90 d $\sigma_y(\tau)$ may go as $\tau^{-1/2}$. Measurement period, $\tau_0 = 1/2$ hour. Micro-stepper set once/week. Future plans: Prefer to use the best of primary standards and decrease the steering time. Accuracy of Cs's 3 and 4 expected to be about the same as Cs2, i.e., 1.5×10^{-14}.

M. Granveaud, LPTF

Are the Independent Atomic Time Scales Worth of Interest?

International atomic time references are available to the time/frequency community and to the potential users. They are computed by the BIPM time section according to three forms: coordinated as UTC, free as EAL — available upon request — and worldwide scientific reference as TAI. End 1987, 13 laboratories compute their own independent atomic time scales which are published in the BIH/BIPM circular. Some of them are based on the same stability concept as EAL; others are of the TAI type.

From the user's point of view, it is worth wondering if these local independent time scales have specific qualities with respect to the international references.

It is intended to point out this problem.

Notes: Cir. T. 39 UTC(k) + 11 TA(lab) + 4 TA(country). TAI less stable than TA(PTB), long-term (80 d to 320 d). APL H-maser same stability as TAI. TAI time-link noise $\sigma_y(\tau = 10d) \simeq 1.9 \times 10^{-14}$. TA(labs) tend to be better than TAI $\tau \lesssim 20d$. TAI better than most TA(labs) at $20\ d \lesssim \tau \lesssim 440\ d$. TA(PTB) better than TAI $\tau > 440\ d$.

A. Shenhar, W. Litman, INPL; A. Lepek, A. Citrinovitch, Time and Frequency

Limited; D.W. Allan, T.K. Peppler, NIST

Israel's New Synchronized Time Scale, UTC(INPL)

Notes: "UTC(INPL): a near real time software clock". 15 day filter on $\overline{(\Delta^2 x^2)}$ with $\tau_0 = 24$ hours—to alias away diurnal variation. Perform interpolated weighting for any point between. Paper scale performed much better than single clock which has been UTC(INPL).

C. Thomas, BIPM

Long-Term Stability of the International Atomic Time

TAI is a weighted average time scale based on the readings of an ensemble of about 180 atomic clocks. Improving its very long term stability needs to have the optimal mode of prediction for the rate of the operating clocks, especially when changes of weights occur. Various studies tend to prove that relatively long samples (two months) and the choice of what is called "linear prediction" are the best suitable for that purpose. In particular, trying to modelize the seasonal variation of TAI or tempting to include a longer past for the clocks seem to be unavailing.

Notes: Predicted frequency equal to a linear combination of five past two- month frequency samples
→ $a_{-5} = 0.1$, a_{-4} *small*, $a_{-2} < 0$, $a_{-1} = 0.3$ *concluded to continue using one–step linear prediction. Weight limit changed to 1000 from 200 for TAI. Compute $\sigma(5, 2mo)$ and infer $\sigma(6, 2mo)$ assuming RWFM then test if $\Delta y > 3\sigma(6, 2)mo$ to turn the weight to zero). 14% (24 clocks) are at 1000, which implies 6 $\sigma_{TA1}((6, 2mo) = 4 \times 10^{-15}$.*

M. Weiss, D.W. Allan, and T.K. Peppler, NIST

A Study of the NBS Time Scale Algorithm

Since 1968 the NBS time scale algorithm has been generating a clock which is theoretically better than any of the individual clocks in its ensemble. In the last few years, thanks to the Global Positioning System, we have been able to measure the time difference between the NBS time scale algorithm and the other time standards around the world. We are able to study long term stability of the order of years, and short term stability of the order of days. We now have estimated fractional frequency stabilities for averaging time out to a year of 1×10^{-14}. This paper studies the behavior of the algorithm from a theoretical point of view, characterizing its performance.

Notes: Defining frequency steps is difficult but very important. 3σ reject yields a factor of 2 improvement in $\sigma_y(\tau = 1/2 \ yr)$ which implies you could get by with 1/4 the number of clocks from a statistical point of view. Redundancy and reliability are separate issues from stability. 2σ reject yields factor of 4 improvement, which would imply 1/16 the number of clocks.

D.W. Allan, M.A. Weiss, and T.K. Peppler, NIST

In Search of the Best Clock

Because of the increased need for better clock performance than is currently available, this paper addresses some fundamental questions regarding clock metrology. Heretofore, most work has focussed on improving the clocks to meet the increased need. Though this is fundamental, we will show that significant gains are also available through the algorithms (computational methods for optimally combining the information) which process the readings of the clocks and through international comparisons now available via satellite. Proper algorithms for processing seem to be more important than the proportionate attention generally given them. In fact, to date, the only way we have been able to investigate some of the outstanding time predictability in long)term of the millisecond pulsar, PSR 1937+21, is by using such optimization algorithms.

D.W. Allan, NIST

In Search of the Best Clock, An Update

Because of the increased need for better clock performance than is currently available, this paper addresses some fundamental questions regarding clock metrology. Heretofore, most work has focussed on improving the clocks to meet the increased need. Though this is fundamental, we will show that significant gains are also available through the algorithms (computational methods for optimally combining the information) which process the readings of the clocks and through international comparisons now available via satellite. Proper algorithms for processing seem to be more important than the proportionate attention generally given them. In fact, to date, the only way we have been able to investigate some of the outstanding time predictability in long)term of the millisecond pulsar, PSR 1937+21, is by using such optimization algorithms.

J.A. Barnes, Austron, Inc.

An Adaptive Algorithm to Evaluate Clock Performance in Real Time

ARIMA Models and Kalman filters allow one to evaluate noises in clocks and oscillators and to forecast clock performance into the future with known confidence intervals. The coefficients used in an ARIMA model (as in Kalman filters) are normally estimated before the clock has been put into service. The parameter estimation procedures often minimize a variance function of many variables which can require significant "number crunching" capabilities. This batch mode of operations can give poor results, especially if the parameters are not absolutely constant in time (i.e., an imperfect physical model). It is possible to quantify the model parameters recursively, in real time, even while the system is running. Of course, the issue is the cost measured in terms of reduced system performance. The evaluation of system performance was accomplished using simulation techniques. The models considered were white noise PM, white FM, random)walk FM, and linear frequency drift (aging) in some realistic combination in a conventional application of ARIMA analysis (i.e., batch mode). The same data sets were used to evaluate the real time (recursive) analysis techniques. Only a single pass through the data was allowed as a simulation of the real time analysis. The parameter estimators were based on the fact that the ARIMA model corresponding to the noise types noted above is the ARIMA(0,2,2,) model. In typical ARIMA analyses, the raw data are processed by trial inverse filters to find those coefficients which minimize the variance of the residuals. In the real)time analysis the correlation coefficients for lags 1 and 2 of the inverse filtered data go to zero when the estimated MA coefficients (inverse filter) approach the "real" coefficients respectively. At each time point estimates of the first two auto correlation coefficients are taken as input to two first order servos which independently control each of the two estimated MA parameters. As the system runs, the MA parameter estimates improve)) they are not constant. The comparison of the batch and real)time modes was based on forecast applications, NOT the variances in the MA parameters themselves since the MA parameters are only one source of uncertainty. In the very long term, the errors due to inaccuracies of the MA parameters become insignificant relative to the normal forecast errors (assuming the models are good). Parameter errors are only transient problems for the real)time analysis.

Wei Guo, Song Jin)An, Shaanxi Astronomical Observatory

Characteristic Analysis of Clock Noise: A Dynamic Model

Power spectral density and autocorrelation function are two indispensable aspects to describe a stochastic signal, in frequency domain and time domain respectively. Of course, the fluctuations of clock time can be treated as a kind of stochastic signal. However, there is an inevitable difficulty in the widely used model—power law model—about this signal, it is merely in frequency domain and does not result in time domain; Atomic time scale algorithm designing depends mainly on the characteristics of the time fluctuations, so the usage of the power law model is limited quite in such field. In fact, some models relating to the time scale algorithm, such as polynomial model and ARIMA model, do not take the advantage of the power law model. All of these are concerned with the defect of the power law model itself: because of the nonstationarity of the noise, Wiener–Khintchine relations are untenable and $R_x(\tau)$ diverges. To counter this question, a dynamic model is introduced in this paper. From this dynamic model, a set of autocorrelation functions is obtained. With the aid of such results, it can be seen that the quantities of stability characterization in time domain (e.g., Allan variance) depend on the stationarity of the statistic. On other hand, a Kalman filter is derived with such model. This filter can be used as a optimal estimator for the clock states (phase and frequency) as well as a predictor of frequency and time. As a comparison, time

predictions of the difference between two commercial cesium clocks are carried out with Kalman model, ARIMA model and quadratics model respectively. It is indicated that Kalman model is superior to the others, especially in prediction multi)step ahead. Also, the relation between the noise covariance matrix and the power law parameters is presented in this paper.

Wei Guo, Song Jin)An, Shaanxi Astronomical Observatory

The Measurement Error of Quadratic Polynomial Parameter in Atomic Clock

Clock Quadratic polynomial is a simplified model in time scale algorithm. The least square method has been used to estimate the quadratics parameters (i.e., a,b and c) all along. When LS is used, it is assumed actually that the noise in the model is white and the conventional error formulae for white noise are used to estimate the accuracy of the measurement. However, facts show that the parameters measured by LS are quite instable and the errors are far beyond the range given by the conventional formulae. A detailed analysis about this question is presented in this paper according to the dynamic model. A new set of error formulae are derived. By these new formulae, it is indicated that the measurement error of parameter a and b increased with sample number N under the nonstationary noise, and even though the error of parameter c can be decreased by increasing N, the error is much greater than the one obtained under the white noise. Such conclusions are verified by computer simulations.

In recent years Kalman filter method has appeared in the field of frequency and time. This result shows clearly that it is reasonable to describe a clock with dynamical variable (i.e., phase, frequency and drift). In fact, clock itself is a dynamic system. It is why a dynamic model is introduced. In this paper, a Kalman filter is derived according to the dynamic model. This filter can be used as a optimal estimator for the clock states (phase and frequency) as well as a predictor of frequency and time. As a comparison, time predictions of the difference between two commercial cesium clocks are carried out with Kalman model, ARIMA model and quadratics model respectively. It is indicated that Kalman model is superior to the others, especially in multiple steps prediction.

Notes: Kalman gives better prediction error than ARIMA? (Appears to have used a non opt. model for ARIMA, used a (1,2,1) model).

B. Guinot, BIPM

Time Scales Established in Retrospect

Notes: TTBIPM (post analysis) Vondrak method used in astronomy '69, modified by Guinot. Yields weighting according to the Fourier components of the clock. PRB Cs 1 ends up being predominant in the long)term. Annual term is assumed to be in EAL or TAI and not in Cs1 and is subtracted to generate TTBIPMXX.

A. Gifford, NRL; F. Varnum, Falcon AFS

The NRL Hydrogen Maser Ensemble and a GPS Time Steer Experiment

The Naval Research Laboratory (NRL) has developed and is installing a Clock Ensemble in the Master Control Station (MCS) of the Global Position System (GPS) at the Consolidated Space Operational Command (CSOC), FALCON AFS, Colorado Springs, CO. This system is described, current performance data are presented, and several methods of integrating the NRL ensemble with GPS are outlined. One method which uses the current steering mechanism of GPS is described in detail. The current method of estimating GPS clock states and steering those states to UTC(USNO) is reviewed.

J. Levine, D.W. Allan, NIST

The Steering of a Real–Time Clock to UTC(NBS) to UTC

We describe the procedures that we use to define UTC(NBS) and to steer it towards UTC(BIPM) using an averaging process with a time constant of about 1 year. In addition, we describe the hardware and software that is used to steer a physical clock so that its output realizes UTC(NBS) in real time. The method uses a micro)stepper whose frequency offset is updated 5 times per hour by the time scale computer. The corrections applied to the micro)stepper yield a physical tick whose average offset from UTC(NBS) is less than 0.5 nanoseconds. The algorithm can cope with various fault conditions and can also provide remote notification of a fault.

M.A. Weiss, NIST

The Design of Kalman Smoothers for Global Positioning System Data

Measurements of clocks aboard Global Positioning System (GPS) satellites as well as GPS system time are made many times per day at time standards laboratories around the world according to a tracking schedule issued by the Bureau International des Poids et Mesures (International Bureau of Weights and Measures). We use common view differences of these data as input to a Kalman smoother. Biases in measurements repeated one per sidereal day produce apparent diurnal effects in the data. A composite time and frequency Kalman estimator is used here. This allows frequency updates of clocks at time intervals less than one day while aliasing diurnal variations, and updating time once per day.

Notes: F + T Kalman smoothed. Diurnal variations in global GPS data. Use frequency estimate once per sidereal day for each SV and daily average of all SV times for F + T Kalman updates. Eliminate diurnal variation and allows an estimate of SV + GPS clock stability for 1/2 day and 1/4 day τ values.

P.A. Clements, JPL; B.P. Gibbs, J.S. Vandergraft, Computational Engineering Inc.

Stable Kalman Filters for Processing Clock Measurement Data

Kalman filters have been used, for some time, to process clock measurement data. Due to instabilities in the standard Kalman filter algorithms, the results can be unreliable requiring manual intervention of the data, the models, or the filter to obtain reasonable estimates. During the past several years stable forms of the Kalman filter have been developed, implemented, and used in many diverse applications. These algorithms, while algebraically equivalent to the standard Kalman filter, exhibit excellent numerical properties. Two of these stable algorithms, the UD filter and the Square Root Information Filter (SRIF) have been implemented to replace the standard Kalman filter used to process data from the Jet Propulsion Laboratory's Deep Space Network's (DSN) Hydrogen Maser clocks. The clocks are located at the DSN tracking complexes located in California, Australia and Spain. The data are time offsets between the clocks in the DSN, the timescale at the National Bureau of Standards and two geographically intermediate clocks. The measurements are made using the GPS navigation satellites in mutual view between clocks. The filter programs allow the user to easily modify the clock models, the GPS satellite dependent biases, and the random noise levels in order to compare different modeling assumptions.

The results of this study, to be presented in this paper, show the usefulness of such software in developing accurate models. Moreover, the results demonstrate that the new filter algorithms are indeed stable, efficient, and flexible. They provide reliable tools for obtaining better estimates of

time, frequency and drift than do current methods. The talk will include a brief overview of these stable filters.

Notes: To avoid instabilities that can occur in Kalman filters, tested UD filter and SRIF modification of Kalman filter approach. UD seems to avoid instabilities and agrees well with 10 day averages. New approach will make system automatic in estimating T/F of masers in Spain, CA, and Australia (DSN tracking sites). With respect to UTC(NBS) to a few nanoseconds and a few parts in 10^{14}. SRIF is more reliable, but with increased compute time. Since that is not a problem, may use SRIF.

B. Guinot, BIPM

Importance and Limitation of the Algorithms for Time Scales

The frequency prediction in usual algorithms is based on frequency samples not referred to an ideal time scale, but to the scale to which the clocks contribute. This is a severe limitation. In particular, if there are no changes of the clock ensemble, no changes of the weights and if the same predictive filter is applied to all clocks, the prediction is absolutely useless. We have tried to overcome this difficulty. However, if one wishes to optimize the stability over a sampling time tau, that requires that the time scale be issued after a delay tau.

M. Mnackri, LPTF; C. Thomas, BIPM

A Comparative Study of Two Procedures for Generating

Ensemble Time: Kalman Filters and Weighted Averages

Different algorithms are being used in construction of local atomic time scales. They are based upon different statistical models; It is often difficult to make "a posteriori" comparison between algorithms results; so, it could be worth of interest to compare time scale algorithms through "a priori" assumptions and computational parameters. Availability of sets of data and time sampling of clock readings are of prime importance in a time scale algorithm.

Notes: A study of Kalman filter approach was performed and it was shown that ALGOS is a special form. The Kalman gives a weighting according to two stochastic parameters. (White FM $\tau^{1/2}$, and random walk FM $\tau^{1/2}$ for $\sigma_y(\tau)$. ALGOS weights the clocks according to random walk FM because τ = 60 days.

D. Percival, APL for USNO

Comparison of Two U.S.N.O. Time Scale Algorithms

Notes: Poisson distr of Δy steps gives reasonable $\sigma_y(\tau)$ diagrams.

Current algorithm

Key points: a) manual procedure, b) smoothing algorithm, c) equal weights for all clocks.

Robust Alg $\Delta^2 x = \epsilon_t(i) - \theta(i)\epsilon_{t-1}(i)$

ARIMA (0,2,1)

Estimate $\mu_+ \equiv x_t(0)$ Weight is constant out to 2σ, then linear decay to ϕ to σ.

Need 186 days to characterize, next 186 to test. Took 80 hrs. to run 20 repetitions.

Real data non normal (heavier task)–more energy in the tails.

$\tau_{min} \sim 2$ *weeks*

1 dB improvement

Conclusion: robust simpler, clock modeling question

S.R. Stein, Ball Aerospace (funded by NRL)

Kalman Ensembling Algorithm: Aiding Sources Approach

Notes: NBS(AT1) is steady state Kalman but is "performing better because of difference in weighting functions".

Advantages of modified Kalman approach:

    a) Estimates states of measurables

    b) Weights explicitly account for correlations between each clock and ensemble

    c) Covariance matrix is bounded

    d) Used robust statistical rejection

Tested with simulation and real data for CSOC clocks

Simulated white FM clock and R.W. FM clock

Output $\sigma_y(\tau)$ reflected white FM level well and long–term within ~ 3 dB of best clock.