In Search of the Best Clock

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Abstract—**There is an increased need for better clock performance than is currently available. Past work has focused on developing better clocks to meet this increased need. Though this is fundamental and should not be deemphasized, we will also show that significant gains have been and can be obtained through the algorithms which optimize the clock readings and through international comparisons now available via satellite. Algorithms for processing are more important than the proportionate attention generally given them. In fact, to date, one of the main ways we have been able to investigate some of the long-term performance aspects of the millisecond pulsar, PSR 1937 + 21, is by using such optimization algorithms. Since there are indications in the pulsar data of variations which could be explained as arising from the influence of gravitational waves, these long-term stability studies take on a new importance. Improved long-term stability of earth-bound clock systems will significantly assist the study of the incredibly stable spin rates of these neutron stars.**

I. INTRODUCTION

IN THE SEARCH for a ''best clock,'' we explicitly mean a clock with the best frequency stability, which in turn can be shown to lead to the minimum time or phase departure over a time interval corresponding to the frequency sampling interval, τ, after calibration. Also, reliability is of great importance in the time and frequency discipline. There are three important steps for producing a quality time scale. *Step one* is to have a primary frequency reference—the more accurate the frequency, the better will be the achievable long-term frequency stability. Short-term frequency stability is not a requirement for long-term performance, though it is often very useful and certainly adds convenience. *Step two* is to have a precise and accurate method of accumulating the time intervals defined by the available frequency standards and of measuring the time differences between these clocks. It is step two that makes a frequency standard a clock. *Step three* is an algorithm, which makes optimum use of the time differences among all the clocks involved, of the stochastic behavior of each of the clocks, of the systematic behavior and environmental sensitivities of each of the clocks, and of periodic frequency calibrations with the primary frequency standards.

Since errors in clocks can be well modeled by systematic deviations and by random deviations, we optimally estimate the systematic deviations in the presence of the random deviations. We also estimate the characteristics of the random deviations in the presence of the systematic deviations. Because of this semicircular nature of the error analysis in clocks, the combining computer algorithms have to be approached judiciously, paying attention to confidence of the estimates and to correlations. That the systematic deviations are somewhat orthogonal to the random deviations assists in the tractability of the error analysis. One of the significant benefits of the output of step three is in regard to the random deviations: we have the very exciting result that on the average the instabilities of the optimally computed time and frequency are less than those of the best contributing member clock [1]. With the computer era upon us, there should be more and more opportunities to integrate step three into the real-time output as well as into post-analysis.

There are now sufficient data available from high-accuracy satellite time and frequency comparison techniques that we can view most of the best clocks (in the sense of long-term stability) in the world as if they were in the same laboratory [2]. Therefore, we are now at a point in international time and frequency metrology where we can combine the time differences and frequency calibrations from the best clocks and frequency standards in the world and generate a world's ''best clock'' using an optimized algorithm [3]. This study in long-term stability of clocks and of ensemble outputs using various algorithms has led to some very interesting findings—including some possible causes of annual variations and of cross-correlations between clocks [3]–[5]. The clocks studied were primary cesium beam frequency standards, hydrogen masers, mercury ion frequency standards, and various commercial atomic clocks including those in space. All of the primary timing centers' outputs, which are available to NBS via the Global Positioning System common-view measurements, were used in the study. The millisecond pulsar signal from PSR 1937+21 was used as well. The efforts to search for a ''best clock'' have the advantage that post-analysis better helps us characterize and deal with systematic deviations.

Theoretically, a perfect clock cannot be built because of quantum mechanical limits in any given atomic resonance and because of the noise inherent in all physical systems. Even if we imagine having a perfect clock, as soon as its time is transformed to some other frame of reference, the uncertainties of that transformation will limit the accuracy of its apparent reading. In addition, the interrogating electronics and associated measurement systems will not only perturb but also limit a clock's apparent performance. However, there is no known theoretical limit to how nearly perfect a clock or a clock system can be.

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This paper investigates both the performance of the best clocks known, and some ways to gain significant improvements. We show that steps one, two, and three, properly combined, all help considerably in moving us toward the ideal clock.

This investigation has been driven in large measure by the observed outstanding long-term clock performance of the millisecond pulsar discovered in 1982 [6], [7]. This millisecond pulsar has already turned out to be an extremely useful timing tool, and improving the performance of earth-bound atomic clocks has the potential for opening up several important measurements—not the least of which is the measurement of gravitational waves [3], [8], [9].

II. Perspective

Simplistically, a clock's time can be thought of as reading an accumulator of equally spaced events counted from some predetermined origin. The first step in time metrology is to employ as precisely periodic a phenomenon as possible, such as the cesium hyperfine frequency. Using an atomic resonance for step one was first envisioned by Rabi in the early 1940's [10]. Lyons brought Rabi's idea to fruition in 1948 with the first ammonia maser atomic clock [10], [11]. Its accuracy, however, was only slightly better than could be obtained from astrometry, and it was not kept in continuous operation. Further work in the 1950's by Lyons demonstrated that the above mentioned cesium hyperfine frequency was a better choice for this first step in time metrology [10].

However, the second important step in atomic time metrology was not realized until June 1955 with the work of Essen and Parry [12]. They were able to accumulate the oscillations associated with this cesium resonance as the periodic events. Atomic time keeping had its birth with their work.

Conceptually, if we had only one atomic clock, we could simply define time as its reading. In practice, however, an atomic clock cannot be kept running indefinitely; physical things fail. In addition, the stochastic behavior of a clock's output cannot be estimated unless we have at least three independent clocks [13]–[15]. If three are always needed to ascertain performance, then at least four are needed should one fail, etc. Since every clock's time differs from every other clock's time, except for at most an instant, the third important step in time metrology is to find an algorithm which will optimally and reliably combine the readings of a set of clocks [1], [16], [17]. In this paper we document the importance, effectiveness, and efficiency of this third step.

III. The Performance of Primary Frequency References

Since the first atomic frequency standard was built, the accuracy of determining an atomic resonance for step one has improved by about an order of magnitude every seven years. This trend is expected to continue, and we should be prepared to take optimal advantage of the new standards as they become available.

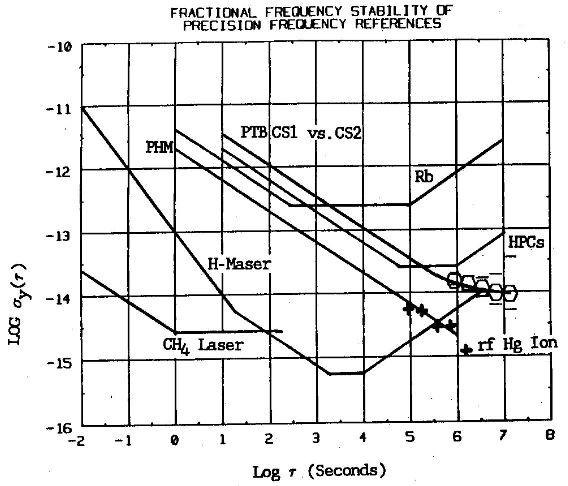

Fig. 1. A $\sigma_y(\tau)$ versus τ plot of the measured stability of some of the best frequency references. The hexagons at the right of the PTB CS1 versus PTB CS2 plot are the actual values from 800 days of data. The Rb is for a commercial rubidium gas-cell frequency standard. Much better long-term stability has been obtained for rubidium under good environmental control and for some types [29]. The "HPCs" stand for a high-performance commercial cesium-beam frequency standard and "PHM" for a passive hydrogen maser. The "+" symbols are compliments of the USNO for their RF trapped mercury-ion standard versus their active hydrogen maser, M19, for 36 days of data. "H-Maser" stands for an active hydrogen maser with the measurement system bandwidth, f_h, set at 500 Hz; the stability shown is $1/\sqrt{2}$ times the stability between a pair—under the assumption of equal contribution. "CH$_4$" is for methane-stabilized laser.

The stochastic behaviors of some of the main frequency references are plotted in Fig. 1 [18], [19]. It is obvious from this figure that what is best for some is not best for others, etc. The proper frequency standard to employ should be a function of the need. In practice, most atomic frequency standards utilize a quartz crystal oscillator electronically servoed to the atomic resonance with a servo time constant chosen to take advantage of the short-term stability of the quartz oscillator and the long-term stability of the atomic resonance. With the advance of computer technology it is now possible to servo (using digital techniques) via an algorithm so that a software time is generated which takes advantage of the best frequency stability at any integration time, τ. Shorter integration times are still impractical because of data storage and computation speed, but that will continue to improve with time.

The frequency instabilities resulting from systematic and environmental perturbations are of basic importance, and often have long-lasting effects. The causes of these instabilities have to be considered in order to approach "best clock" performance. This aspect is treated elsewhere, and so only generally relevant aspects are discussed here [20].

Fig. 2 shows some of the possible frequency standards of the future [18]. Quartz oscillators will probably not be adequate as the local oscillator without special considerations and higher order servo loops [21]. One of the big-

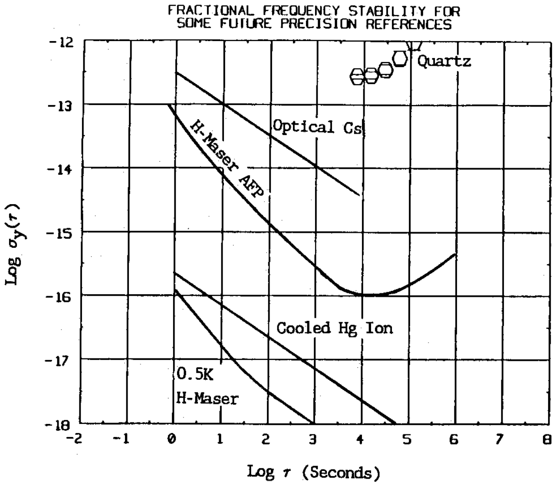

Fig. 2. "Optical Cs" is the estimated stability for a cesium-beam frequency standard using optical energy state selection and detection. The numbers are estimated from some collaborative work between NBS and NRLM [30]. "Cooled Hg Ion" refers to a future laser cooled-mercury trapped-ion standard with a theoretical accuracy of a few parts in 10^{18}. The "H-M AFP" denotes an adiabatic fast passage technique applied to a Hydrogen Maser [31]. "0.5 K-H-Maser" refers to a future super-cooled hydrogen maser [32]. Some state-of-the-art stability values for a quartz crystal oscillator are plotted to illustrate the challenge of developing an adequate local oscillator [21].

gest challenges of the future is performing step two for the infrared and optical frequency standards being developed; i.e. accurately counting these frequencies.

Of course, a frequency standard need not be operated continuously as a clock. In fact, some standards need to be turned off during the evaluation process to determine their accuracy. If better accuracy is obtainable in this mode of evaluation than can be achieved by running them continuously, and if adequate fly-wheel clocks are available, then these periodic frequency calibrations should be built into the algorithm [22].

IV. A Clock from a Frequency Standard

Frequency is a fundmental quantity in physics. It is measurable with respect to some theoretical absolute value, such as the cesium hyperfine frequency. An accuracy can be specified as to how well a frequency is known. Since time is the integral of frequency, it is a function of the constant of integration, which is arbitrary, and of the frequency inaccuracy, which causes a ramping of the time error.

Einstein has shown that time is relative and has no meaning independent of some reference frame. Some have said that time is the reading of a clock. We, certainly, can measure the time of an event against some clock reading, such as a picture of the limb of the moon and background star field against the clock that tripped the camera shutter in our frame of reference. But a given event can have a different time of occurrence from every independent frame of reference. In fact, we know of no experiment in which we can measure this illusive and abstract quantity, time.

We often measure the time difference between two clocks, and this difference can be recorded with a confidence limited by the precision of the measurement. But unlike frequency, there is no theoretical absolute value for time. And "time accuracy" only has meaning with respect to some defined or agreed upon standard in a specified frame of reference. "Time accuracy" has no meaning for a primary reference clock.

Interestingly, the realization of time difference measurements with a current technology involves important considerations, both theoretical and practical, that were not originally considered by Einstein in his thought experiments with perfect clocks. Since the highly accurate era of atomic clocks postdates the theory of relativity, the limitations to frequency accuracy due to both the Heisenberg uncertainty principle and the finite number of atoms available in a cesium beam primary frequency standard, for example, were not appreciated. These limitations alone will cause two independent, perfectly designed, primary-frequency standards to have a time difference which is unbounded (random walk) and always nonzero except at an instant. In addition to the above-mentioned finite frequency inaccuracy, which will always exist in any standard, there are all the systematic, environmental, and stochastic perturbations which cause a clock's time to walk away from the conceptualized ideal [15]. The above-mentioned random-walk-time modulation will cause the time difference between two clocks to disperse as the integration time to the one-half power. Other important, long-term-limiting stochastic models for atomic clocks are flicker-noise-frequency modulation (FM) and random-walk FM. These will drive the root-mean-square (rms) time difference dispersion as the integration time and as the integration time to the three-halves power, respectively [15].

Knowing the frequency stability, as illustrated above, an rms time interval error can be estimated [15], [23], [24]. The error in how well we can predict the time difference between a pair of clocks, for example, will be a function of how we use the past time difference readings. If we use the data properly, we can minimize the error of prediction. By optimum prediction we usually mean to minimize these errors in an rms sense. A nominal estimate of this rms error as driven by the random perturbations affecting the time difference between a pair of clocks is given by $k\tau\sigma_y(\tau)$, where k depends on the model for the random perturbations and is usually near unity. This is a measure of the time prediction capability of a clock or the rms way its time will tend to walk away.

One sees that very sophisticated equipment is needed to measure the time or phase deviations over short intervals. Phase error multiplication via frequency multiplication coupled with double heterodyne techniques, such as the dual-mixer-time-difference method, may be necessary to measure future clocks as we move into subpicosecond-time-difference requirements.

We see from the above that time from a clock is the accumulation (integral) of periodic events (inherent frequency reference) from some agreed upon value (constant of integration); and further, that its time will be perturbed with respect to another clock both by relativistic movements and by systematic and random perturbations. Because of these perturbations every clock is wrong with respect to some conceptualized ideal, and correct time is a matter of agreement and definition.

V. Ensemble Time

The NBS ensemble algorithm from which UTC(NBS) is derived features two weighting factors per clock [1]. In theory, the algorithm's free-running output, NBS(AT1), should have better short-term and better long-term stability than the best clock in short-term and the best clock in long-term, respectively, in that ensemble. This algorithm has been running since 1968, and much experience has been gained. Another feature of the NBS algorithm is that a clock's weighting factors are adaptive, conforming to its performance as it ages—sometimes getting better and sometimes getting worse toward end of life. This algorithm's output has been tested with respect to other time scales and using simulated data. The theory appears to be consistent with these tests.

Recently, a global Kalman ensemble time was generated utilizing the data from all the principle timing centers available via GPS [25]. Fig. 3 is a plot of the frequency stability between this ensemble time and NBS(AT1). For sample times shorter than one day the time/frequency Kalman smoother biases the stability values too low. For τ greater than or equal to one day the plotted values obviously fit the theoretical curve very well.

Some people object to having a software clock, even though it is better than the best physical clock. To satisfy those we have built a digital-computer-controlled servo to control the time of a physical clock. Fig. 4 is a frequency stability plot showing how well this micro-phase-stepper output tracks the software clock.

The Global Position System is anticipating the use of ensemble time both at the Master Control Center and for the satellite vehicle (SV) clocks. We have encouraged and supported these plans, and to show the quality of such an effort we employed the NBS time scale algorithm used in generating AT1 and applied it to the SV clocks [5]. We used SV cesium-beam clocks and the one temperature-controlled rubidium gas-cell clock for the SV ensemble. This set of space clocks also gave us an independent reference with a significantly decoupled environment. The $\sigma_y(\tau)$ versus τ plot of the stability of this space ensemble against NBS(AT1) gave rise to a performance better than most of the other timing centers in the world [3]. The long-term stability was well modeled by a flicker noise FM ($\sigma_y(\tau)$ was independent of the sample time, τ) for 10 days $< \tau <$ 1/2 year at a level of about 2×10^{-14}. Apparently the environment plus the time-scale algorithm make the difference.

We had previously reported some possible annual variations in the PTB CS1 primary frequency standard using the millisecond pulsar and NBS(AT1) as references [3].

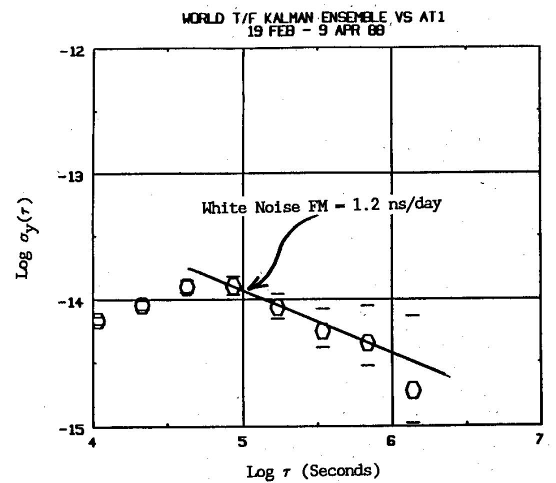

Fig. 3. A 50-day analysis of the relative stability between a new time/frequency Kalman algorithm for calculating global time from all the principal timing centers using GPS in common view and NBS(AT1) [25]. The Kalman smoother characteristics cause the stability values to be too optimistic at $\tau = 1/8, 1/4, 1/2$ day. The theoretical frequency stability of the NBS(AT1) ensemble output is plotted for comparison, and it agrees very well where it should.

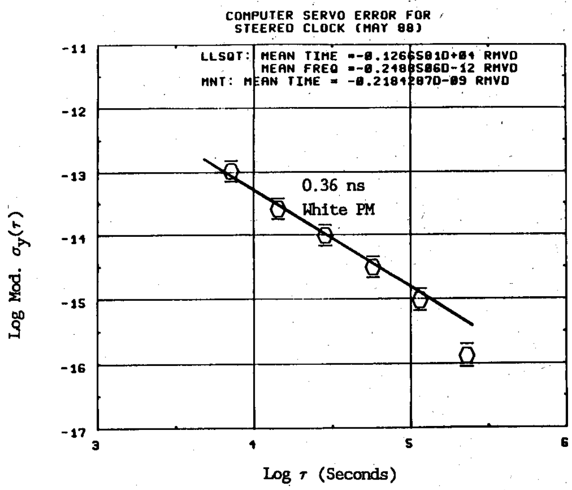

Fig. 4. A plot of the Mod.$\sigma_y(\tau)$ stability of the computer servoed real-time output of UTC(NBS) as derived from the software clock computation of NBS(AT1); The slope of the solid line model is indicative of white noise PM and it seems to be a good model at a level of about 341 ps [15].

Recent investigations have revealed a significant dependence of some commercial clocks' frequencies on the relative humidity [5]. This effect would give rise to a possible correlation mechanism for Northern or Southern hemisphere clocks. Since most of the clocks used in the generation of TAI are Northern hemisphere clocks, this could be a possible cause of annual variations. Further study will be necessary before we can determine with some confidence the cause of these annual variations.

Terrestrial time as generated at the BIPM during 1988 and denoted TTBIPM88 is generated in retrospect. It is based on an algorithm with special weightings for the set

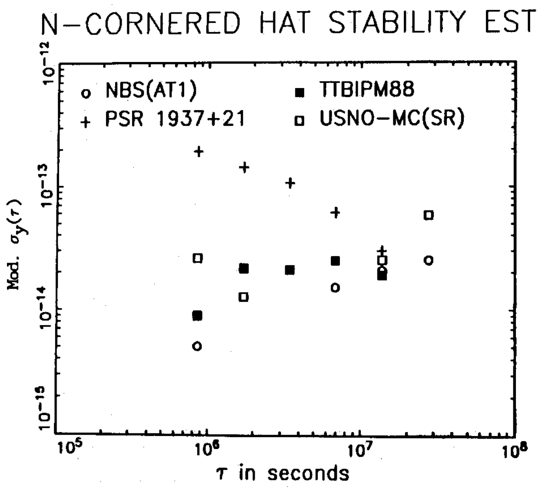

Fig. 5. An N-cornered hat frequency stability estimate with five nominally independent contributing clocks; four are ensemble times from a set of clocks. The clocks are not perfectly independent, as NBS and USNO both contribute in the computation of TAI. The same clocks are used in the computation of TTBIPM88, but with a different weighting algorithm. The contributions of the NBS and USNO clocks to the computation of this latter scale are probably less than 5 percent and 10 percent, respectively. Because of the correlations and apparent correlations, due to finite data lengths (this data set covered 1060 days), negative variances may result [13]. If the clocks are uncorrelated and a negative variance for any given clock, we can infer that that clock is better than the best one plotted at that particular value of τ. That was the case for NBS(AT1) at $\tau = 20$ and 40 days, and for TTBIPM88 and PSR 1937+21 at $\tau = 320$ days. If a frequency drift is subtracted from the USNO-MC(SR) data, the stability values improve at $\tau = 160$ and 320 days. The "(SR)" means that the coordinated steering corrections have been subtracted so that the time scale is independent of these.

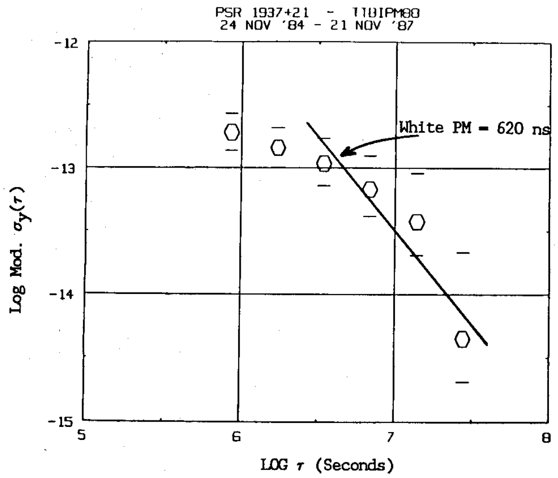

Fig. 6. A plot of Mod.$\sigma_y(\tau)$ for the millisecond pulsar, PSR 1937+21, versus TTBIPM88. The pulsar residuals were originally fitted against UTC(NBS), which is also a steered coordinated scale. A measurement noise of 620 nanoseconds is plotted for comparison. The average sampling interval for the pulsar data was 17 days. Ten-day, equally spaced interpolated values were used in order to have matching pulsar values with the other time scales. Thus interpolating the data causes a filtering effect, and hence, the stability values at $\tau = 10$ and 20 days are too optimisitc.

of some 160 clocks in order to meet the most stringent requirements for long-term stability as for the studies of millisecond pulsars. Most of the clocks in TTBIPM88 are commercial cesium clocks, a few are primary cesium clocks and few are hydrogen masers. TTBIPM88 ensemble time along with the USNO master clock with the coordinated time steering removed to make it an independent ensemble time, the compensated arrival times of the millisecond pulsar (PSR1937+21) and the NBS(AT1) ensemble time were used in an n-cornered hat relationship to estimate the stability of each of the four. The estimated stabilities are plotted in Fig. 5. Fig. 6 is a frequency stability plot between TTBIPM88 and PSR 1937+21, confirming the probable precision is about 1×10^{14} for $\tau = 320$ days. The significant drop in the stability from $\tau = 160$ days to $\tau = 320$ days is probably due, in part, to the annual variations in TTBIPM88. Both Figs. 5 and 6 confirm the outstanding long-term stabilities of TTBIPM88 and PSR 1937+21.

VI. Conclusions

Using an optimized algorithm for determining the time and the rate of a time scale has several advantages. Some of these include: redundancy and reliability to shield against clock failures, optimum frequency stability performance, optimum use of calibrations with a primary frequency standard or time reference, optimum error detection and rejection, minimum adverse effects on the time scale output due to abnormal performance of a member clock, optimum steering to a remote time scale, the inclusion of robust statistical approaches in dealing with data outliers [26]–[28], minimum and optimum calibration interval for characterizing the performance of new clocks or visiting clocks, minimum perturbation effects as clocks are added or deleted from the time-scale clock ensemble [29], significant economic savings in achieving the same level of performance and optimum estimation of time scale perturbations due to systematic and environmental effects.

There are, of course, disadvantages as well, including the increased complexity as compared to a single clock, single points of failure, and clock models which mismatch reality. But so far it appears that the advantages far outweigh the disadvantages, and none of the disadvantages are serious.

Due to space limitations, only a few of the more significant time and frequency metrology advantages have been discussed in the body of this paper. One of the purposes of this paper is to point out these opportunities. Without the use of clock ensemble algorithms it would have been impossible to estimate the stability of the millisecond pulsar PSR 1937 + 21 at one year sample time (Mod. $\sigma_y(\tau = 360 \text{ days}) \lesssim 1.5 \times 10^{-14}$). Within about ten years we project that, by properly using our clock as outlined at the start of this paper, it should be possible to arrive at the 1×10^{-15} level of stability for $\tau = 1$ year. At this level of stability there is a finite possibility of observing gravitational waves in comparing an optimum-combined ensemble of earth-bound atomic clocks with millisecond pulsars.

In a more general sense, as we look to the future, the integration of frequency standards, clock measurement systems, computers, algorithms and international data networking will become increasingly important. It may take substantial worldwide cooperation on the part of time and frequency metrologists—properly combining all three steps outlined above to reach these goals. Future algorithms will need to make better use of information theory, maximum likelihood estimation techniques, Kalman filters, efficient statistical processing, models for systematic effects, and of ways to compensate for environmental perturbations. For example, evidence now indicates that there are annual variations of a few hundred nanoseconds in some time scales.

We have not discussed the advantages of post analysis of the data, an idea which can be very useful for research. The TTBIPM88 is such a scale—constructed for the most stringent requirements in long-term stability. It appears to have achieved that goal. In general, time keeping is a problem of determing "what time it is now!" This, of course, precludes rerunning an algorithm with the benefit of hindsight to generate a new time. Clearly, with that additional information, a smoother time scale than can be done in real time can be generated in retrospect. The UTC(k) time scales generated by the various timing centers in the world are as near as practicable real-time scales; this includes NBS(AT1), discussed in the paper, from which UTC(NBS) is derived. The NBS algorithm and clock ensemble for generating NBS(AT1), as a real-time time/frequency metrology tool both for research into new standards and for a basic reference for calibrations, have now been used for about two decades. Better algorithms are being studied; insights into environmental perturbing effects are being gained. With continued data sharing and international cooperative effort, all time scales stand to benefit. Significant gains are available, even now, by better utilizing our clocks as cited above—both for real-time scales and for postprocessed time scales. The output of such cooperation is the "best clock" possible.

The spin offs for industrial applications may also be significant. What is now operating in the laboratory could well be produced as a computerized instrument. We would envision a timing box which would monitor more than one clock producing a time and frequency output better than any of the contributing input clocks. The box would properly process aperiodic inputs from a variety of sources such as from a primary frequency standard or from a local time reference, or from a satellite reference. The box could act as a source of time and frequency or could be synchronized and/or syntonized to a remote reference. The effects of failures and of time and freqency steps in individual clocks would be minimized by such a device. The systematic and environmental sensitivities of individual clocks could be programmed into the box. As time elapsed, after a clock was plugged in, the box would improve its knowledge of the performance characteristics of that clock—hence, it could serve as a calibration facility. With the speed, data storage, and processing capability of future desktop computers, we could well imagine such a machine interacting with this timing box and providing all the information we need to know the performance of all relevant aspects of the system.

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