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ION TRAPS FOR LARGE STORAGE CAPACITY*

D. J. Wineland
Time and Frequency Division
National Bureau of Standards
Boulder, Colorado 80303

ABSTRACT

Ion storage in Penning-type or rf (Paul)-type traps is discussed. Emphasis is given to low-energy, long-term confinement of high densities and large numbers of ions. Maximum densities and numbers are estimated using a low-temperature, static model of the ion plasmas in the traps. Destabilizing mechanisms are briefly discussed.

INTRODUCTION

The following notes are concerned with the storage of large densities and numbers of charged particles or ions in electromagnetic traps. Certainly, this is a small part of the problem concerned with the accumulation, storage, and manipulation of antimatter. However, this particular problem appears to be interesting by itself and may have interesting applications elsewhere.[1-3] Devices such as tokamaks and tandem-mirror machines for fusion plasmas are not discussed because we concentrate on long-term, low-energy confinement which is desirable for antimatter storage. For brevity, only Paul (rf) and Penning-type traps[4,5] are discussed. Maximum densities and maximum numbers of ions for one or a mixture of species with different charge to mass ratio are investigated.

GENERAL CONSIDERATIONS

We will assume that thermal equilibrium of the stored ion sample has been achieved. This condition may not always be achieved or desirable. For example, antihydrogen might best be made by passing antiprotons through positrons without thermal equilibrium between these two species being achieved. For long term storage, however, thermal equilibrium appears likely.

If we assume that the ion trap has symmetry about the z axis, the thermal equilibrium distribution function for the ith species can be written[6-9]

$$f_i = n_i \left(\frac{m_i}{2\pi k_B T}\right)^{3/2} \exp[-(H_i - \omega \ell_{zi})/k_B T]. \tag{1}$$

m_i is the ion mass, k_B is Boltzmann's constant, T is the ion temperature, and ω and n_i are constants (determined below). If a magnetic field is superimposed along the z axis ($\vec{B} = B\hat{z}$), then

$$H_i = m_i v_i^2/2 + q_i\phi(x) \tag{2}$$

is the ion energy and

$$\ell_{zi} = m_i v_{\theta i} r_i + q_i A_\theta(\vec{x}) r_i/c \tag{3}$$

is the ion canonical angular momentum. q_i and $\vec{v}_i$ are the ion charge and velocity, and $v_i = |\vec{v}_i|$. $v_{\theta i}$ and $A_{\theta i}$ are the θ components of the velocity and the vector potential, and r_i is the radial coordinate of the ion in cylindrical coordinates. $\phi = \phi_I + \phi_T + \phi_{ind}$ is the total potential, which is written as the sum of the potential that is due to ion space charge ϕ_I, the applied trap potential ϕ_T, and the potential, ϕ_{ind}, that is due to the induced charges on the trap electrodes.[9] We choose the symmetric gauge where $\vec{A}(\vec{x}) = \vec{B} \times \vec{x}/2$, so that $A_{\theta i} = Br_i/2$. We can therefore write the distribution function as

$$f_i(\vec{x},\vec{v}) = n_i(\vec{x}) \left(\frac{m_i}{2\pi k_B T}\right)^{3/2} \exp[-\tfrac{1}{2}m_i(\vec{v}_i - \omega r_i\hat{\theta})^2/k_B T], \tag{4}$$

where $n_i(\vec{x})$ is the ion density given by

$$n_i(\vec{x}) = n_i \exp\{-[q_i\phi(r,z) + \tfrac{1}{2}m_i\omega(\Omega_{ci} - \omega)r_i^2]/k_B T\}. \tag{5}$$

We assume that the trap potential is adjusted to make $\phi = 0$ at the origin, in which case n_i is the ion density at the center of the trap for a single ion species. From Eq. 4, we see that the ion cloud, or non-neutral ion plasma, rotates at frequency ω. $|\Omega_{ci}| = |q_i|B/m_i c$ is the cyclotron frequency. The signs of ω and Ω_{ci} indicate the sense of circulation where we use the right-hand rule. For example, Ω_{ci} is negative for positive ions ($q_i>0$) and is positive for $q_i<0$.

In the limit $T\to 0$, we must have $q_i\phi - m_i\omega(\omega - \Omega_{ci})r_i^2/2 \to 0$ for f_i to be well behaved. This condition and Poisson's equation imply that the charge density, ρ, at the position of the ions be given by

$$\rho = -\rho_i' + m_i\omega(\Omega_{ci} - \omega)/(2\pi q_i), \tag{6}$$

where ρ_i' is a fictitious charge density arising for rf traps since the pseudopotential does not satisfy Poisson's equation[9] (see below). By low temperature, we mean that the Debye length, λ_D, for the plasma given by[6-9]

$$\lambda_D^2 = k_B T/(4\pi n_i q_i^2), \tag{7}$$

is short compared to the plasma dimensions. From Eq. 7, we can write

$$\lambda_D \approx 6.9(T/n_i)^{1/2}/Z \text{ cm}, \tag{8}$$

where T is expressed in kelvin, n_i is expressed in cm^{-3}, and Z is the ion charge in units of the proton charge. For T = 4 K, Z = 1, and $n_i = 10^8/cm^3$, $\lambda_D \approx 14\ \mu m$. Therefore, for the low temperatures desired for long term storage, the T→0 limit will usually be valid.

PENNING TYPE TRAPS

By Penning-type trap, we will mean that the trap is formed by a static magnetic field $\vec{B} = B\hat{z}$ and an axially symmetric electric trap potential, ϕ_T, of the form (in spherical coordinates)

$$\phi_T = \sum_k C_k r^k P_k(\cos\theta), \tag{9}$$

where C_k are constants and P_k is a Legendre polynomial of order k. Axial symmetry is particularly important for the Penning trap. If the trap is not axially symmetric, angular momentum can be coupled into the ions and they will diffuse out of the trap. If the trap is axially symmetric, angular momentum is a constant of the motion and stable ion clouds are maintained.[10] For spectroscopy,[4,5] particularly mass spectroscopy,[5,11,12] we desire $\phi_T \propto r^2 P_2(\cos\theta)$ as indicated in Fig. 1. This is because the ion motions (neglecting relativistic effects) will be harmonic. A fourth order trap where $\phi_T \propto r^4 P_4(\cos\theta)$ would look something like that in Fig. 2.

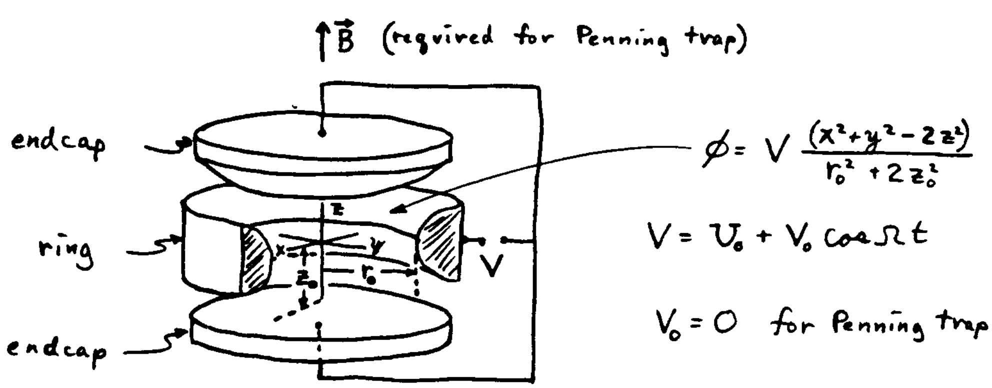

Fig. 1. Electrode configuration for the quadrupole rf (Paul) or Penning trap. Inner surfaces of electrodes are assumed to be equipotentials of ϕ, and the effect of truncating the electrodes is neglected. r_0 is the inner radius of the ring electrode and $2z_0$ is the endcap to endcap spacing.

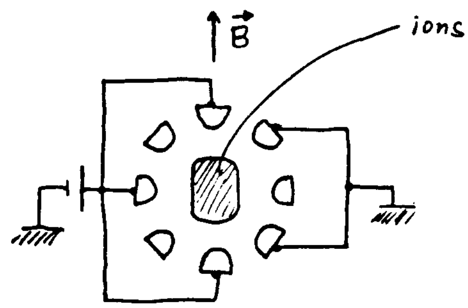

Fig. 2. Sketch of fourth order Penning-type trap for positive ions. The view shown is a cross section in the y-z plane.

An experimentally convenient geometry[1] might be a "cylinder trap" as shown in Fig. 3.

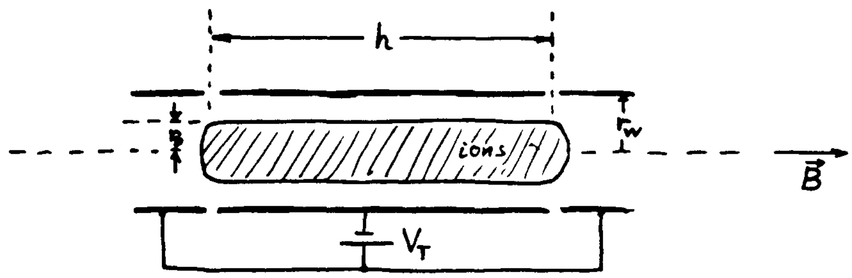

Fig. 3. Sketch of cylinder Penning-type trap for positive ions.

With $\rho_i'=0$, Eq. 6 can be written

$$n_i = m_i\omega(\Omega_{ci}-\omega)/(2\pi q_i^2). \tag{10}$$

Therefore, if we assume axial symmetry, n_i is independent of ϕ_T. Because of this, we will assume throughout that we are using the experimentally convenient geometry of the cylinder trap (Fig. 3). n_i is maximum for $\omega = \Omega_{ci}/2$, which is called the Brillouin limit.[6] ω may be determined by initial loading conditions or may be altered as discussed below. If we assume a single ion species and drop the indices,

$$n(max) = B^2/(8\pi mc^2) \simeq 2.7 \times 10^9\ B(T)^2/M, \tag{11}$$

where B(T) is the magnetic field in tesla and M is the ion mass in u (atomic mass units). n(max) is independent of Z. From Fig. 3 and Eq. 11, the maximum number of ions, N(max), is given by

$$N(max) \simeq 8.4 \times 10^9\ r_p^2 h B(T)^2/M, \tag{12}$$

where r_p and h are the plasma radius and height in cm, respectively. The potential, ϕ_c, at the center of the plasma (assuming the central cylinder is grounded as in Fig. 3) is easily calculated to be[1]

$$\phi_c(V) \simeq 1.4 \times 10^{-7}\ NZ(1+2\ell n(r_w/r_p))/h, \tag{13}$$

where ϕ_c is given in volts. The magnitude of V_T must be higher than the magnitude of this potential to confine ions along the z axis. If we eliminate N from Eqs. 12 and 13, we obtain

$$r_p^2 = 8.3 \times 10^{-4} M\phi_c/(B(T)^2 Z(1+2\ell n r_w/r_p)). \tag{14}$$

For $\phi_c = 30$ kV, B = 10T, $r_p/r_w = 0.5$, and Z = 1, we find $r_p^2 \simeq M/10$.

A few examples of n(max) and N(max) for these values of ϕ_c, B, and r_p/r_w are illustrated in table I. From the above considerations and table I, large mass storage would favor ions with large values of M.

Table I. Example parameters for a cylinder Penning-type trap. Fig. 3 applies with $V_T > \phi_c = 30$ kV, B = 10 T, $r_p/r_w = 0.5$, and Z = 1.

M(u)	h(cm)	r_p(cm)	n(max)(cm^{-3})	N
1	500	0.32	2.7×10^{11}	$4.3 \times 10^{13} \simeq 0.7 \times 10^{-7}$ mg
1000	500	10	2.7×10^{8}	$4.3 \times 10^{13} \simeq 0.7 \times 10^{-4}$ mg
1/1836	500	0.0075	5.0×10^{14}	4.3×10^{13}

rf(PAUL) - TYPE TRAPS

Paul or rf - type traps provide trapping of charged particles in oscillating, spatially-nonuniform electric fields. It is also possible to use static electric fields to alter the trapping geometry somewhat,[4,5] but for simplicity, we assume trapping in pure oscillating fields here. If the trapping potential is given by

$$\phi_T = \phi_0(\vec{r})\cos\Omega t, \tag{15}$$

then, for Ω sufficiently high (defined below), ions are confined in a pseudo potential, ϕ_{pi}, given by[4]

$$\phi_{pi} \simeq q_i |\vec{\nabla}\phi_0(\vec{r})|^2/(4m_i\Omega^2). \quad (16)$$

The fictitious charge density is given by $\nabla^2\phi_{pi} = -4\pi\rho_i'$. For spectroscopy, the usual choice is the quadrupole rf trap where $\phi_0 \propto r^2 P_2(\cos\theta)$ (spherical coordinates).[4,5] This makes the trap pseudopotential harmonic in all directions. In this case, Ω sufficiently high means[4,5]

$$q_{zi} = \frac{8|q_i|V_0}{m_i\Omega^2(r_0^2+2z_0^2)} \lesssim 1. \quad (17)$$

V_0 is the magnitude of the rf voltage applied between ring end endcaps whose dimensions are indicated in Fig. 1.

If we assume that the "secular" motion in the pseudopotential well of Eq. 16 can be cooled, and if we assume $\omega = 0$, the density of ions for a single species is determined by space charge repulsion and is given by [4,5] (dropping the subscripts)

$$n = 8.3 \times 10^8\ q_z V_0(kV)/(Z(r_0^2+2z_0^2)). \quad (18)$$

If we neglect ϕ_{ind}, the shape of the ion plasma is a spheroid of revolution.[9] If we also assume $r_0 = 2z_0$, and assume that the dimensions of the ion cloud are equal to inner trap dimensions, the maximum number of ions is given by

$$N(max) = 2.3 \times 10^9\ q_z\ z_0 V_0(kV)/Z. \quad (19)$$

From Eq. 17, we have

$$\Omega/2\pi(MHz) = 5.7(V_0(kV)Z/(Mq_z))^{1/2}/z_0. \quad (20)$$

From Eqs. 18-20, we tabulate a few example parameters in table II.

Table II. Example parameters for a quadrupole rf trap.

M	V_0(kv)	q_z	Z	r_0(cm)	n(cm^{-3})	N	$\Omega/2\pi$(MHz)
1000	10	0.5	1	1	2.8×10^9	5.8×10^9	0.81
1	"	"	"	"	"	"	25
1/1836	"	"	"	"	"	"	1100
"	"	0.005	"	"	2.8×10^7	5.8×10^7	11 GHz
M	"	0.5	"	20	6.9×10^6	1.2×10^{11}	$1.3/M^{1/2}$

We note the independence of n and N(max) on M. For electrons and positrons, the practical limits on n and N(max) may be due to the required large values of Ω.

For higher order rf traps, $\phi_0(\vec{r})$ might take the form given by Eq. 9. Perhaps a more useful form of ϕ_0 is given by the potential (in cylindrical coordinates)

$$\phi_0(r,\theta,z) = V_0 r^{k+1} \cos[(k+1)\theta]/r_0^{k+1}, \tag{21}$$

where θ is the azimuthal angle and k is an integer. A sketch of such a trap for k=2 is given in Fig. 4. Ideally, the electrode surfaces are equipotentials of Eq. 21; in Fig. 4, they are shown as cylindrical rods for simplicity. k=1 describes the Paul quadrupole mass filter. The ends of the trap could be closed off by bending the rods in towards the axis of the trap or using endcaps with appropriate static potentials thereby making a cylinder rf-type trap. Alternatively, the individual rods could be connected end-on-end to make a kind of race track.[13]

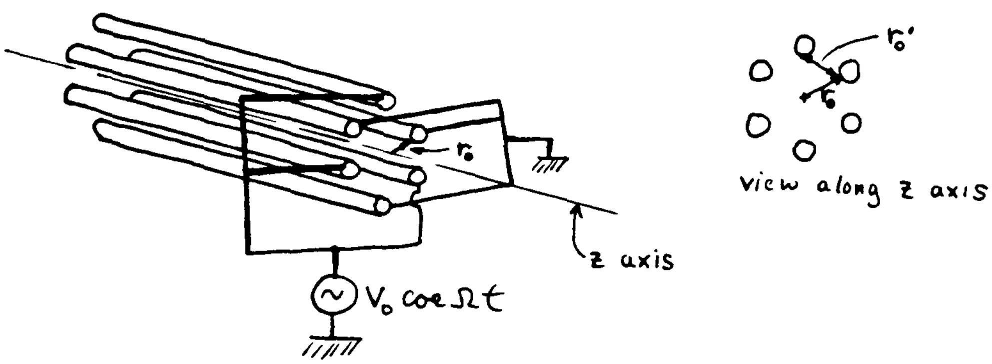

Fig. 4. Sketch of cylinder rf-type trap for k = 2.

Experimental examples of higher order traps are given in refs. 14 and 15. From Eqs. 16 and 21,

$$\phi_{pi} = C_{ki} r^{2k}, \tag{22}$$

where

$$C_{ki} = q_i V_0^2 (k+1)^2/(4m_i \Omega^2 r_0^{2(k+1)}). \tag{23}$$

ϕ_{pi} is cylindrically symmetric.

From Gauss's law, the density of stored ions is determined by space charge repulsion and is given by (if we assume $\omega = 0$)

$$n_i(r) = k^2 C_{ki} r^{2(k-1)}/(\pi q_i). \tag{24}$$

Therefore, for $k >> 1$, the ions tend to form cylindrical shells near the inner edge of the trap electrodes where $r = r_0$. If the ions occupy the space out to $r = r_0$, integration of Eq. 24 gives

$$N(\max) = \frac{k(k+1)^2}{32} \frac{V_0 h}{q_i} \left(\frac{r_0'}{r_0}\right)^2 q_{xi}, \qquad (25)$$

where q_{xi} is the stability parameter analogous to Eq. 17, and r_0' is defined in Fig. 4. We have

$$q_{xi} = 8 V_0 q_i/(m_i \Omega^2 r_0'^2). \qquad (26)$$

For $k = 1$, $r_0'/r_0 = \sqrt{2}$ and $q_{xi} \sim 1$ for stability. For $k = 1$, the ratio of N(max) for the cylinder rf - trap to N(max) for the rf quadrupole trap of Eqs. 15 - 20, is equal to $3h/(4z_0)$ where h is the length of the plasma column as in Fig. 3. For $k >> 1$, we have

$$N(\max)_k/N(\max)_{k=1} \simeq k\pi^2/8, \qquad (27)$$

where we have assumed that the required value of the trap stability parameter, q_x, is independent of k. Thus, N(max) increases approximately as kh/z_0.

ION MIXTURES

For brevity, we consider only two ion species, with charge-to-mass ratios $q_1/m_1 \neq q_2/m_2$. The results are easily generalized to more species. When $q_1/q_2 > 0$, we consider only cylinder-type traps for simplicity. However the qualitative aspects of the results apply to other geometries.

Cylinder Penning-type trap, $(q_1/q_2 > 0)$

In Eq. 6, with $\rho' = 0$ for the Penning-type trap, ρ can be single valued only if the two ion species occupy different spatial regions. Qualitatively, since thermal equilibrium implies that ω is the same for all ions, if $q_1 = q_2$ and $m_2 > m_1$, species 2 is forced to larger radii than species 1 because of the larger centrifugal force acting on it. This separation has been theoretically and experimentally studied.[16,17] In the T = 0 limit, the separation should be complete with a gap between the species. If we look along the z axis of the trap, the spatial distributions appear as in Fig. 5.

The ion densities are still given by Eq. 10. The maximum density of the outer species is again given by $\omega = \Omega_{c2}/2$. Using this value of ω, we can find the corresponding value of n_1. The separation of the plasmas is given by solving the equation of motion for the rotation motion of species 2 at the radius a_2. We find

$$a_2/b_1 = (\rho_1/\rho_2)^{\frac{1}{2}} . \qquad (28)$$

For $a_2 - b_1 > \lambda_D$, we expect the thermal contact between the two species to be reduced.[17]

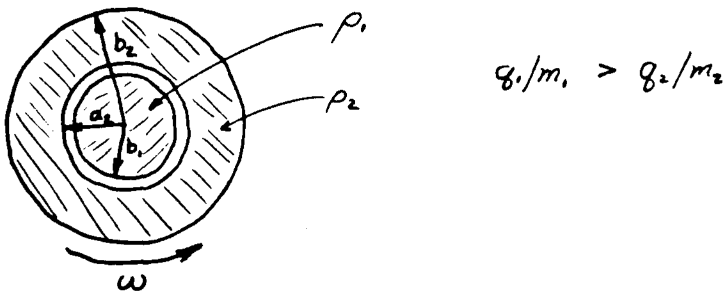

Fig. 5. Centrifugal separation of ion species in cylinder Penning or rf-type trap. The view is along the z axis, $\vec{B}$ is into the paper, and q_1, $q_2 > 0$.

A possible configuration for maintaining cold trapped positrons[3] in a Penning trap might be the following. Consider simultaneous trapping of positrons and $^9Be^+$ ions. The $^9Be^+$ ions would form a hollow cylinder around the central e^+ column. Using laser cooling, the $^9Be^+$ ions and therefore the positrons could be cooled to temperatures much less than 1 K.[17] In addition, angular momentum can be imparted to the ions in the laser cooling process to prevent radial diffusion of both species.[17] In this way, a stable, low temperature positron sample could be maintained. It is likely that ω would be limited to the value for maximum $^9Be^+$ density; that is $\omega(\max) = \Omega_c(^9Be^+)/2$. If we assume $B = 10$ T this would limit the maximum positron density to $n(e^+) \simeq 5.9 \times 10^{10}/cm^3$.

It would be nice to realize a similar scheme for antiprotons; unfortunately, negative ions suitable for laser cooling do not seem to be available. Cooling of antiprotons by thermalized electrons should work[1,18-20]. In the condition of thermal equilibrium, the antiprotons form a hollow cylinder around the electrons. In order to prevent radial diffusion, angular momentum of the appropriate sign might be imparted to the ions by particle beams,[21] positive or negative feedback on the ion motion[21,22] or angular momentum transfer from plasma waves.[23,24]

<u>Cylinder rf-type trap ($q_1/q_2 > 0$)</u>

For the rf-type trap, assumming $\omega = 0$, separation of the species with $q_1/m_1 \neq q_2/m_2$ occurs because the pseudopotential wells are unequal. This phenomenon has been investigated[25] for the quadrupole trap configuration. A simpler case to analyze is that of the cylinder rf-type trap where ϕ_0 is given by Eq. 21. If we assume $q_1/m_1 > q_2/m_2$, Fig. 5 again applies. Ion densities are still given by Eq. 24 and the separation of species can be easily determined by noting that at $r = a_2$, the space-charge electric field outward from species 1 is just equal to the pseudopotential field inward acting on species 2. We find

$$\frac{a_2}{b_1} = \left(\frac{Z_1 M_2}{Z_2 M_1} \right)^{\frac{1}{2k}} . \tag{29}$$

Low order traps (e.g., k = 1 (quadrupole trap)) can give rise to a significant separation of the species.

Penning-type traps ($q_1/q_2 < 0$)

Simultaneous trapping of charges of opposite sign is particularly interesting in the context of this meeting. An important example is the simultaneous trapping of positrons and antiprotons for the production of antihydrogen by radiative recombination.

For the Penning-type trap, simultaneous trapping of charges of opposite sign is precluded because it is impossible to simultaneously trap both species along z. Therefore, to achieve a mixture of positive and negative ions, one species must be injected through the other. A possible arrangement might be to stack Penning traps for charges of opposite sign along the z axis in a "nested" arrangement as suggested in ref. 26. If positrons are stored in the central trap, antiprotons could be passed back and forth between traps adjacent to the central trap. Angular momentum transfer would tend to spread the positrons, but this might be overcome as described in the previous section.

rf - type traps ($q_1/q_2 < 0$)

In the pseudopotential approximation, it is apparent that if $N_1 q_1 = N_2 q_2$ the ions shrink to a point! The practical limit would appear to be due to rf heating which tends to keep the ions very hot. This problem is accentuated for simultaneous storage of positive and negative ions since they can come very close together due to Coulomb attraction and the rf micromotions[4,5] are 180° out of phase for the two species. Some experimental work has been reported in ref. 27, where $T\ell^+$ and I^- ions were simultaneously stored in a quadrupole rf trap. In this work, an increase in density over the maximum value for a single stored species was observed, but overall densities were still fairly low. Clearly more work needs to be done on this interesting possibility.

Penning - rf trap combinations

An example of such a combined trap might be an rf-type trap for positrons superimposed on a Penning-type trap for antiprotons. Such a scheme appears valid for small numbers of positrons in an antiproton sample. When the number of positrons exceeds the number of antiprotons, trapping is more likely to occur because the positrons are held by the rf trap pseudopotential and antiprotons are held by the attractive space charge of the positron sample.

PROBLEMS

The analysis in this paper has considered only the static properties of nonneutral ion plasmas held in Penning-type or rf-type traps. The calculated maximum densities and numbers will be reduced by various effects which we might classify as plasma instabilities.

Penning-type traps

The most serious problem appears to be due to asymmetry induced transport. Radial transport can also occur due to ion collisions with background gas, but at very high vacuum, this process is negligible. The basic idea of asymmetry induced transport is that if the trap is not axially symmetric, angular momentum can be coupled into the ions which causes them to spread radially in the trap. Eventually, the ions will strike the electrodes and be lost. During this spreading process, the electrostatic space-charge energy will be converted to heat. In a cylinder Penning-type trap, spreading rates proportional to $(h/B)^2$ have been observed,[28] where h is the plasma length (Fig. 2). Although the spreading mechanism is not understood, one possibility might be resonant particle transport.[9,28,29] Resonant particle transport might occur if the trap geometric axis and magnetic field axis are misaligned. As the ion density increases, ω becomes larger. As the temperature becomes lower, the individual ion axial frequency, ω_z', defined as the mean axial velocity, v_z, divided by h/π, becomes smaller. When $\omega_z' \simeq \omega$, a coupling exists[9,29] which can simultaneously heat the axial motion and increase the ion radii. At the densities required for this condition, ω_z' becomes less well defined due to collisions, but the same mechanism may still apply. For $\omega = \omega_z'$, the densities may be considerably less than the maximum density given by Eq. 11. For small samples, some experimental evidence for such a resonant effect exists.[9]

Independently of the mechanism, such transport is likely caused by trap asymmetries. Conversely, if axial symmetry is preserved, angular momentum is a constant of the motion and confinement is assured.[10] Therefore, axial symmetry in the Penning-type traps appears to be at a premium.

rf-type traps

Historically, the mechanism of rf heating[30] has prevented the attainment of low temperatures and long confinement times in the rf traps. Therefore, attainable densities have typically been less than those dictated by space charge limitations (Eqs. 18 and 24). Cooling with a buffer gas allows the space-charge limited densities to be obtained, but this option seems to be precluded for antimatter storage.

The basic mechanism for ion-ion rf heating is that the rf driven micromotion can impart energy to the secular motion, that is, the motion in the pseudopotential well. In many experiments, this has limited the ion kinetic energy to a value of about 1/10 of the well depth of the trap due to evaporation.[4,5,30] This problem is often accentuated since it is desirable to operate the trap with large q_z values (Eq. 17) in order to obtain deep well depths. For the quadrupole rf trap (Eqs. 15-20), the relation $\omega_z/\Omega = q_z/(2\sqrt{2})$ holds, where ω_z is the single ion secular frequency.[4,5,30] Therefore, for large values of q_z, ω_z approaches Ω and subharmonic excitation of the secular motion becomes more likely.

For stored ions of the same sign of charge, it would appear that rf heating could be made less by using much lower values of q_z ($q_z < 0.01$). This reduces the ion micromotion amplitude at a given distance from the center of the trap and it increases the mean spacing between ions. Both of these effects would reduce the rf heating mechanism and thereby allow the space charge density limit to be approached. Unfortunately, reducing q_z also reduces n (Eq. 18), so that a reasonable compromise would have to be met to achieve maximum stored numbers.

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