From: *Proceedings of the 42nd Annual Symposium on Frequency Control*, 1988.

EXTENDING THE RANGE AND ACCURACY OF PHASE NOISE MEASUREMENTS

F.L. Walls, A.J.D. Clements, C.M. Felton, M.A. Lombardi, and M.D. Vanek

Time and Frequency Division
National Institute of Standards and Technology
Boulder, Colorado 80303

Summary

This paper describes recent progress in extending high accuracy measurements of phase noise in oscillators and other devices for carrier frequencies from the rf to the millimeter region and Fourier frequencies up to 10% of the carrier (or a maximum of about 1 GHz). A brief survey of traditional precision techniques for measuring phase noise is included as a basis for comparing their relative performance and limitations. The single oscillator techniques, although conceptually simple, require a set of 5 to 10 references to adequately measure the phase noise from 1 Hz to 1 GHz from the carrier. The two oscillator technique yields excellent noise floors if, for oscillator measurements, one has a comparable or better oscillator for the reference and, for other devices, either pairs of devices or a reference oscillator with comparable or better noise. We have developed several new calibration techniques which, when combined with previous two oscillator techniques, permits one to calibrate all factors affecting the measurements of phase noise of oscillator pairs to an accuracy which typically exceeds 1 dB and in favorable cases can approach 0.4 dB. In order to illustrate this expanded two oscillator approach, measurements at 5 MHz and 10 GHz are described in detail. At 5 MHz we achieved accuracies of about ±0.6 dB for phase noise measurements from 20 Hz to 100 kHz from the carrier. At 10 GHz we achieved an accuracy of ±0.6 dB for phase noise measurements a few kHz from the carrier degrading to about ±1.5 dB, 1 GHz from the carrier.

I. Introduction

This paper describes recent progress at the National Bureau of Standards (NBS) in extending high accuracy measurements of phase noise in oscillators, amplifiers, frequency synthesizers, and passive components at carrier frequencies from the rf to the millimeter region and Fourier frequencies up to 10% of the carrier (or a maximum of about 1 GHz). An examination of existing techniques for precision phase noise measurements of oscillators[1-11] showed that present approaches which don't require a second "reference" oscillator have good resolution or noise floor for Fourier frequencies extending over only 1 or 2 decades.[7-10] Consequently, these approaches require a set of 5 to 10 references, either delay lines or high Q factor cavities, in order to adequately measure the phase noise from 1 Hz to 1 GHz from the carrier. Using the cavity approach would require a entire set of reference cavities for each carrier frequency measured. Similar considerations also apply to phase noise measurements of the other devices unless one can measure pairs of devices.

The limitations of the single oscillator techniques led us to adopt a two oscillator method for all measurements. This approach yields good resolution or noise floor from essentially dc to the bandwidth of the mixer if, for oscillator measurements, one has a comparable or better oscillator for the reference and, for other devices, either pairs of devices or a reference oscillator with comparable or better noise.

The major limitations in the accuracy of the two oscillator method (which also apply to the single oscillator methods) are the calibration of: the mixer phase-to-voltage conversion factor, the amplifier gain versus Fourier frequency, and the accuracy of the spectrum analyzer. We have developed several new calibration techniques which, when combined with previous techniques, allow us to address each of these limitations. The net result is the development of a complete measurement concept that permits one to calibrate all factors affecting the measurements of phase noise of oscillator pairs to an accuracy which typically exceeds 1 dB and in favorable cases can approach 0.4 dB. For other types of devices the limitations are similar if the noise of the reference oscillator can be neglected. The ultimate accuracy that can be easily achieved with this approach is now limited by the accuracy of the attenuators in available spectrum analyzers.

Measurements at 5 MHz and 10 GHz are described in detail, in order to illustrate this expanded two oscillator approach. Specifically we measure the mixer phase-to-voltage conversion factor multiplied by amplifier gain on all channels versus Fourier frequency using a new ultra-wideband phase modulator. We then measure the absolute mixer conversion factor multiplied by the amplifier gain at one Fourier frequency, the effect of the phase-lock loop on the measured noise voltage, and spectral density function of the spectrum analyzers. These measurements are used to normalize the relative gains of the noise measurements on the various channels. At 5 MHz we achieved accuracies of about ±0.6 dB for phase noise measurements from 20 Hz to 100 kHz from the carrier. At 10 GHz we achieved an accuracy of ±0.6 dB for phase noise measurements a few kHz to 500 MHz from the carrier. The accuracy degrades to about ±1.5 dB, 1 GHz from the carrier.

II. Model of a Noisy Signal

The output of an oscillator can be expressed as

$$V(t) = [V_o + \epsilon(t)] \sin(2\pi\nu_o t + \phi(t)), \qquad (1)$$

where V_o is the nominal peak output voltage, and ν_o is the nominal frequency of the oscillator. The time variations of amplitude have been incorporated into $\epsilon(t)$ and the time variations of the instantaneous frequency, $\nu(t)$, have been incorporated into $\phi(t)$. The instantaneous frequency is

$$\nu(t) = \nu_o + \frac{d[\phi(t)]}{2\pi dt} . \qquad (2)$$

The fractional frequency deviation is defined as

$$y(t) = \frac{\nu(t) - \nu_o}{\nu_o} = \frac{d[\phi(t)]}{2\pi\nu_o dt} . \qquad (3)$$

Power spectral analysis of the output signal V(t) combines the power in the carrier ν_o with the power in $\epsilon(t)$ and $\phi(t)$ and therefore is not a good method to characterize $\epsilon(t)$ or $\phi(t)$.

Since in many precision sources understanding the variations in $\phi(t)$ or y(t) is of primary importance, we will confine the following discussion to frequency-domain measures of y(t), neglecting $\epsilon(t)$ except in cases where it sets limits on the measurement of y(t). The amplitude fluctuations, $\epsilon(t)$, can be reduced using limiters whereas $\phi(t)$ can be reduced in some cases by the use of narrow band filters.

Spectral (Fourier) analysis of y(t) is often expressed in terms of $S_\phi(f)$, the spectral density of phase fluctuations in units of radians squared per Hz bandwidth at Fourier frequency f from the carrier ν_o. Alternately $S_y(f)$, the spectral density of fractional frequency fluctuations in a 1 Hz bandwidth at Fourier frequency f from the carrier ν_o can be used [1]. These are related as

$$S_\phi(f) = \frac{\nu_o^2}{f^2} S_y(f) \quad rad^2/Hz \quad 0<f<\infty . \qquad (4)$$

$S_\phi(f)$ can be intuitively understood as,

$$S_\phi(f) = \frac{\Delta\phi^2(f)}{BW} \qquad (5)$$

where $\Delta\phi(f)$ is the rms phase deviation measured at Fourier frequency f from the carrier in a bandwidth BW. It should be noted that these are single-sided spectral density measures containing the phase or frequency fluctuations from both sides of the carrier. The mean squared phase modulation in a measurement bandwidth, BW, is given by

$$\Delta\phi^2(f) = \int_{-BW/2}^{+BW/2} S_\phi(f)\, df . \qquad (6)$$

Other measures sometimes encountered are $\mathcal{L}(f)$, dBC/Hz, and $S_{\Delta\nu}(f)$. These are related by [1-3]

$$S_{\Delta\nu}(f) = \nu_o^2 S_y(f) \quad Hz^2/Hz$$

$$S_\phi(f) = \mathcal{L}(\nu_0 - f) + \mathcal{L}(\nu_0 + f) \equiv 2\mathcal{L}(f) \qquad (7)$$

$$dBC/Hz = 10 \log \mathcal{L}(f)$$

$\mathcal{L}(f)$ and dBC/Hz are single sideband measures of phase noise. These are the revised definitions of $\mathcal{L}(f)$ and dBC as per the most recent recommendation of the Standards Coordinating Committee for Time and Frequency of the IEEE [3]. With these revised definitions $\mathcal{L}(f)$ and dBC/Hz are now defined for arbitrarily large phase modulation, whereas previously they were restricted to small angle modulation. In some situations, especially where f becomes a sizeable fraction of ν_0, the phase noise spectrum is asymmetric about the carrier. In these cases one should specify whether the upper sideband noise $\mathcal{L}(\nu_0+f)$ or the lower sideband noise $\mathcal{L}(\nu_0-f)$ is being referenced. These distinctions are becoming increasingly important as users require the specification of phase noise near the carrier where the phase excursions are large compared to 1 radian, or at Fourier frequencies which exceed the bandwidth of typical circuit elements.

The above measures provide the most powerful (and detailed) analysis for evaluating types and levels of fundamental noise and spectral density structure in precision oscillators and signal handling equipment as it allows one to examine individual Fourier components of residual phase (or frequency) modulation.

* See Appendix Note # 6

III. A. Two Oscillator Method *

Fig. 1 shows the block diagram for a typical scheme used to measure the phase noise of a precision source using a double balanced mixer and a reference source. Fig. 2 illustrates a similar technique for measuring only the phase noise added in multipliers, dividers, amplifiers, and passive components. It is very important that the substitution oscillator be at the same drive level, impedance, and at the equivalent electrical length from the mixer as the signal coming from the reference oscillator. This is dramatically illustrated in Fig. 3, discussed below. The output voltage of the mixer as a function of phase deviation, $\Delta\phi$, between the two inputs to the mixer is nominally given by

$$V_{out} = K \cos \Delta\phi \qquad (8)$$

Near quadrature (90° phase difference) this can be approximated by

$$V_{out} = K_d \delta\phi, \text{ where } \delta\phi \equiv [\Delta\phi - \frac{2n-1}{2}\pi] < 0.1 \qquad (9)$$

where n is the integer to make $\delta\phi \sim 0$. The voltage to phase conversion ratio sensitivity, K_d, is dependent on the frequency, the drive level, and impedance of both input signals, and the IF termination of the mixer [4]. The combined spectral density of phase noise of both input signals, the noise of the mixer, and the amplitude noise from the IF amplifiers is

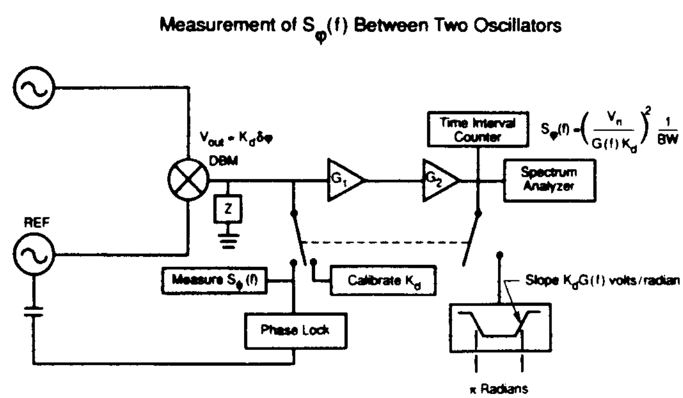

Fig. 1. Precision phase measurement system using a spectrum analyzer. Calibration requires a recording device to measure the slope at the zero crossing. The accuracy is better than 0.4 dB from dc to 0.1 ν_o Fourier frequency offset from the carrier ν_o. Carrier frequencies from a few Hz to 10^{11} Hz can be accommodated with this type of measurement system [4].

given by

$$S_\phi(f) = \left[\frac{V_n(f)}{G(f)K_d}\right]^2 \frac{1}{BW}, \qquad (10)$$

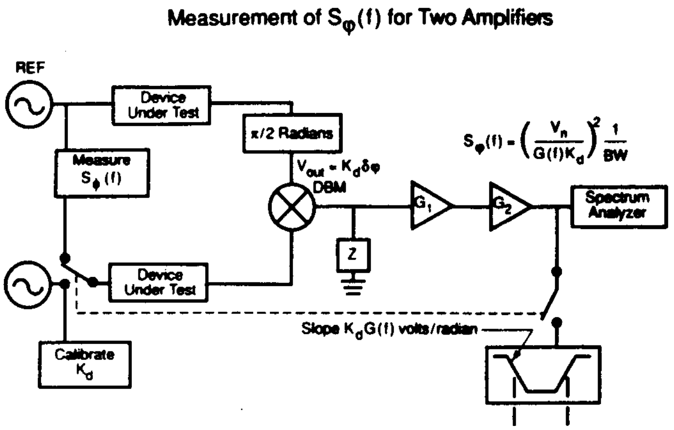

Fig. 2. Precision phase measurement system featuring self calibration to 0.4 dB accuracy from dc to 0.1 ν_0 Fourier frequency offset from carrier. This system is suitable for measuring signal handling equipment, multipliers, dividers, frequency synthesizers, as well as passive components [4].

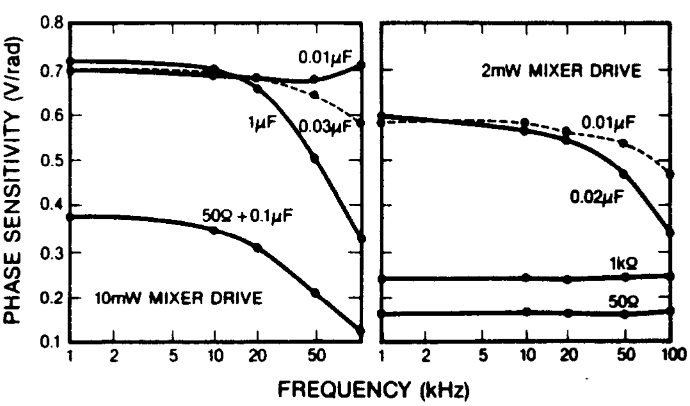

Fig. 3. Double-balanced mixer phase sensitivity at 5 MHz as a function of Fourier frequency for various output terminations. The curves on the left were obtained with 10 mW drive while those on the right were obtained with 2 mW drive. The data demonstrate a clear choice between constant, but low sensitivity or much higher, but frequency dependent sensitivity [4].

where $V_n(f)$ is the RMS noise voltage at Fourier frequency f from the carrier measured after IF gain G(f) in a noise bandwidth BW. Obviously BW must be small compared to f. This is very important where $S_\phi(f)$ is changing rapidly with f, e.g., $S_\phi(f)$ often varies as f^{-3} near the carrier. In Fig. 1, the output of the second amplifier following the mixer contains contributions from the phase noise of the oscillators, the noise of the mixers, and the post amplifiers for Fourier frequencies much larger than the phase-lock loop bandwidth. In Fig. 2, the phase noise of the oscillator cancels out to a high degree (often more than 20 dB). Termination of the mixer IF port with 50 Ω maximizes the IF bandwidth, however, termination with reactive loads can reduce the mixer noise by ~ 6 dB, and increase K_d by 3 to 6 dB as shown in Fig. 3 [4]. Accurate determination of K_d can be achieved by measuring the slope of the zero crossing in volts/radian with an oscilloscope or other recording device when the two oscillators are beating slowly. For some applications the digitizer in the spectrum analyzer can be used to measure both the beat period and the slope in V/s at the zero crossing. The time axis is easily calibrated since one beat period equals 2π radians. The slope in volts/radian is then calculated with a typical accuracy of 0.2 dB. Estimates of K_d obtained from measurements of the peak to peak output voltage induced can introduce errors as large as 6 dB in $S_\phi(f)$ even if the amplitude of the other harmonics is measured unless the phase relationship is also taken into account [4]. $S_\phi(f)$ can be made independent of the accuracy of the spectrum analyzer voltage reference by comparing the level of an externally IF signal (a pure tone is best), on the spectrum analyzer used to measure V_n with the level recorded on the device used to measure K_d.

The noise bandwidth of the spectrum analyzer also needs to be verified. This calibration procedure is sufficient for small Fourier frequencies but looses precision and accuracy due to the problems illustrated in Fig. 3 and the variations of amplifier gain with Fourier frequency. If measurements need to be made at Fourier frequencies near or below the phase-lock-loop bandwidth, a probe signal can be injected inside the phase-lock-loop and the attenuation measured versus Fourier frequency.

Some care is necessary to assure that the spectrum analyzer is not saturated by spurious signals such as the power line frequency and its multiples. Sometimes aliasing in the spectrum analyzer is a problem. If narrow spectral features are to be measured it is usually recommended that a flat top window function (in the spectrum analyzer) be used. In the region where the measured noise is changing rapidly with Fourier frequency, the noise bandwidth should be much smaller than the measurement frequency. The approximate level of the noise floor of the measurement system should be measured in order to verify that it does not significantly bias the measurements or, if necessary, to subtract its effect from the results.

Typical best performance for various measurement techniques is shown in Fig. 4. The two oscillator approach exceeds the performance of almost all available oscillators from below 0.1 MHz to over 100 GHz and is generally the technique of first choice because of its versatility and simplicity. Figs. 10 - 12 give some examples. Phase noise measurements on pairs of signal sources can be made with an absolute accuracy better than 1 dB using the above calibration procedure. Such accuracy is not always attainable when the phase noise of the source exceeds that of the added noise of the components under test (see Fig. 2). The use of specialized high level mixers with multiple diodes per leg increases the phase to voltage conversion sensitivity, K_d and therefore reduces the contribution of IF amplifier noise [5] as shown in Fig. 4. Phase noise measurements can generally be made at Fourier frequencies from approximately dc to 1/2 the source frequency. The major difficulty is designing a mixer terminations to remove the source frequency from the output signal, which would generally saturate the low noise amplifiers following the mixer, without degrading the signal-to-noise ratio. As mentioned earlier the phase noise spectrum is quite likely asymmetric when f exceeds the bandwidth of the tuned circuits in the device under test. For example one expects that the phase noise at $1/2\nu_0$ is different than the phase noise at $3/2\nu_0$.

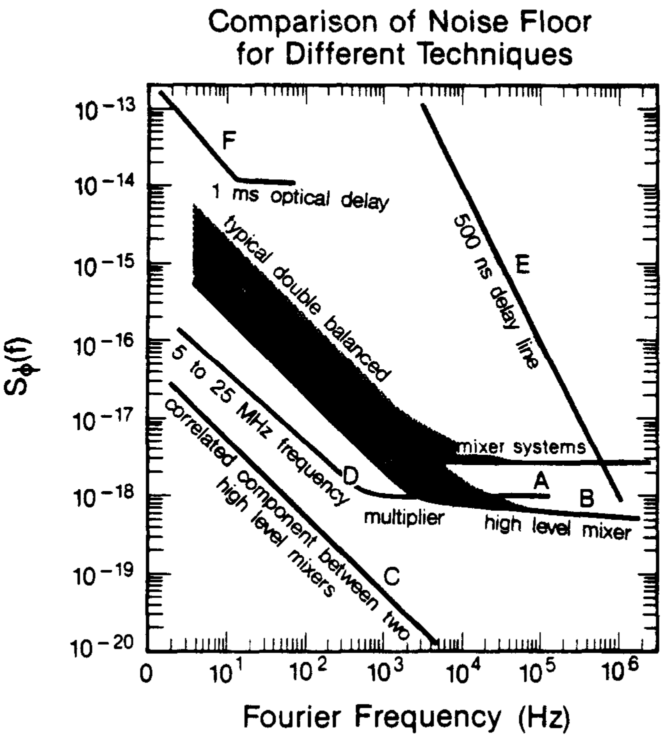

Fig. 4.

Curve A. The noise floor $S_\phi(f)$ (resolution) of typical double balanced mixer systems (e.g. Fig. 1 and Fig. 2) at carrier frequencies from 0.1 MHz to 26 GHz. Similar performance possible to 100 GHz [5].

Curve B. The noise floor, $S_\phi(f)$, for a high level mixer [5].

Curve C. The correlated component of $S_\phi(f)$ between two channels using high level mixers [5].

Curve D. The equivalent noise floor $S_\phi(f)$ of a 5 to 25 MHz frequency multiplier.

Curve E. Approximate phase noise floor of Fig. 8 using a 500 ns delay line.

Curve F. Approximate phase noise floor of Fig. 8 where a 1 ms delay has been achieved by encoding the signal on an optical carrier and transmitted it across a long optical fiber to a detector.

Most double balanced mixers have a substantial non-linearity that can be exploited to make phase comparison between the reference source and odd multiples of the reference frequency. Some mixers even feature internal even harmonic generation. The measurement block diagram looks identical to that of in Fig. 1, except that the source under test is at an odd (even) harmonic of the reference source. This method is relatively efficient (as long as the harmonics fall within the bandwidth of the mixer) for multiples up to x5 although multiples as high as 25 have been used. The noise floor is approximately degraded by the amount of reduction in the phase sensitivity of the mixer. The phase noise of the reference source is also higher at the multiplied frequency as shown in Section III.F below.

III. B. Enhanced Performance Using Correlation Techniques

The resolution of the many systems can be greatly enhanced (typically 20 dB) by using correlation techniques to separate the phase noise due to the device under test from the noise in the mixer and IF amplifier [5, 11].

For purposes of illustration, consider the scheme shown in Fig. 5. At the output of each double balanced mixer there is a signal which is proportional to the phase difference, $\Delta\phi$, between the two oscillators and a noise term, V_N, due to contributions from the mixer and amplifier. The voltages at the input of each bandpass filter are

$$V_1(\text{BP filter input}) = G_1\,\Delta\phi(t) + C_1 V_{N1}(t) \qquad (11)$$

$$V_2(\text{BP filter input}) = G_2\,\Delta\phi(t) + C_2 V_{N2}(t),$$

where $V_{N1}(t)$ and $V_{N2}(t)$ are substantially uncorrelated and C_1 and C_2 are constants. Each bandpass filter produces a narrow band noise function around its center frequency f:

$$V_1(\text{BP filter output}) = G_1[S_\phi(f)]^{1/2} B_1^{1/2} \cos[2\pi ft + \phi(t)]$$

$$+ C_1[S_{VN1}(f)]^{1/2} B_1^{1/2} \cos[2\pi ft + n_1(t)] \qquad (12)$$

$$V_2(\text{BP filter output}) = G_2[S_\phi(f)]^{1/2} B_2^{1/2} \cos[2\pi ft + \psi(t)]$$

$$+ C_2[S_{VN2}(f)]^{1/2} B_2^{1/2} \cos[2\pi ft + n_2(t)]$$

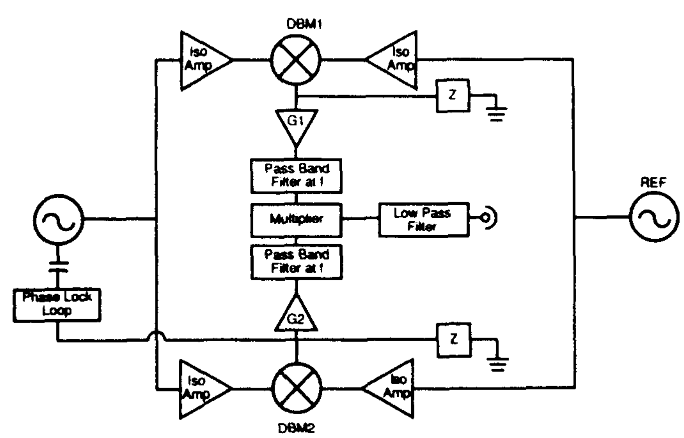

Fig. 5. Correlation phase noise measurement system.

where B_1 and B_2 are the equivalent noise bandwidths of filters 1 and 2 respectively. Both channels are bandpass filtered in order to help eliminate aliasing and dynamic range problems. The phases $\psi(t)$, $n_1(t)$ and $n_2(t)$ take on all values between 0 and 2π with equal likelihood. They vary slowly compared to 1/f and are substantially uncorrelated. When these two voltages are multiplied together and low pass filtered, only one term has finite average value.

The output voltage is

$$V^2_{out} = \tfrac{1}{2}\, G_1 G_2 S_\phi(f)\, B_1^{\,\frac{1}{2}} B_2^{\,\frac{1}{2}} + D_1\langle\cos[\phi(t)-n_1(t)]\rangle \quad (13)$$

$$+\, D_2\langle\cos[\phi(t) - n_2(t)]\rangle + D_3\langle\cos[n_1(t)-n_2(t)]\rangle,$$

so that $S_\phi(f)$ is given by

$$S_\phi(f) = \frac{(2)V_N^2(f)}{G_1 G_2 \sqrt{B_1 B_2}}\,. \quad (14)$$

For times long compared to $B_1^{-\frac{1}{2}} B_2^{-\frac{1}{2}}$ the noise terms D_1, D_2 and D_3 tend towards zero as $\sqrt{t}$. Limits in the reduction of these terms are usually associated with harmonics of 60 Hz pickup, dc offset drifts, and non-linearities in the multiplier. Also if the isolation amplifiers have input current noise, they will pump current through the source resistance. The resulting noise voltage will appear coherently on both channels and cannot be distinguished from real phase noise between the two oscillators. One half of the noise power appears in amplitude and one half in phase modulation.

Obviously the simple single frequency correlator used in this example can be replaced by a fast digital system which simultaneously computes the correlated phase noise for a large band of Fourier frequencies. Typical results show a reduction in noise floor of order 20 dB over the noise floor of a single channel (See Fig. 4). The great power of this technique is that it can be applied at any carrier frequency where are available double balanced mixers. The primary limitations come from the bandwidth and non-linearities in the cross correlator [5,11].

III. C. Reference Phase Modulation Method

Another method of determining $S_\phi(f)$ uses phase modulation of the reference oscillator by a known amount. The ratio of the reference phase modulation to the rest of the spectrum then can be used for a relative calibration. This approach can save an enormous amount of time for measurements which are repeated a great many times. An adaptation of this approach is utilized in the new NBS phase noise system described in section IV below.

III. D. Frequency Discriminator Methods

It is sometimes convenient to use a high-Q resonance directly as a frequency discriminator as shown in Fig. 6. The oscillator can be tuned 1/2 linewidth ($\nu_o/2Q$) away from line center yielding a detected amplitude signal of the form

$$V_{out} = G(f) k_d Q dy(f)\ [V + \epsilon(t)] \quad (15)$$

This approach mixes frequency fluctuations between the oscillator and reference resonance with the amplitude noise of the transmitted signal. By using amplitude control (e.g. by processing to normalize the data), one can reduce the effect of amplitude noise. [5] The measured noise at the detector is then related to the phase fluctuation of the reference resonance by

$$S_\phi(f) = \left(\frac{\nu_o V_N(f)}{f Q k_d G(f)}\right)^2 \frac{1}{BW} \quad \text{for } f < \frac{\nu_o}{2Q}\,. \quad (16)$$

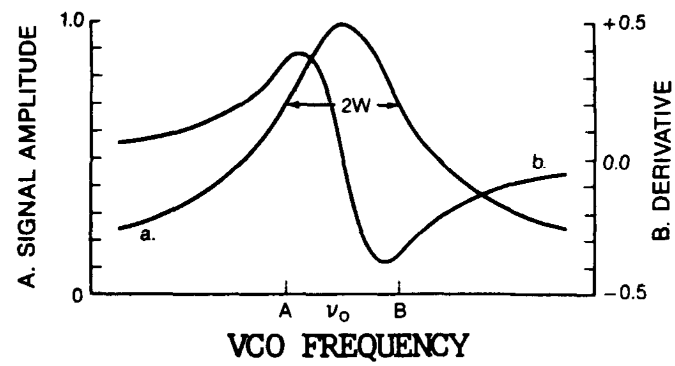

Fig. 6. High-Q resonance used as a frequency discriminator. Note that the peak response is displaced from the center of the resonance by about the half bandwidth.

This approach normally has the limitations that f must be small compared to the linewidth of the cavity, and the effect of residual amplitude noise is difficult to remove; however, no reference source is needed. The calibration factors G(f)KdQ can be measured even for Fourier frequencies larger than $\nu_0/2Q$ by stepping the source frequency an amount dy (which is small compared to 1/2Q) and measuring the output voltage versus the modulation frequency, f.

Differential techniques can be used to measure the inherent frequency (phase) fluctuations of two high-Q resonators as shown in Fig. 7 [7]. The output voltage is of the form $V_{out} = 2QK_d G(f)\, dy(f)$. The phase noise spectrum of the resonators is then obtained using Eq. 4.

$$S_\phi(f) = \left(\frac{\nu_o V_N(f)}{2Q f K_d G(f)}\right)^2 \frac{1}{BW} \qquad f < \frac{\nu_o}{2Q} \quad (17)$$

Measurement of the Inherent Phase Noise in High-Q Resonators

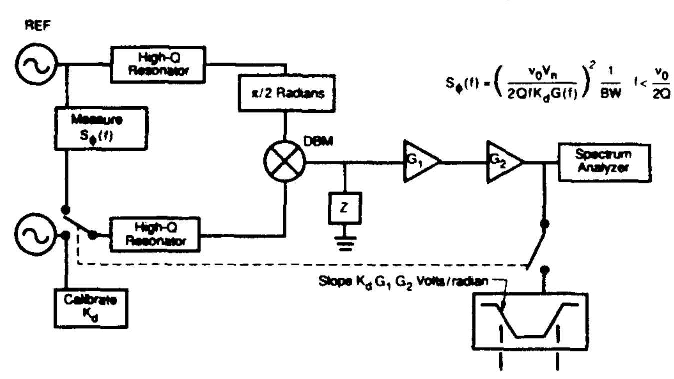

Fig. 7. Differential frequency discriminator using a pair of high-Q resonators. In this approach the phase noise of the source tends to cancel out.

The phase noise in the source can cancel out by 20 to 40 dB depending on the similarity of resonant frequencies Q's and the transmission properties of the two resonators. This approach was first used to demonstrate that the inherent frequency stability of precision quartz resonators exceeds the performance of most quartz crystal controlled oscillators [7].

If only one resonance is used, the output includes the phase fluctuations of both the source and the resonator. The calibration is accomplished by stepping the frequency of the source and measuring the output voltage, i.e., $\Delta V = G(f)K^1(f)\ \Delta\nu_1$. From this measurement the phase spectrum can be calculated as

$$S_\phi(f) = \left[\frac{V_N(f)}{f\ K^1(f)}\right]^2 \left[\frac{1}{BW}\right] . \qquad (18)$$

Fig. 8A shows one method of implementing this approach at X-band. The cavity has a loaded quality factor of order 25,000. Fig. 8B shows the measured frequency discriminator curve. Note that $K^1(f)$ is constant for $f<\nu/(2Q)$ and decreases at values of f larger than the half bandwidth of the resonance as

$$K^1(f) \sim \frac{1}{1 + \left(\frac{2Qf}{\nu_o}\right)^2} \qquad (19)$$

The frequency dependence shown in eq. (19) can be accurately determined by measuring ΔV when stepping the source an amount $\Delta\nu$, which is small compared to ν_0/Q, versus the modulation frequency, f. This approach has poor resolution near the carrier and limited high frequency response. Therefore the measurement of phase from close to the carrier out to 10% of the carrier could require a large set of cavities with different Q factors. In order to achieve the required Q factor for close in measurements it may even be necessary to use cryogenic techniques.

III. E. Delay Line Method

Another different approach uses a delay line to make a pseudo reference which is retarded relative to the incoming signal [7-10] as shown in Fig. 9.

The mixer output is of the form

$$V_{out} = 2\pi t_d K_d \nu_o dy \ , \qquad (20)$$

and the input phase noise is given by

$$S_\phi(f) = \left[\frac{V_N(f)}{2\pi f\tau_d G(f) K_d}\right]^2 \frac{1}{BW} , \quad f< \frac{1}{\tau_d} \qquad (21)$$

This approach is often used at microwave frequencies when only one oscillator is available. In this technique the ability to resolve phase noise close to the carrier depends on the delay time. For example, if f = 1 Hz and τ_d = 500 ns, then, $(2\pi f\tau_d)^2 \sim 10^{-11}$. The noise floor is 110 dB higher at f = 1 Hz than that of the two oscillator method, decreasing as $1/f^2$. Recent advances make it possible to encode the rf signal on an optical signal which then can be transmitted down an optical fiber to achieve delays up to the order of 10^{-3} s with an increase in the noise floor to approximately -140 dB relative to 1 rad^2/Hz. The noise floor can be reduced by ~20 to 40 dB using the correlation techniques described above [11]. Note that the range of Fourier frequencies is usually limited to less than ~ $1/\tau_d$. This technique normally has good resolution over 1½ to 2 decodes in Fourier frequency. Therefore, measurements of phase noise from close to the carrier out to 10% of the carrier require a large set of different delay lines and hardware including optical delay lines, associated lasers, modulators, and detectors.

Measurement of Phase Noise using a High-Q Cavity

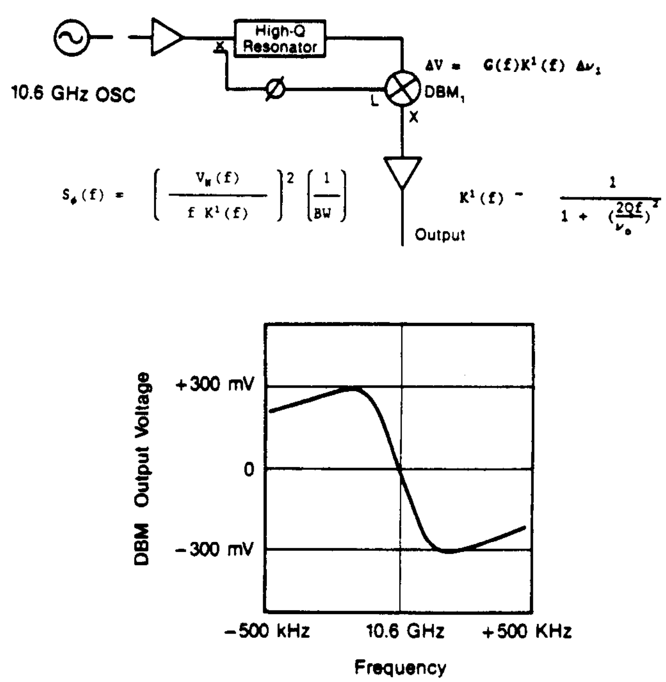

Fig. 8. A. block diagram of a high Q resonator used as a frequency discriminator. B. Frequency discriminator curve for the scheme shown in A used at X-band with a cavity having a loaded quality factor of approximately 25,000.

Measurement of S_ϕ (f) Using a Delay Line

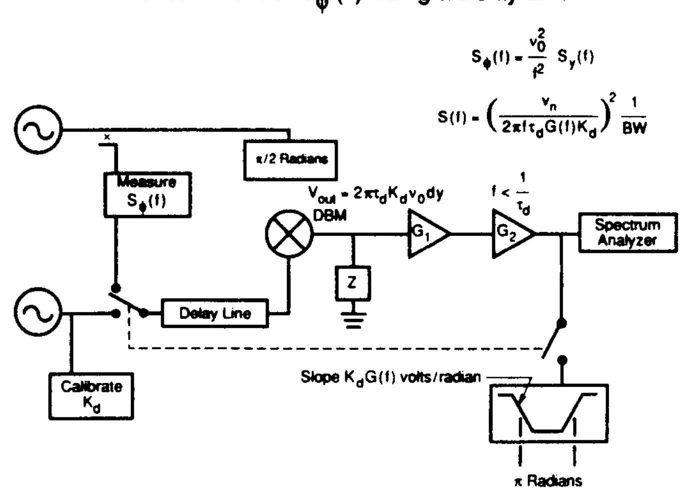

Fig. 9. Delay line frequency discriminator.

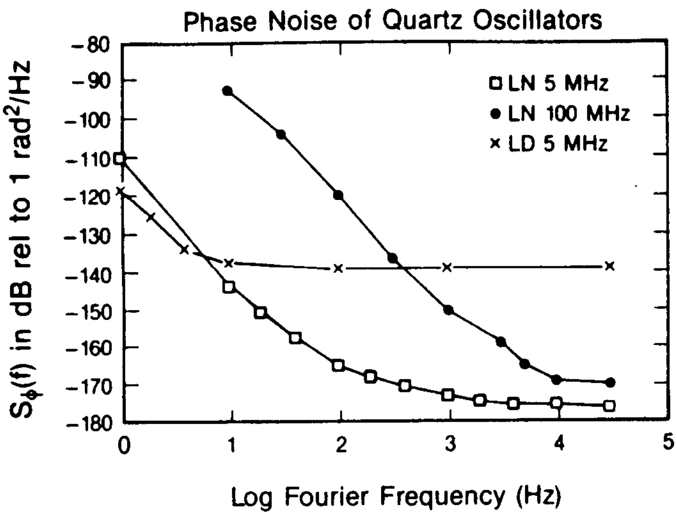

Fig. 10. Phase noise performance of selected quartz oscillators. The LN 5 MHz oscillator is driven at a high level to reduce the wideband noise while the LD 5 MHz oscillator is driven at a lower level to obtain low phase noise close to the carrier.

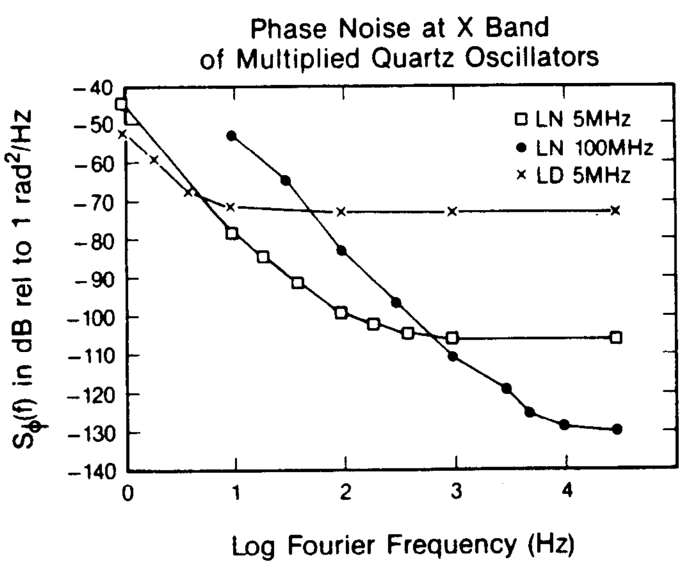

Fig. 11. Phase noise of the oscillators of Fig. 10 if multiplied to X-band in a perfect frequency multiplier.

III. F. Multiplication/Division

The use of perfect frequency multipliers (or dividers) between the signal source and the double balanced mixer increases (decreases) the phase noise level [12] as

$$S_\phi\nu_1(f) = \left(\frac{\nu_2}{\nu_1}\right)^2 S_\phi\nu_1(f) \qquad (22)$$

where ν_1 is the initial carrier frequency and ν_2 is the final carrier frequency. This can be used to either increase or decrease the phase sensitivity of the mixer system. Fig. 4 shows the noise of a specialized 5 to 25 MHz multiplier referred to the 5 MHz input. A potential problem with the use of the multiplier approach comes from exceeding the linear range of the mixer. Once the phase excursion, $\Delta\phi$, exceeds about 0.1 radian, non-linearities start to become important and at $\Delta\phi \sim 1$ radian, the measurement is no longer valid [12]. An additional practical problem is that low noise multipliers are usually narrowband devices. Each significantly new frequency generally requires a new set of frequency multipliers.

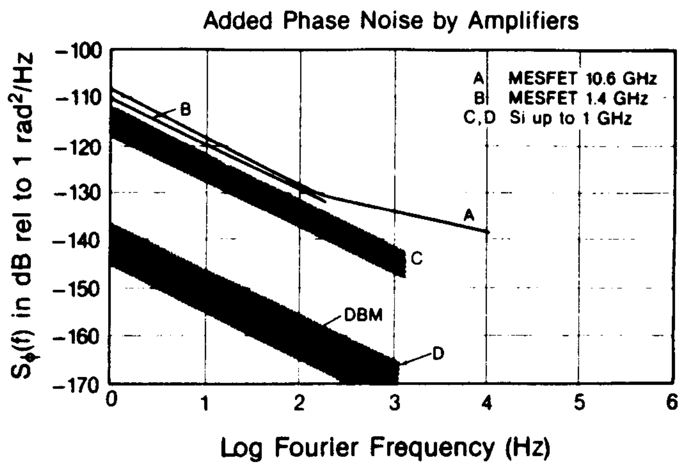

Fig. 12. Curves A and B show the phase noise added by selected GaAs MESFET amplifiers. Curve C shows the phase noise added by a typical common emitter silicon bipolar transistor with a "good" rf bypass on the emitter lead. Curve D shows the typical performance of the same amplifier with a small unbypassed impedance (approximately 1/transconductance) in the emitter lead. The added phase noise is generally independent of frequency over a very large range.

IV. A. The New NBS Phase Noise Measurement Systems

The new NBS phase noise measurement systems are a combination of the traditional two oscillator approach shown in Figs. 1 and 2 plus the reference phase modulation technique mentioned in section IIIC. The complete block diagram is sketched in Fig. 13. This approach yields the widest possible bandwidth and the lowest phase noise of a single channel system. It does, however, require the use of two sources for oscillator measurements. From hardware considerations we generally use 3 different phase noise measurement systems. Test set A accepts carrier frequencies from 5 to 1300 MHz and can measure the phase noise from 1 Hz to about 10% of the carrier or a maximum of 100 MHz. Test set B accepts carrier frequencies from 1 GHz to 26 GHz and can measure the phase noise from 0.01 Hz to about 500 MHz from the carrier. Test set C accepts carrier frequencies from about 2 to 26 GHz and can measure the phase noise from 0.01 Hz to 1 GHz from the carrier. Test set D accepts carrier frequencies from 33 to 50 GHz in WR22 waveguide and can measure the phase noise from 0.01 Hz to about 1.3 GHz from the carrier.

The construction of the phase modulator between the reference source and the mixer will be described in detail elsewhere [12]. The low pass filter section is used in order not to saturate the amplifiers with the carrier feedthrough signal from the mixer. One dc amplifier is used for phase noise measurements from dc to 100 kHz from the carrier. One ac amplifier is used for phase noise measurements from 50 kHz to 32 MHz. This range is well matched for one of our spectrum analyzers. The wideband ac amplifier has a bandwidth of 50 kHz to 1.3 GHz and is used when the desired measurement bandwidth exceeds 32 MHz. The wide bandwidth spectrum analyzer also provides a convenient way to observe the gross features of the output phase noise and to identify any major spurious outputs if present. In order to obtain the most accurate measurement of the phase noise it is, however, necessary to measure the amplitude of the first IF signal at about 21 MHz in order to avoid the variations in gains of the log amplifiers with various environmental factors.

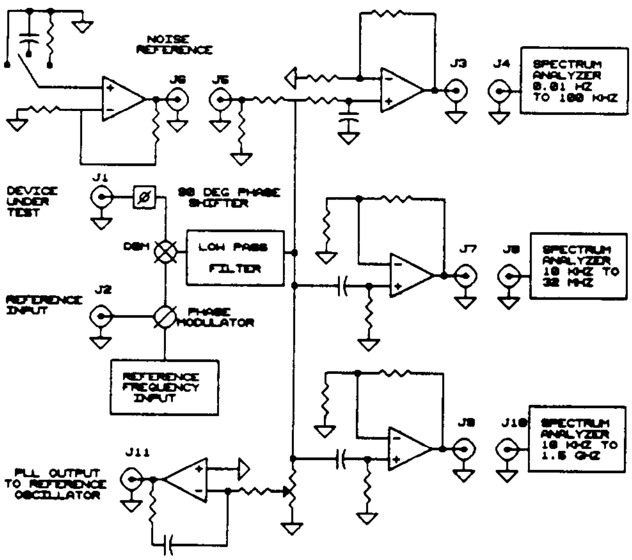

Fig. 13. Generalized block diagram of the new NBS phase noise measurement systems. The phase noise of carrier frequencies from 1 MHz to 100 GHz can be measured by varying the components in the phase shifters and mixers. Dedicated measurement systems covering 5 MHz to 50 GHz are described in the text.

IV. B. Measurement Sequence

1) The output power of the two sources to be measured is typically set to between +5 and + 13 dBm at the mixer. This takes into account the insertion loss of the phase modulator. If the oscillators don't posses sufficient internal isolation to prevent unwanted frequency pulling, isolators (or isolation amplifiers) are generally inserted between the sources and the mixer.

2) The absolute sensitivity of the mixer and the dc amplifier for converting small changes in phase to voltage changes is determined in a way similar to the traditional method, namely by allowing the two oscillators to slowly beat. The output of the dc amplifier is recorded by the digitizer in the FFT connected to the dc amplifier in order to accurately determine the period of the beat. In test sets C and D an additional 50 MHz digitizer is used to average and record the beat frequency. The time scale of the digitizer is then expanded to approximately 10% of the beat period and pretrigerred at about -2V in order to accurately determine the slope of the output in volts per radian. This calculation, shown in eq. (23) below, is typically accurate ± 2% or 0.2 dB.

$$K = \text{Volts/Second})(\text{Period}/(2\pi)\qquad (23)$$

3) The two sources to be measured are phase locked together with sufficient bandwidth that the phase excursions at the mixer are less than 0.1 radian. The necessary phase lock gain is calculated using an estimate for the noise of the oscillators and the tuning rate for the reference oscillator. This is then verified by noting the peak to peak excursions of the dc amplifier and using the measured conversion sensitivity measured in step 2 above. If the peak phase excursions are in excess of 0.1 radian, then the phase lock loop bandwidth is increased (if possible) in order to satisfy this condition.

4) The modulator is driven by a reference frequency (typically at +7 to +10 dBm) which steps through the Fourier frequencies of interest and the detected RMS voltage recorded on the appropriate spectrum analyzer. This approach accurately yields the relative gains of each amplifier and its respective spectrum analyzer since it automatically accounts for the effect of the phase lock loop and residual frequency pulling as well as the termination of the mixer and the variations in gain of the various amplifiers with Fourier frequency. The amplitude of the phase modulation on the carrier is constant in amplitude to better than ± 1.5 dB (typically ±0.2dB for f < 500 MHz) for reference frequencies dc to about 10% of the carrier frequency or a maximum of 1 GHz. Initial measurements of the prototype modulator are shown in Fig. 14.

This measurement is then combined with the measurement of the absolute mixer sensitivity multiplied by the gain of the dc amplifier described in step 2 above. The absolute gain of all the amplifiers shown in Fig. 13 can generally be determined to an accuracy of ± 0.3 dB (1.5 dB for Fourier frequencies from 500 MHz to 1 GHz) over the Fourier frequencies of interest.

5) Next the spectral density function of the FFT is verified. The level of the noise determined by the FFT for the input of the noise source amplifier sequentially shorted to ground, connected to ground through a 100 kΩ metal film resister, and connected to ground through 200 pF, is then recorded. From these data one can determine the inherent noise voltage and noise current of the noise source amplifier plus the FFT as well as the noise of the resistor to about ± 0.25 dB which is the stated accuracy of the FFT. This primary calibration of the FFT can be carried out from about 20 Hz to over 50 kHz. Above 50 kHz, the noise gain of the amplifier we used contributes a significant amount of noise. With some compensation the noise is flat to within ±0.2 dB from 20 Hz to 100 kHz. Next the noise source is switched into the output of the mixer and the relative noise spectrum of all the spectrum analyzers is calibrated by knowing their relative gains. This procedure verifies the voltage references and noise bandwidths of the various spectrum analyzers.

6) The noise voltage is recorded on the three spectrum analyzers over the Fourier frequencies of interest, generally the same one used in step 4

above. The measured noise voltages are scaled using the measured gains and the spectral density of phase noise calculated. The overlapping ranges of the various spectrum analyzers allows one the opportunity to compare the measurements on the three spectrum analyzers. Typically one can obtain $S_\phi(f)$ of the oscillator pair to an accuracy of about ± 0.6 dB at Fourier frequency above 100 Hz. The agreement with repeat measurements is often of order ± 0.2 dB as shown in Fig. 15.

7) The noise floor of the system is determined by driving both sides of the measurement system with one oscillator having a similar power and impedance level as that used in these measurements. If the noise floor is within 13 dB of that measured in step 7 above, corrections are made to the measurement data to remove the bias generated by the noise floor.

Measurements of the amplitude accuracy of the phase modulation side bands generated by the prototype phase modulators are summarized in Fig. 15. The same modulator was used for carrier frequencies from 5 to 300 MHz. The error in the phase modulation amplitude is less than 0.5 dB for modulation frequencies from dc to 10% of the carrier. The 10 GHz modulator also maintains an accuracy of better than 0.5 dB out to 500 MHz from the carrier. At 1 GHz the modulation amplitude is 1.5 dB high. Once this is measured it can be taken into account in the calibration procedure. The 45 GHz modulator results shown in Fig. 15 should be attainable over the entire WR22 wavequide bandwidth.

The performance that can be obtained with this measurement technique is illustrated by actual phase noise data on oscillator pairs shown in Figs. 15 and 16. Typical accuracies are ±0.6 dB with a noise floor at about - 175 dB relative to 1 radian^2/Hz. The corrections applied to the raw data at 10 GHz are shown in Fig. 17. At low frequencies the effect of the phase-locked loop is apparent while at the higher frequencies the roll-off of the amplifiers are important. These effects have been emphasized here in order to examine the ability of the calibration process to correct for instrumentation gain variations.

V. Conclusion

We have analyzed several traditional approaches to making phase measurements and found that they all lacked some element necessary for making phase noise measurements from essentially dc out to 10% of the carrier frequency with good phase noise floors and an accuracy of order 1 dB. By combining several of the techniques and adding a phase modulator which is exceptionally flat from dc to about 10% of the carrier frequency, we have been able to achieve excellent phase noise floors, bandwidths of at least 10% of the carrier, and accuracies of order ±0.6 dB.

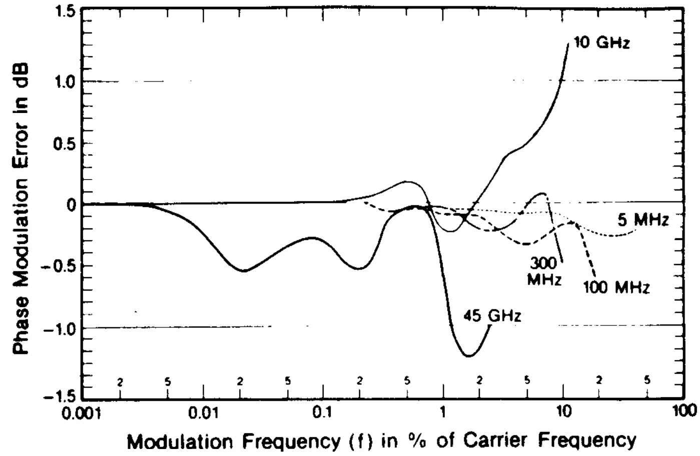

Fig. 14 Measurement of the amplitude error of modulation signal versus Fourier frequencies f, for these prototype phase modulators. Curves labeled 5, 100, and 300 MHz were obtained with the modulator used in 5 to 1,300 MHz test set. The curve labeled 10 GHz was obtained with the modulator for the 2 to 26 GHz test set. The curve labeled 45 GHz was obtained with the WR22 test set.

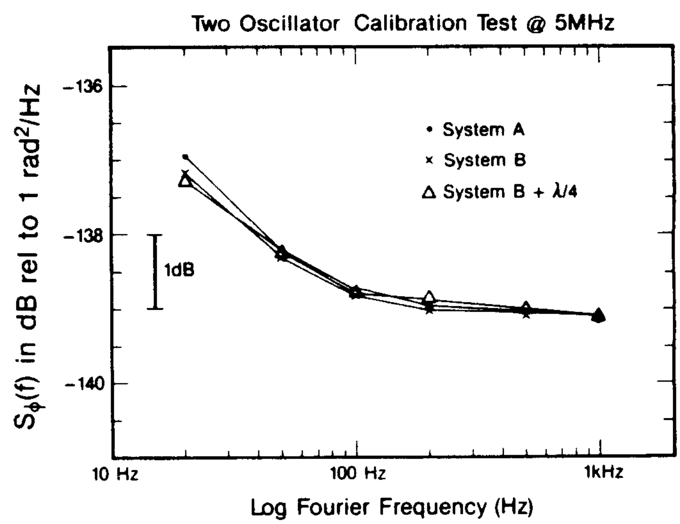

Fig. 15 Demonstration of calibration accuracy for two oscillator concept. The curve labeled System A shows the measured phase noise of a pair of 5 MHz oscillators using the test set shown in Fig. 13. The curve labeled System B shows the measured phase noise of the same pair of 5 MHz oscillators using a totally separate measurement system with the oscillators held in phase quadrature with the measurement test set of A. The curve labeled System B + λ/4 shows the phase noise of the same pair of oscillators using test set B with and extra cable length of λ/4 inserted into each signal path. The agreement between the three curves is in the worse case ± 0.15 dB.

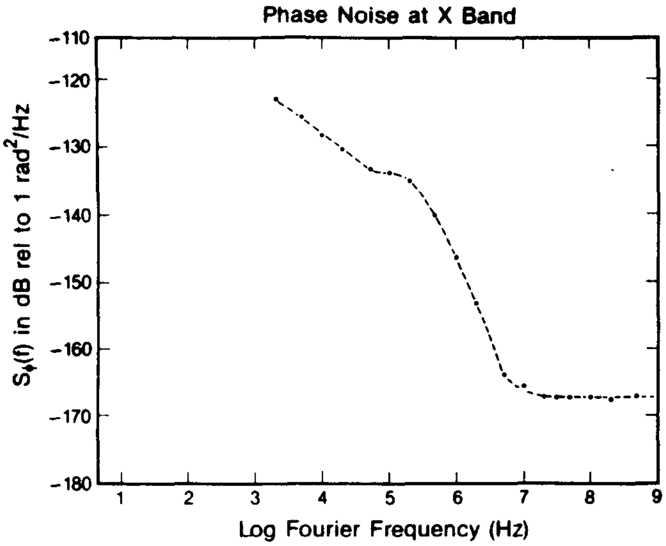

Fig. 16 Phase noise measurement on a pair of 10.6 GHz sources using the new NBS measurement technique.

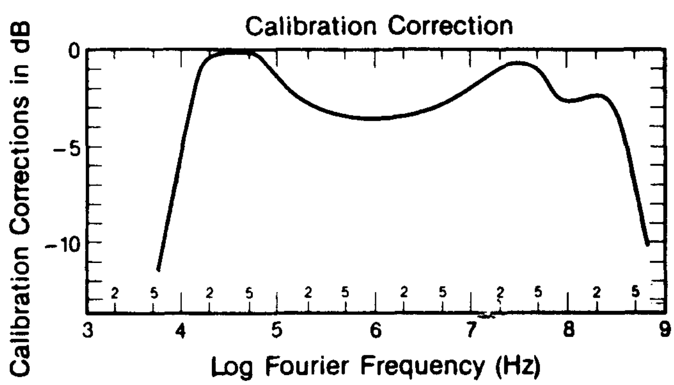

Fig. 17 Correction factor applied to the measurement data made to obtain the results of Fig. 16.

Acknowledgements

The authors are grateful to many colleagues, especially David W. Allan, James C. Bergquist, Andrea DeMarchi, David J. Glaze, James E. Gray, David A. Howe, John P. Lowe, Samuel R. Stein and Charles Stone for many fruitful discussions on this topic and the Calibration Coordination Group for the funding to improve the accuracy and bandwidth of phase noise metrology.

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