USING MULTIPLE REFERENCE STATIONS TO SEPARATE THE VARIANCES OF NOISE COMPONENTS IN THE GLOBAL POSITIONING SYSTEM

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Abstract

The separation of variance technique has been applied to measurements of a clock against received signals from Global Positioning System (GPS) satellites to separate out various noise components in the system. In this paper we extend the previous work in several ways. First, we show how measurements can be taken from several different locations to obtain estimates of more components of the GPS system, and to obtain better estimates of the components previously studied. We show how to estimate the variances of the following five components: the GPS system clock, the error in the transmitted correction term between the satellite clock and the GPS system clock, propagation noise in the measurement including ionospheric and tropospheric modelling errors, error in the transmitted ephemeris for the satellite, and the local reference clock. We consider the effects of correlations between elements of the data, and analyze the confidence one may have in our estimates in light of these. Finally, this multi-station separation of variance technique is applied to recent GPS data. We discuss new insights into the GPS system that have been learned using this technique.

Introduction

The Separation of Variance Technique as applied to the Global Positioning System has been reported previously (1) using data taken from one location. This paper reports an expansion of that technique using data taken from multiple reference stations and considering the effect of correlations. Using multiple references allows one to separate propagation noise from ephemeris error variances, as well as providing greater confidence in the estimates of the other noise components: the GPS master clock, the error in the satellite clock correction terms, and the satellite vehicle (SV) clocks. From the study of correlation effects we find better ways to use the data for our estimates as well as understand better some of the limitations in the separation of GPS noise components technique.

We first discuss the method employed for taking data. Next we consider the possible variances one may compute using this data and their components. There are two kinds of variances we discuss: Allan variances of data types, and Allan variances from applying the "N-corner hat" technique to Allan variances of differences of data. Thirdly we discuss the correlation terms contained in these computed variances. Finally we show how to solve these equations for individual noise components in a way which minimizes the effect of correlations as well as indicating the magnitude of some of these correlations. We then apply this technique to recent data, January and February of 1986, and discuss the results.

Data

There are two kinds of time measurements we obtain from a GPS receiver: (GPS-Ref)', a measurement of the GPS system clock against the local reference clock with various kinds of noise, and (SV-Ref)', a measurement of the particular GPS satellite vehicle clock against the local reference clock with noise. In practice the (SV-Ref)' is obtained from direct measurements of the received signal against the local reference clock, then corrected by the GPS receiver to account for the motion of the satellite vehicle (SV) using the ephemeris as transmitted from the SV in the real time bit stream. The (GPS-Ref)' value is obtained by decoding the SV clock correction term from the bit stream, and adding it to the (SV-Ref)' value along with a relativistic correction. We have

$$(\text{GPS-Ref})' = \text{GPS} + \text{Clx}' + \text{Prop} + \text{Eph} - \text{Ref},$$

where

GPS	= the GPS master clock time variation,
Clx'	= error in the transmitted correction term between the SV clock and the GPS system clock,
Prop	= propagation noise in the measurement including ionospheric and tropospheric modelling errors, receiver noise, and local coordinate errors,
Eph	= error in the ephemeris for the SV as transmitted,
Ref	= the local reference clock time variation

Also:

$$(\text{SV-Ref})' = \text{SV} + \text{Prop} + \text{Eph} - \text{Ref},$$

where

SV	= the satellite vehicle clock phase variation

and where other terms are defined as above.

We obtain these two numbers by tracking and averaging a particular satellite for 13 minutes: (GPS-Ref)' and (SV-Ref)'. Since the satellites are in 12 hour sidereal orbits, we may repeat this measurement one sidereal day later and maintain the same geometry. This is important since the control segment's estimates and uploads of system parameters are all tied to the sidereal period of the orbits. In this way we obtain a time series whose noise components are well defined. The Clx' and Eph terms, as well as the Prop term, to the extent it reflects ionospheric modelling errors, are all defined as errors in the control segment's estimates. We can solve for the variance of these only because our measurements are made with the period of the system being one sidereal day, the same for the control segment and for the users.

Computed Variances

Allan Variances of Data Types

Since we have two time series, the (GPS-Ref)' and (SV-Ref)' data, we may compute three independent Allan variances: the variance of each data type and the variance of their difference. Let us denote these fractional frequency variances as

AG = the variance of the (GPS-Ref)' data type,

AS = the variance of the (SV-Ref)' data type,

AGS = the variance of the (GPS-Ref)' - (SV-Ref)' data,

which is simply the variance of the transmitted clock correction term.

Let us denote the variances of the noise components as

G = the fractional frequency variance of GPS,

C = the fractional frequency variance of Clx',

S = the fractional frequency variance of SV,

P = the fractional frequency variance of Prop,

E = the fractional frequency variance of Eph,

R = the fractional frequency variance of Ref.

With this notation we may write the variance of each data type in terms of the variances of the components. The variance of a sum or difference of random variables is the sum of the variances plus cross-correlations. We will consider correlations later. For now let us assume independence of the components, and we have

$$AG = G + C + P + E + R$$

$$AS = S + P + E + R$$

$$AGS = G + C + S$$

From these three equations we may solve for G+C, S, and P+E+R. If the characteristics of the reference clock are known, we may estimate R independently yielding an estimate of P+E.

The other variances we compute are those comparing differences from at least three SV's. To describe these we first discuss the "N-corner hat" technique.

N-Corner Hat Theory

The separation of variance technique grew out of the desire to know the stability of a particular clock given the fact that measurements of clocks must be made in pairs. The "three corner hat" technique (2) has been used, where pair-wise measurements are made among three clocks and the fractional frequency variances of the individual clocks can be found under the assumption of independence of the noise processes of the three clocks. This works as follows:

Let X_{ij} be the time difference measurement clock i minus clock j. Then if the S_{ij}^2 is the variance of the time series X_{ij}, under the assumption of independence, we have

$$S_{ij}^2 = S_i^2 + S_j^2 ,$$

where S_i^2 is the variance of clock i alone.

In this way we find

$$S_i^2 = (S_{ij}^2 + S_{ik}^2 - S_{jk}^2)/2 .$$

Note that, due to finite data sets and correlations in the data, estimates of variances here can be negative. This can usually be taken to imply that the true level of that clock is significantly below the levels of the worst clock.

This technique can be expanded to allow for N-clocks (3). This is an improvement since the above is an exact solution only if there is perfect independence. In practice, because of a finite data length there is some apparent correlation between clocks even if they are physically independent. Also, there are often mechanisms for real correlations. We will discuss later how these appear in GPS data.
The "N-corner hat" is defined as a least squares estimate. We minimize:

$$A = \sum_{j=2}^{N} \sum_{i=1}^{j-1} (S_{ij}^2 - S_i^2 - S_j^2)$$

in order to obtain the solution

$$S_i^2 = (\sum_{j=1}^{N} S_{ij}^2 - B)/(N-2) ,$$

where

$$B = (\sum_{i=1}^{N} \sum_{j=1}^{N} S_{ij}^2)/2(N-1) .$$

Note that here, again, estimates of variances can be negative.

N-Corner Hat Computed Variances

We apply the N-corner hat technique to the problem of separating GPS noise components first by differencing our time series between pairs of SV's, and later by differencing our data between pairs of locations. Let us denote our (GPS-Ref)' data via SV "i" minus SV "j" as $(GPS\text{-}Ref)'_{ij}$, and similarly $(SV\text{-}Ref)'_{ij}$ and their difference $(GPS\text{-}SV)'_{ij}$, this last being simply the difference of the transmitted clock corrections from the two SV's. In terms of our previous notation, and continuing our use of "ij" for data via SV "i" minus SV "j" we have

$$(\text{GPS-Ref})'_{ij} = \text{Clx}'_{ij} + \text{Prop}_{ij} + \text{Eph}_{ij} \,,$$

$$(\text{SV-Ref})'_{ij} = \text{SV}_{ij} + \text{Prop}_{ij} + \text{Eph}_{ij} \,,$$

$$(\text{GPS-SV})'_{ij} = \text{Clx}'_{ij} - \text{SV}_{ij} \,.$$

We see that the clocks in common across different SV's, i.e. the GPS and the reference clock, cancel when we difference the data. This is true exactly only if the measurements are taken simultaneously. If they are not, as is true in the NBS case, then there is a constant time offset in the above phase data from both clocks plus a random fluctuation due to any stochastic nature of the two clocks. These terms can be neglected when we take variances of the data if the noise of the clocks is small enough over the time intervals between tracks. Since the reference clocks we use are those in primary time standards labs around the world this is true at the nanosecond level in our case. Also we find the GPS clock is good enough to allow time intervals between tracks up to 1/3 to 1/2 day.

If we take the Allan variance of each of these time series we obtain variances of the sum of the components plus any cross-correlation terms. Since we will consider correlations later let us assume independence for now. Then we simply have a sum of variances, each being the variance of the difference of a single noise component between two SV's. These we may separate using N-corner hat, again under the assumption of independence, if we have differences among at least three SV's. For example if we start with

$$\begin{aligned}\text{AGS}_{ij} &= \text{the Allan variance of } (\text{GPS-SV})'_{ij} \text{ data,}\\ &= C_{ij} + S_{ij}\end{aligned}$$

we may use the N-corner hat technique to solve for

$$\text{NGS}_i = C_i + S_i.$$

Similarly, we may solve for NG_i and NS_i by using the N-corner hat technique on Allan variances of $(\text{GPS-Ref})'_{ij}$ and $(\text{SV-Ref})'_{ij}$ data respectively. We may now suppress the index "i" to see what computed variances are available for a specific track of an SV. Thus in addition to the AG, AS, and AGS terms listed above we have

$$\begin{aligned}\text{NG} &= C + P + E\,,\\ \text{NS} &= S + P + E\,,\\ \text{NGS} &= C + S\,.\end{aligned}$$

Here we see we could separately solve for C, S, and P+E. Since we saw before when we considered AG, AS, and AGS that we could solve for G+C, S and P+E+R, by using data from several satellites and the N-corner hat we can separate G from C and P+E from R, and we have redundancy in estimating S. Unfortunately, we shall see later that there are correlation terms to consider which effectively remove the redundancy. Also, we find the confidence of the estimate in N-corner hat depends on the relative size of the noise components. This makes it difficult to estimate R, the reference clocks, if they are significantly quieter than the SV clocks. Before we look in detail at these ideas, we show how we may use common view data from several locations to separate P from E.

We go back to consider (SV-Ref)' data, but now we look at how we can use this data when we have the same SV measured against several locations tracking simultaneously. If we now subtract (SV-Ref)' data taken at location "A" from data taken in common view at location "B" we find

$$(\text{SV-Ref})'_{AB} = \text{Prop}_{AB} + \text{Eph}_{AB} - \text{Ref}_{AB}.$$

We see that the SV clock cancels. Further, the Eph_{AB} term is the differential ephemeris error which tends to cancel dependent on the baseline between locations (4). If we compute the variance of this common view data and apply our N-corner hat technique we find that the original ephemeris variance is reduced by products of common view cancellations (see appendix). The result is that, if we solve for a particular location from N-corner hat, the variance of the ephemeris error is reduced by at least an order of magnitude, which effectively makes it negligible compared to the other terms. If we denote this variance for a given location as NL, we have:

$$\text{NL} = P + R.$$

Since we have estimated P+E+R previously, we may use this to separate E from P+R.

Where Correlations Occur in the Computed Variances

Let us now consider the effect of correlations between various noise components in the system. The physical clocks we believe to be statistically independent. The elements which are estimated by the GPS control segment, however, can have correlations dependent on the ways in which they are estimated. The estimates of SV clock and ephemeris are made based on measurements of signals of the satellites against ground station clocks which are referenced to the GPS master clock. Thus there should be correlations among noise components we have denoted Eph, Clx', and GPS. In practice, the ephemeris is estimated in advance for about 10 days. The GPS real time Kalman Filter then estimates the clock correction value and corrects the long term estimate of ephemeris. For this reason we expect the Clx' and Eph terms to be more highly correlated than other terms, and the Clx' and GPS to be more correlated than Eph and GPS. Let us denote these various cross variance terms as G,C; G,E; and C,E corresponding to the correlations between GPS and Clx', GPS and Eph, and Clx' and Eph, respectively. Then we have

$$\begin{aligned}\text{AG} &= G + C + P + E + R + G,C + G,E + C,E\,,\\ \text{AS} &= S + P + E + R\,,\\ \text{AGS} &= G + C + S + G,C\,,\\ \text{NG} &= C + P + E + C,E\,,\\ \text{NS} &= S + P + E\,,\\ \text{NGS} &= C + S\,,\\ \text{NL} &= P + R\,.\end{aligned}$$

We have 7 equations in 9 unknowns. If we estimate R separately this does not improve, since the equation for NS separates nothing more from AS than precisely R. Thus, if we remove R from the equations above NS provides no new information and we have 6 equations in 8 unknowns.

Solutions

In practice what we need to do in order to obtain the variances computed above is to select tracks from each location so that each track is made in common view with at least three locations, and each location tracks every satellite at least once. We also estimate the Allan variance of each reference station clock. Then we need to select special tracks so that for each location we have a set composed of exactly one track for each SV. For each location we may then compute NG and NGS using the set of special tracks (and NS if we want to estimate the reference clocks), as well as AG, AS and AGS for each of these tracks. And, finally, for each of these tracks we compute NL using common view data from all the locations involved for the track. We solve for the following Allan variances of components for each of these special tracks, for each location

$$
\begin{aligned}
G + G,C &= AGS - NGS \ , \\
C + G,E/2 + C,E/2 &= (AG - AS - AGS)/2 + NGS \ , \\
S + G,E/2 - C,E/2 &= (-AG + AS + AGS)/2 \ , \\
E + G,E/2 + C,E/2 &= (AG + AS - AGS)/2 - NL \ , \\
P &= NL - R \ , \\
G,E &= AG - AGS - NG + NGS \ .
\end{aligned}
$$

We see that we solve for the five unknown noise components and one correlation term leaving two correlation terms still affecting our solutions. We note that we could remove the G,E term from our estimates of C, S and E. We choose not to for the following reason. Since the Eph term is a vector error and each track is made from at least three locations, and often these are over large baselines, the Eph term will enter with opposite sign for distant locations. To the extent we have an orthogonal look at the satellite, the G,E and the C,E terms should cancel when we average them over all locations. If we could subtract the G,E term from all of these, we would and attempt to average out only the C,E values. But remember, the above equations can only be used for the special set of one track for each SV for a given location so that we could compute NG and NGS. Thus it is only for these special tracks that we can estimate G,E. However we estimate G + G,C for each of these tracks for each location and average to improve the estimate. As we average tracks with different SV's the G,C correlation is different. It affects our final estimate of G only as an average correlation between the GPS master clock and all clock correction errors.

For the rest of the tracks we compute AG, AS, AGS, and NL. We use our estimated G + G,C also, but only averaged over the special track at each location for a given SV. We solve for the following Allan variances of noise components with possible correlations

$$
\begin{aligned}
S + G,E/2 - C,E/2 &= (-AG + AS + AGS)/2 \ , \\
C + G,E/2 + C,E/2 &= (AG - AS + AGS)/2 - (G + G,C), \\
E + G,E/2 + C,E/2 &= (AG + AS - AGS)/2 - NL \ , \\
P &= NL - R \ .
\end{aligned}
$$

The correlation terms enter here exactly in the form as for the special tracks above where we can use the NG and NGS terms. Thus we may average them, using the orthogonality of our measurement base to cancel some of their effect. We may check this by looking at the G,E/2 + C,E/2 residuals after averaging, and then subtract the G,E estimates for those that are among the special tracks.

Thus, we believe we have good estimates of the Allan variances of the GPS system clock, each individual SV clock, the clock correction error for each SV clock, and the ephemeris error for each SV. In addition the Allan variance for these components can be seen as a function of time of the sidereal day, thus allowing one to look for variations in noise components as a function of orbital position. We also have an estimate of propagation noise for each track of each SV from each location. These may be combined in various ways to look for different aspects of the propagation noise. It is a combination of both ionospheric and tropospheric modelling errors and turbulence, as well as multipath effects, coordinate errors at the antenna and receiver noise.

Results

This technique was applied to data taken over the period January 2 - February 27, 1986. Our computer program limited our study to 6 SV's measured from 9 locations. We studied SV's 6, 8, 9, 11, 12, and 13 (Navstars 3, 4, 6, 8, 10, and 9, respectively). The reference stations were the National Bureau of Standards (NBS) in Boulder, Co, the Jet Propulsion Laboratory's Deep Space Tracking Station at Goldstone, Ca (JPL), the National Research Council (NRC) in Ottawa, Canada, the U.S. Naval Observatory (USNO) in Washington D.C., the Paris Observatory (OP) in Paris, France, the Physikalisch-Technische Bundesanstalt (PTB) in Braunschweig, West Germany, the Tokyo Astronomical Observatory (TAO) and the Radio Research Laboratory (RRL), both in Tokyo, Japan, and the NBS radio station WWVH in Kauai, Hawaii. Thus, we have two stations in each of the following areas of the globe: West North America, East North America, Europe, and East Asia, and one station in Hawaii. This provided coverage of all satellites throughout the day with redundancy. The SV's were tracked simultaneously at NBS, JPL, NRC, and USNO, then later at NRC, USNO, OP and PTB, continuing around to OP, PTB, TAO, and RRL then to being tracked by TAO, RRL, and WWVH or TAO, RRL, NBS, and JPL. There were also combinations involving NBS, and JPL with OP and PTB or combining TAO, RRL, WWVH, NRC and USNO. In all there were 122 tracks per sidereal day taken from the 9 sites, all taken in common view among at least 3 sites with a total of 28 different common view track times per sidereal day. There was much information in the output concerning the SV's, the ground stations, and the GPS in general. We discuss some of it here.

First, in figure 1, we see the square root of the Allan variance, the Allan deviation, of the GPS master clock. This is the root mean square (rms) of all estimates taken over the 6 special tracks (1 for each SV) at each of the 9 locations. This behavior at a level of a part in 10^{13} is higher than one would expect from that clock in a good environment.

Figure 2 compares the levels of the three Block I satellites with rubidium clocks. The frequency drift was removed from each of these using a mean second difference estimator. The values reflect rms of estimates over all tracks of each SV at each location. The level we see for SV# 8 is typical of all three when studied over a year, and reflects the lack of constant linear drift over the period in question. This can be seen in figure 3 where we show the phase plot of SV# 8 against NBS with a drift removed. We see that this was a quiet period for SV's 6 and 9. Figures 4, 5, and 6 give more detailed information concerning these three spacecraft. We put the performance of the SV clock along with our estimates of clock correction error and ephemeris error variances on the same plot. Ideally, the clock correction error and ephemeris error levels should be somewhat below the noise level of the clock. This is because it is measurements against the clock that are used to make these estimates, and the redundancy of the measurements should bring the estimates below the clock noise. We see in all three cases that the clock correction error is somewhat above this ideal at one day of integration time. For the ephemeris, however, we see excellent behavior.

Next we look in figure 7 at our estimates of the clocks aboard SV's 11, 12, and 13 (Navstars 8, 10, and 9). SV# 11 was at that time using its 0.1 degree temperature controlled Rubidium clock, while SV's 12 and 13 were Cesium clocks. The performance level we see is consistent with other estimates. We note, however, that SV# 13 has an increase in variance at the two day integration time. This suggests a periodic behavior in the SV phase with a period of about four days. A study of phase plots suggests this is the case, though a cause is unknown. Figures 8, 9, and 10 give the SV clock noise levels on the same plots as the clock correction error and ephemeris error for these SV's, 11, 12, and 13, respectively, as before for the other SV's. Again we see that the clock correction error is somewhat less than ideal, while the ephemeris error level is excellent.

In this run of multi-station separation of variance we also examined cross correlation effects. In solving for the G,E and residual C,E terms we expected to see a reversal of sign for locations widely separated on opposite sides of a satellite. This occurred to some extent only for integration times of 1 day. Data taken at NBS and JPL in West North America and OP and PTB in Europe worked well for analyzing this effect. We conclude that the level of these correlation terms are below the confidence of our estimates for integration times longer than one day. Since we expect the C,E term to be among the highest correlations, this suggests that correlations in the system are not corrupting our estimates. Our limitations for now are the finite data length, and the use of N-corner hat which employs differences of variances.

Other results of note include comments on the estimates of reference clocks, the propagation noise, and on the behavior of the GPS over Asia. We found we were unable to estimate the behavior of the reference clocks, except for the clock at WWVH whose stability was in the 10^{13} level. In general we find that reference clocks can be estimated only to the level of the SV clocks. Our estimates of the propagation noise showed us that some of the locations in our ensemble are slightly noisier than the others. This could be due to multipath or antenna coordinate problems, or, as in WWVH, being closer to the equator where the ionosphere has more effect. Finally, by looking at the clock correction error variance as it behaved throughout the day we saw some tendency for it to be worse as satellites were over Asia or the Pacific.

Conclusion

In conclusion we see that this separation of variance technique has grown from being powerful in its inception to a technique providing a wealth of information about many important aspects of the GPS. We have found that the use of multiple references gives us greater confidence in the estimates of the physical clocks in the system, the GPS master clock and the SV clocks, as well as providing better estimates of the GPS Kalman estimation error: the error in the satellite clock correction terms and the ephemeris error variances which we can now in large part separate from the propagation noise. From the study of correlation effects we have found better ways to use the data for our estimates as well as to better understand limitations of the separation of GPS noise components technique.

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Appendix: Common View Cancellation of Ephemeris Error

Let us consider cancellation of ephemeris error among 3 sites in common view, the general case among N > 3 sites only having better cancellation. Let us label the sites A, B, and C. We already understand that (SV-Ref)' data contains an ephemeris error term which we denoted Eph. But to understand cancellation we must view the Eph term for location A as a vector ephemeris error, $\underline{Eph}$, projected in the direction e_A from the SV to the ground station A. Then

$$(SV\text{-}Ref)'_A = SV + Prop_A + \underline{Eph}*e_A - Ref_A$$

and

$$(SV\text{-}Ref)'_B = SV + Prop_A + \underline{Eph}*e_B - Ref_B,$$

so

$$(SV\text{-}Ref)'_{AB} = Prop_{AB} + \underline{Eph}*(e_A-e_B) - Ref_{AB}.$$

If we take the Allan variance of this expression we have

$$AS_{AB} = P_A + P_B + (e_A-e_B)*\mathbf{E}*(e_A-e_B) + R_A + R_B\ ,$$

where $\mathbf{E}$ is the covariance matrix of the ephemeris error. A simple computation using the linearity of the covariance matrix $\mathbf{E}$ shows that we have, in the three-corner hat solution

$$(AS_{AB} + AS_{AC} - AS_{BC})/2 =$$

$$P_A + (e_A-e_B)*\mathbf{E}*(e_A-e_C) + R_A.$$

The expression

$$(e_A-e_B)*\mathbf{E}*(e_A-e_C) \le (e_A-e_B)*(e_A-e_C)*E_{max},$$

where E_{max} is the maximum ephemeris error in an orthogonal coordinate system. This follows since, with $\mathbf{E}$ symmetric, we can choose a coordinate system which diagonalizes it. Thus the ephemeris error is reduced in the variance not simply by the common mode cancellation term (e_A-e_B), but by the dot product of a pair of cancellation terms! In the general case for N locations this expression becomes an average of pairs of cancellation terms. The worst possible case for this term occurs when 3 ground stations are located 120 degrees apart around a great circle with the SV on the orthogonal axis. In that case we find

$$(e_A-e_B)*(e_A-e_C) < 0.2 .$$

Of course, this case cannot occur in practice since the SV would be below the horizon at all sites. Thus we see that the variance of the ephemeris error cancels to at least an order of magnitude in the N-corner hat computed variance across locations, NL.

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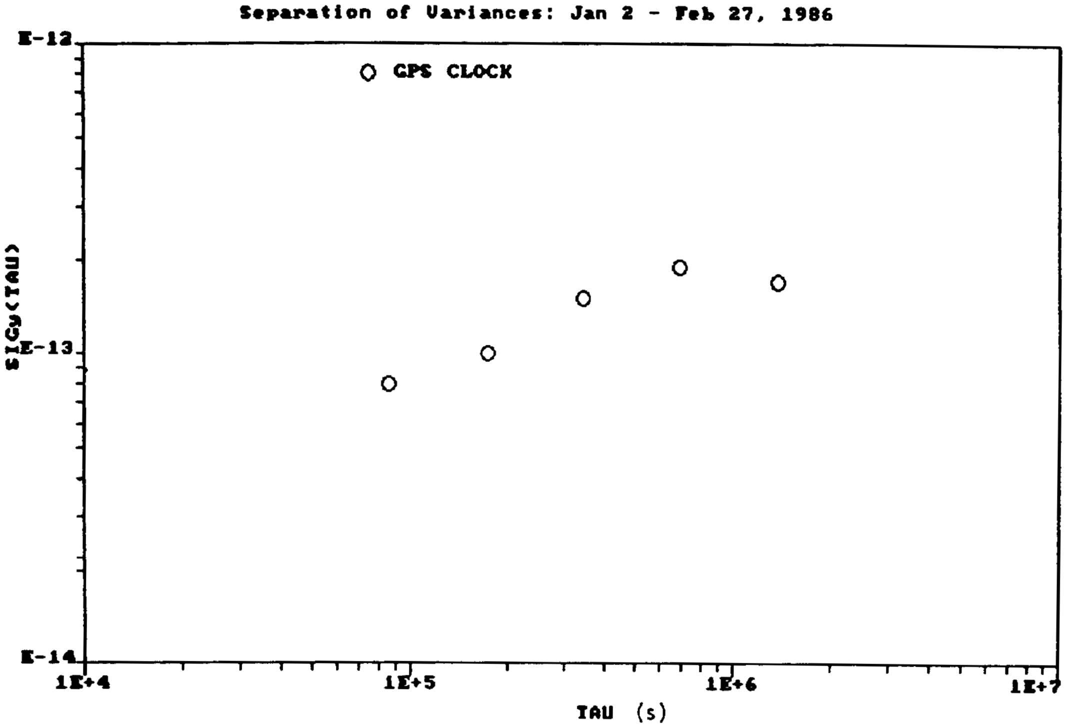

Figure 1: We see the square root of the Allan variance, the Allan deviation, of the GPS master clock. This is the root mean square (rms) of all estimates taken over the 6 special tracks (1 for each SV) at each of the 9 locations. This behavior at a level of a part in 10^{13} is higher than one would expect from that clock in a good environment.

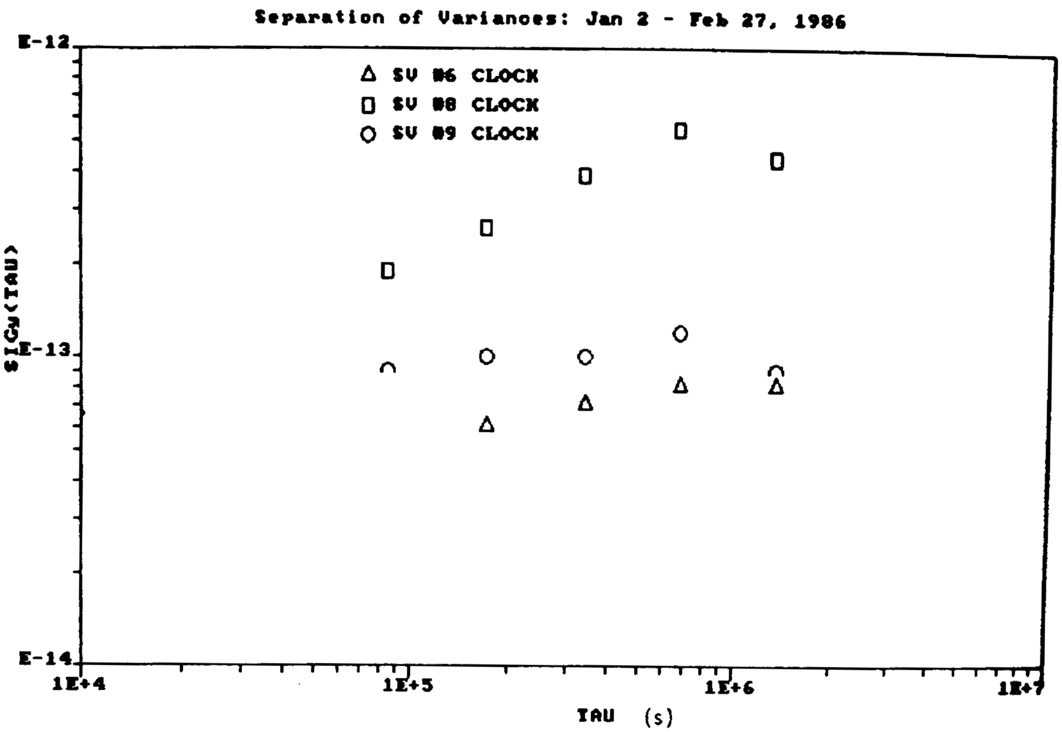

Figure 2: The levels of the three Block I satellites with rubidium clocks. The frequency drift was removed from each of these using a mean second difference estimator. The values reflect rms of estimates over all tracks of each SV at each location. The level we see for SV# 8 is typical of all three when studied over a year, and reflects the lack of constant linear drift over the period in question.

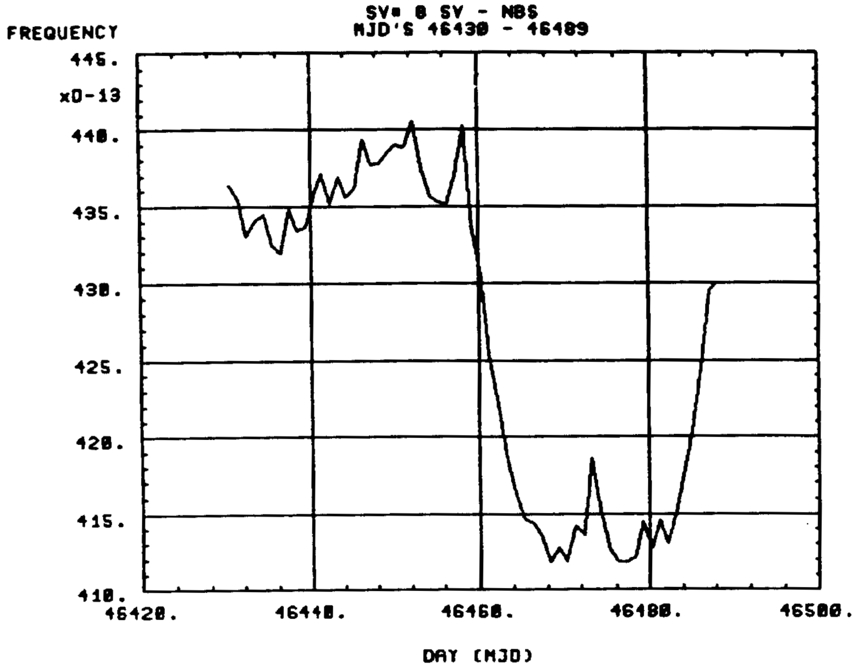

Figure 3: The phase plot of SV# 8 against NBS with a drift removed showing the lack of a constant drift over the period of this analysis.

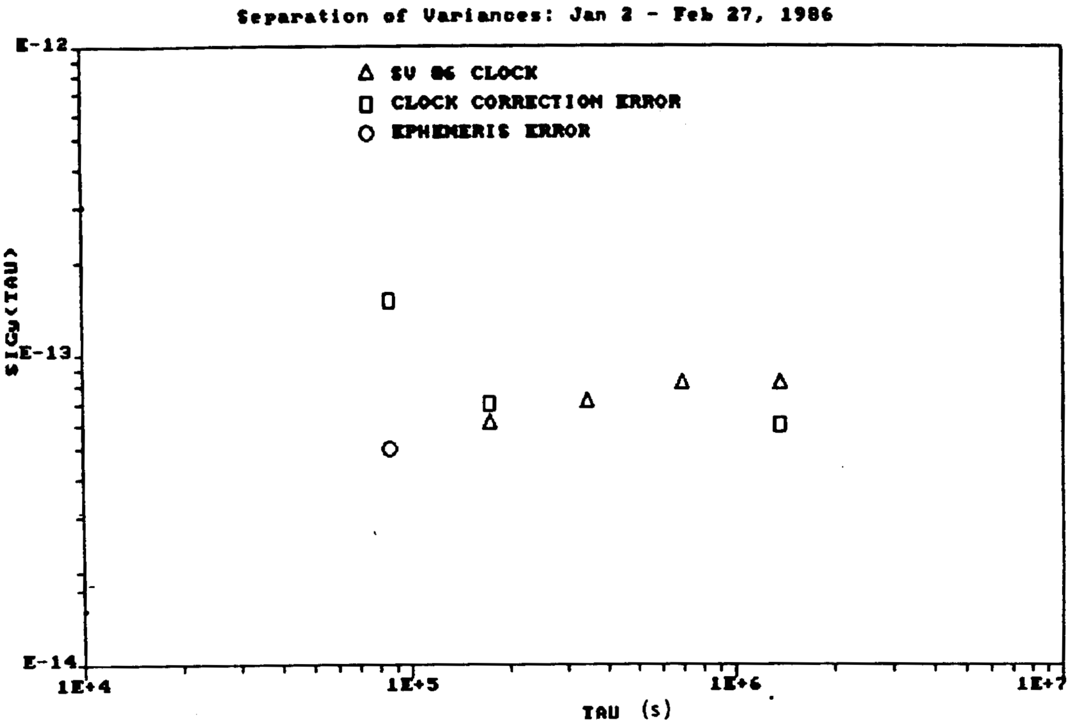

Figure 4: The noise or error levels of SV# 6 (Navstar 3) components. The clock correction error level is worse than the noise level of the SV Rubidium clock at one day.

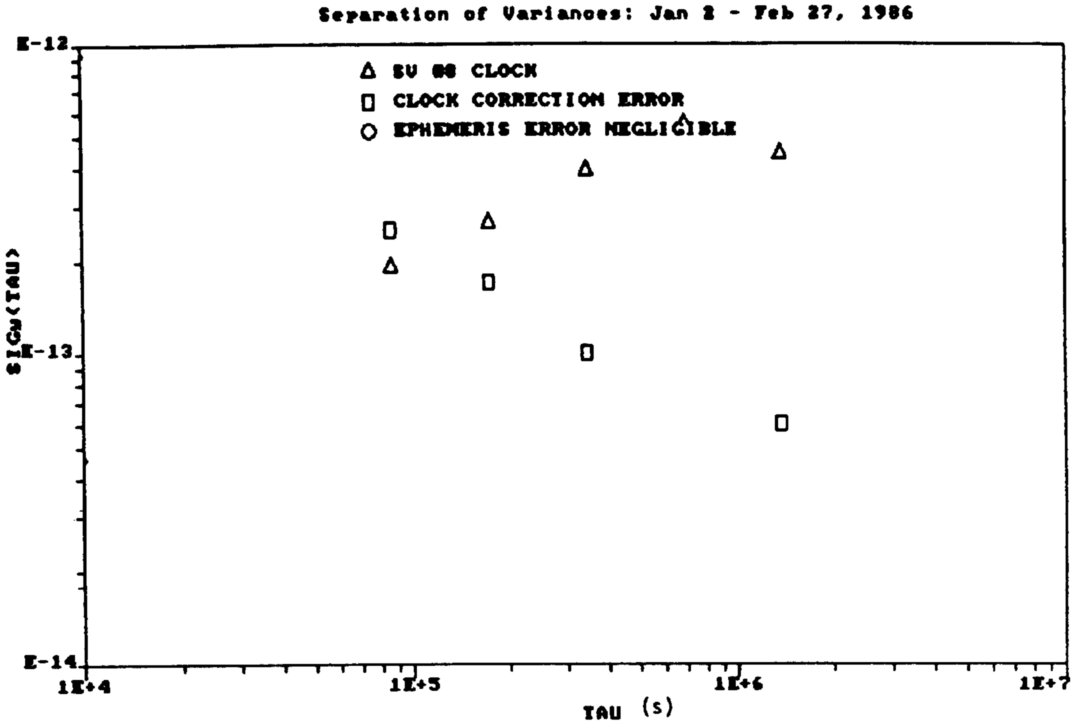

Figure 5: The noise or error levels of SV# 8 (Navstar 4) components. The clock correction error level is worse than the noise level of the SV Rubidium clock at one day.

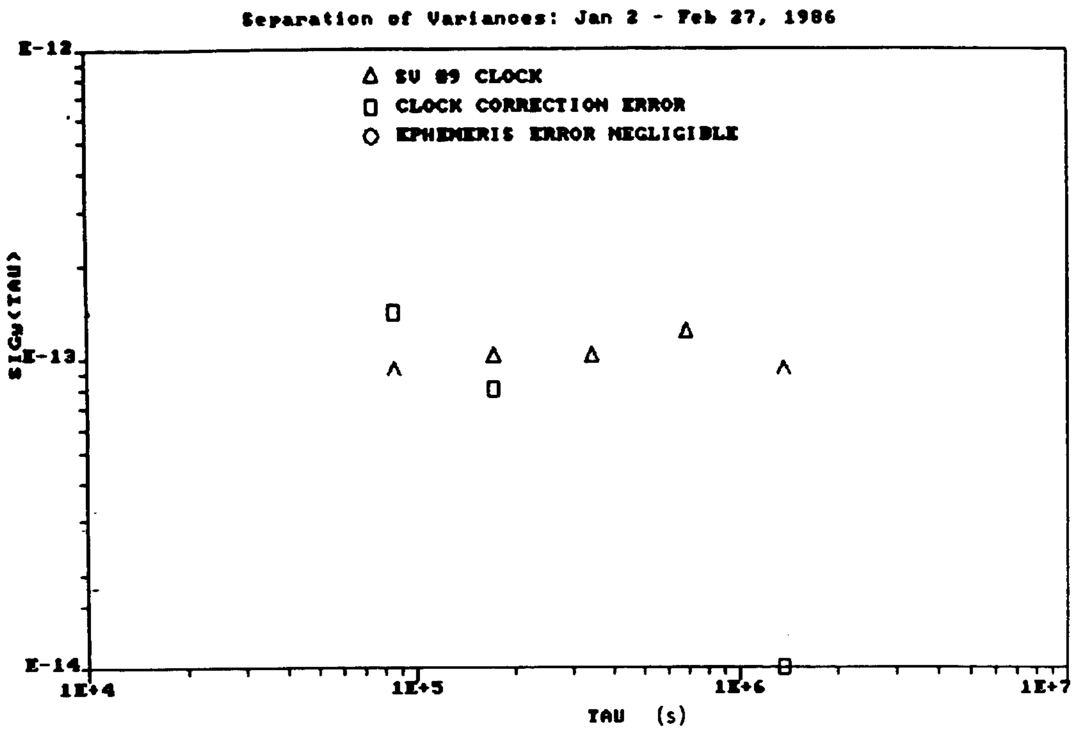

Figure 6: The noise or error levels of SV# 9 (Navstar 6) components. The clock correction error level is worse than the noise level of the SV Rubidium clock at one day.

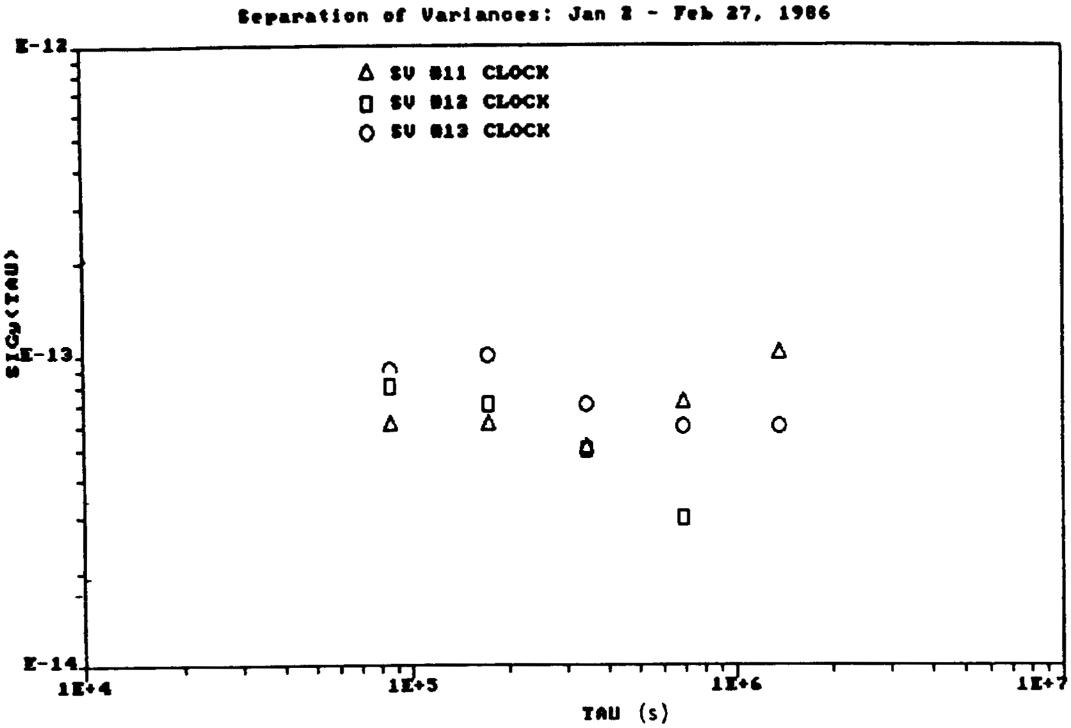

Figure 7: Our estimates of the clocks aboard SV's 11, 12, and 13 (Navstars 8, 10, and 9). SV# 11 was at that time using its temperature controlled Rubidium clock, while SV's 12 and 13 were Cesium clocks. The performance level we see is consistent with other estimates.

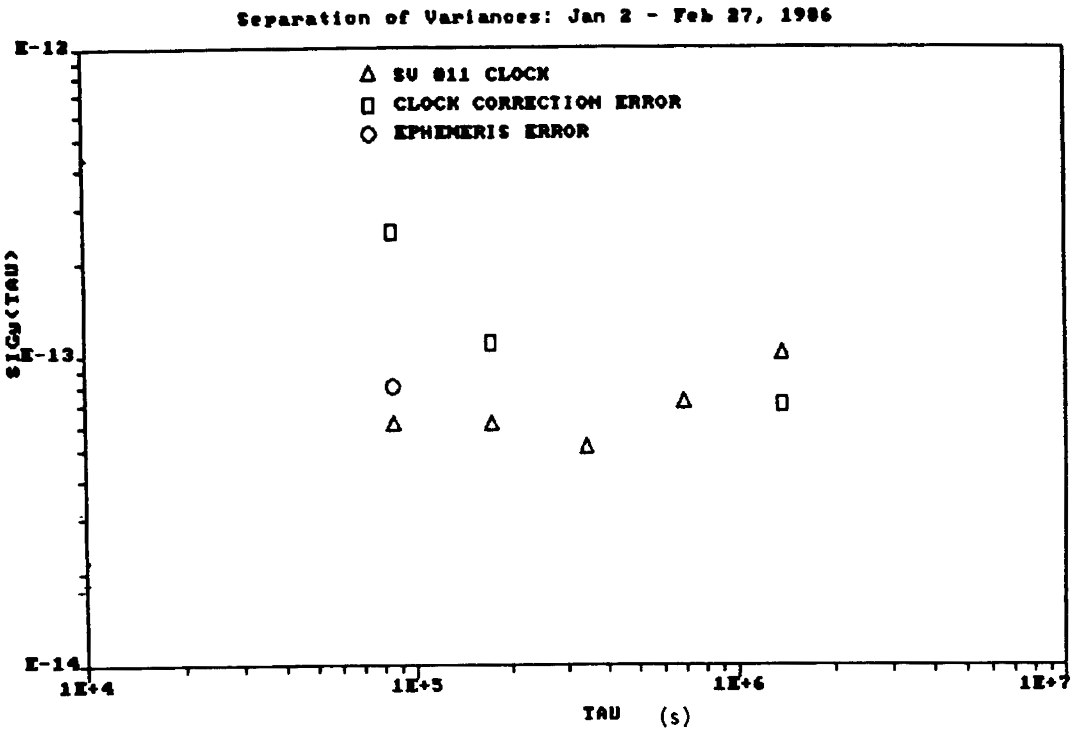

Figure 8: The noise or error levels of SV# 11 (Navstar 8) components. The clock correction error level is worse than the noise level of the SV teperature controlled Rubidium clock at one day.

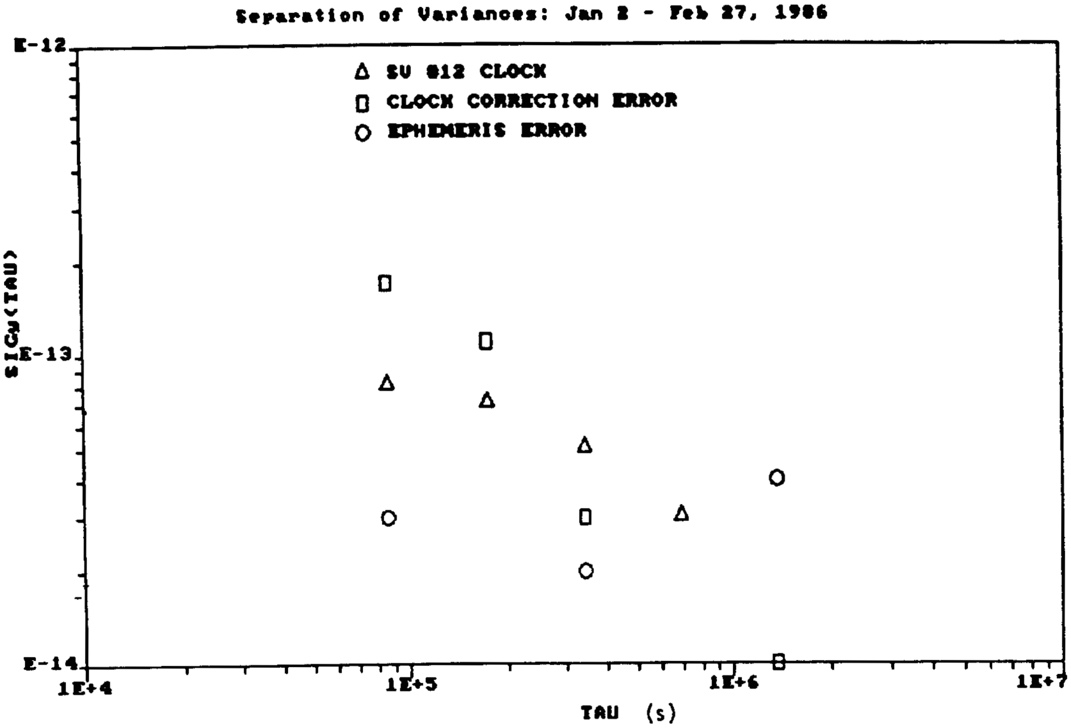

Figure 9: The noise or error levels of SV# 12 (Navstar 10) components. The clock correction error level is worse than the noise level of the SV Cesium clock at one day.

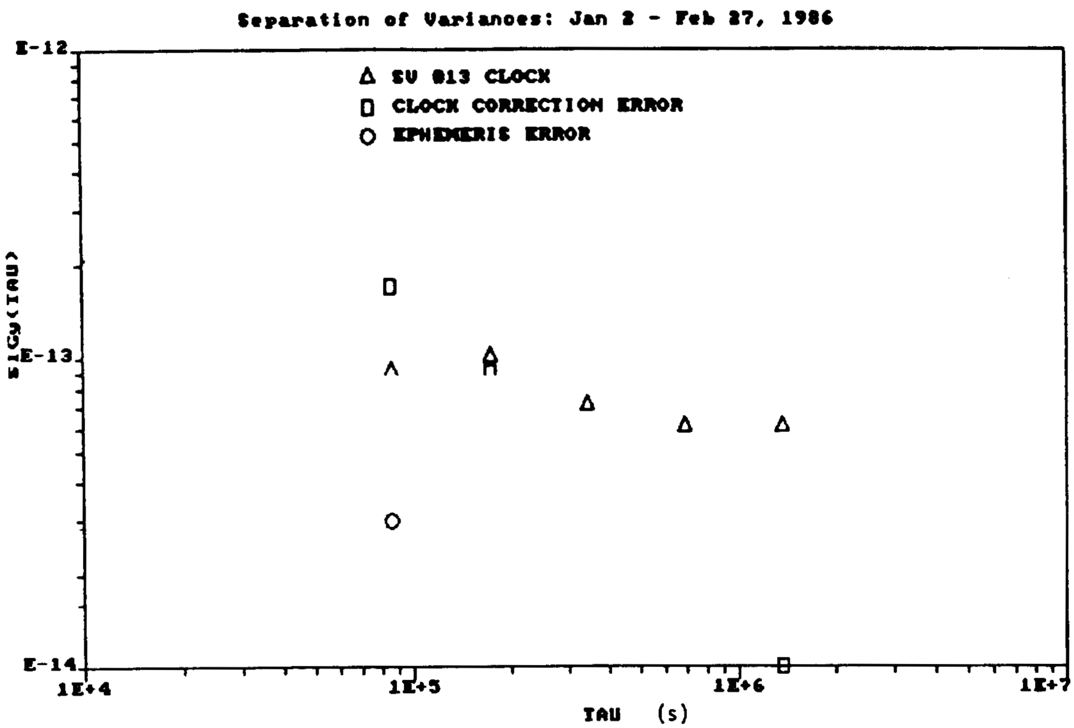

Figure 10: The noise or error levels of SV# 13 (Navstar 9) components. The clock correction error level is worse than the noise level of the SV Cesium clock at one day.