

STANDARD TERMINOLOGY FOR
FUNDAMENTAL FREQUENCY AND TIME METROLOGY

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David Allan, National Bureau of Standards, Boulder, CO 80303
Helmut Hellwig, National Bureau of Standards,
Gaithersburg, MD 20899

Peter Kartaschoff, Swiss PTT, R&D, CH 3000 Bern 29, Switzerland
Jacques Vanier, National Research Council, Ottawa, Canada K1A 0R6
John Vig, U.S. Army Electronics Technology and Devices Laboratory,
Fort Monmouth, NJ 07703

Gernot M.R. Winkler, U.S. Naval Observatory, Washington, DC 20390
Nicholas F. Yannoni, Rome Air Development Center, Hanscom AFB,
Bedford, MA 01731

1. Introduction

Techniques to characterize and to measure the frequency and phase instabilities in frequency and time devices and in received radio signals are of fundamental importance to all manufacturers and users of frequency and time technology.

In 1964, a subcommittee on frequency stability was formed within the Institute of Electrical and Electronics Engineers (IEEE) Standards Committee 14 and, later (in 1966), in the Technical Committee on Frequency and Time within the Society of Instrumentation and Measurement (SIM), to prepare an IEEE standard on frequency stability. In 1969, this subcommittee completed a document proposing definitions for measures on frequency and phase stabilities (Barnes, et al., 1971). These recommended measures of instabilities in frequency generators have gained general acceptance among frequency and time users throughout the world.

In this paper, measures in the time and in the frequency domains are reviewed. The particular choice as to which domain is used depends on the application. However, the users are reminded that conversions using mathematical formulations (see Appendix I) from one domain to the other can present problems.

Most of the major manufacturers now specify instability characteristics of their standards in terms of these recommended measures. This paper thus defines and formalizes the general practice of more than a decade.

2. Measures of Frequency and Phase Instability

Frequency and phase instabilities shall be measured in terms of the instantaneous, normalized frequency departure $y(t)$ from the nominal frequency ν_0 and/or by phase departure $\phi(t)$, in radians, from the nominal phase $2\pi\nu_0 t$ as follows:

$$y(t) = \frac{1}{2\pi\nu_0} \frac{d\phi(t)}{dt} = \frac{\dot{\phi}(t)}{2\pi\nu_0}$$

$$x(t) = \frac{\phi(t)}{2\pi\nu_0}$$

where $x(t)$ is the phase departure expressed in units of time.

3. Characterization of Frequency and Phase Instabilities

a. Frequency Domain:

In the frequency domain, frequency and phase instability is defined by any of the following one-sided spectral densities (the Fourier frequency ranges from 0 to ∞):

$S_y(f)$ of $y(t)$

$S_\phi(f)$ of $\phi(t)$

$S_\phi^2(f)$ of $\dot{\phi}(t)$

$S_x(f)$ of $x(t)$.

These spectral densities are related by the equations:

$$S_y(f) = \frac{f^2}{\nu_0^2} S_\phi(f)$$

$$S_\phi^2(f) = (2\pi f)^2 S_\phi(f)$$

$$S_x(f) = \frac{1}{(2\pi\nu_0)^2} S_\phi(f)$$

A device or signal shall be characterized by a plot of spectral density vs. Fourier frequency or by tabulating discrete values or by equivalent means such as a statement of power law(s) (Appendix I).

According to the conventional definition (Kartaschoff, 1978) of $\mathcal{L}(f)$ (pronounced "script ell"), $\mathcal{L}(f)$ is the ratio of the power in one sideband due to phase modulation by noise (for a 1 Hz bandwidth) to the total signal power (carrier plus sidebands), that is,

$$\mathcal{L}(f) = \frac{\text{Power density, one phase-noise modulation sideband/Hz}}{\text{Total signal power}}$$

The conventional definition of $\mathcal{L}(f)$ is related to $S_\phi(f)$ by

$$\mathcal{L}(f) = 4S_\phi(f)$$

only if the mean squared phase deviation, $\langle \dot{\phi}^2(f) \rangle =$ the integral of $S_\phi^2(f)$ from f to ∞ , is much smaller than one radian. In other words, this relationship is valid only for Fourier frequencies f far enough from the carrier frequency and is always violated near the carrier.

Since $S_\phi(f)$ is the quantity that is generally measured in frequency standards metrology, and $\mathcal{L}(f)$ has become the prevailing measure of phase noise among manufacturers and users of frequency standards, $\mathcal{L}(f)$ is redefined as

$$\mathcal{L}(f) = 4S_\phi(f)$$

This redefinition is intended to avoid erroneous use of $\mathcal{L}(f)$ in situations where the small angle approximation is not valid. In other words, $S_\phi(f)$

is the preferred measure, since, unambiguously, it always can be measured.

b. Time-Domain:

In the time domain, frequency instability shall be defined by the two-sample deviation $\sigma_y(r)$ which is the square root of the two-sample variance $\sigma_y^2(r)$. This variance, $\sigma_y^2(r)$, has no dead-time between the frequency samples and is also called the Allan variance. For the sampling time r , we write:

$$\sigma_y^2(r) = \frac{1}{2} \langle (\bar{y}_{k+1} - \bar{y}_k)^2 \rangle$$

where

$$\bar{y}_k = \frac{1}{r} \int_{t_k}^{t_{k+1}} y(t) dt = \frac{x_{k+1} - x_k}{r}$$

The symbol $\langle \rangle$ denotes an infinite time average. In practice, the requirement of infinite time average is never fulfilled; the use of the foregoing terms shall be permitted for finite time averages. The x_k and x_{k+1} are time residual measurements made at t_k and $t_{k+1} = t_k + r$, $k=1,2,3,\dots$, and $1/r$ is the nominal fixed sampling rate which gives zero dead time between frequency measurements. "Residual" implies the known systematic effects have been removed.

If dead time exists between the frequency departure measurements and this is ignored in the computation of $\sigma_y(r)$, resulting instability values will be biased (except for white frequency noise). Some of the biases have been studied and some correction tables published [Barnes, 1969; Lesage, 1983; Barnes and Allan, 1988]. Therefore, the term $\sigma_y(r)$ shall not be used to describe such biased measurements. Rather, if biased instability measures are made, the information in the references should be used to report an unbiased estimate.

If the initial sampling rate is specified as $1/r_0$, then it has been shown that, in general, we may obtain a more efficient estimate of $\sigma_y(r)$ using what is called "overlapping estimates." This estimate is obtained by computing

$$\sigma_y^2(r) = \frac{1}{2(N-2m)r^2} \sum_{i=1}^{N-2m} (x_{i+2m} - 2x_{i+m} + x_i)^2$$

where N is the number of original time residual measurements spaced by r_0 ($N=M+1$, where M is the number of original frequency measurements of sample time r_0) and $r = mr_0$.

From the above equation, we see that $\sigma_y^2(r)$ acts like a second-difference operator on the time deviation residuals--providing a stationary measure of the stochastic behavior even for nonstationary processes. Additional variances, which may be used to describe frequency instabilities, are defined in Appendix II.

c. Clock-Time Prediction

The variation of the time difference between a real clock and an ideal uniform time scale, also known as time interval error, TIE, observed over a time interval starting at time t_0 and ending at t_0+t shall be defined as:

$$TIE(t) = x(t_0+t) - x(t_0) = \int_{t_0}^{t_0+t} y(t') dt'$$

For fairly simple models, regression analysis can provide efficient estimates of the TIE (Draper and Smith, 1966; CCIR, 1986). In general, there are many estimators possible for any statistical quantity. Ideally, we would like an efficient and unbiased estimator. Using the time domain measure $\sigma_y^2(r)$ defined in (b), the following estimate of the standard deviation (RMS) of TIE and its associated systematic departure due to a linear frequency drift (or its uncertainty) can be used to predict a probable time interval error of a clock synchronized at $t=t_0=0$ and left free running thereafter:

$$RMSTIE_{est} = t \left[\frac{a^2}{4} t^2 + \sigma_{y_0}^2 + \sigma_y^2(r=t) + \left(\frac{x(t_0)}{t} \right)^2 \right]^{1/2}$$

where "a" is the normalized linear frequency drift per unit of time (aging) or the uncertainty in the drift estimate, σ_{y_0} the two-sample deviation of the initial frequency adjustment, $\sigma_y(r)$ the two-sample deviation describing the random frequency instability of the clock at $t=r$, and $x(t_0)$ is the initial synchronization uncertainty. The third term in the brackets provides an optimum and unbiased estimate (under the condition of an optimum (RMS) prediction method) in the cases of white noise FM and/or random walk FM. The third term is too optimistic, by about a factor of 1.4, for flicker noise FM, and too pessimistic, by about a factor of 3, for white noise FM.

This estimate is a useful and fairly simple approximation. In general, a more complete error analysis becomes difficult; if carried out, such an analysis needs to include the methods of time prediction, the uncertainties of the clock parameters, using the confidence limits of measurements defined below, the detailed clock noise models, systematic effects, etc.

4. Confidence Limits of Measurements

An estimate for $\sigma_y(r)$ can be made from a finite data set with M measurements of y_j as follows:

$$\sigma_y(r) = \left| \frac{1}{2(M-1)} \sum_{j=1}^{M-1} (\bar{y}_{j+1} - \bar{y}_j)^2 \right|^{1/2}$$

or, if the data are time readings x_j :

$$\sigma_y(r) = \left| \frac{1}{2r^2(M-1)} \sum_{j=1}^{M-1} (x_{j+2} - 2x_{j+1} + x_j)^2 \right|^{1/2}$$

The 68 percent confidence interval (or error bar), I_a , for Gaussian noise of a particular value $\sigma_y(r)$ obtained from a finite number of samples can be estimated as follows:

$$I_a = \sigma_y(r) \kappa_a M^{-1/2}$$

where:

- M = total number of data points used in the estimate,
- α = an integer as defined in Appendix I,
- $\kappa_2 = \kappa_1 = 0.99$.

$$\begin{aligned} \kappa_0 &= 0.87, \\ \kappa_{-1} &= 0.77, \\ \kappa_{-2} &= 0.75. \end{aligned}$$

As an example of the Gaussian model with $M=100$, $\alpha = -1$ (flicker frequency noise) and $\sigma_y(r = 1 \text{ second}) = 10^{-12}$, we may write:

$$I_{\pm} = \sigma_y(r) \cdot (0.77) \cdot (100)^{\pm 1/2} = \sigma_y(r) \cdot (0.077)$$

which gives:

$$\sigma_y(r = 1 \text{ second}) = (1 \pm 0.08) \times 10^{-12}.$$

If M is small, then the plus and minus confidence intervals become asymmetric and the κ_{\pm} coefficients are not valid; however, these confidence intervals can be calculated (Lesage and Audoin, 1973).

If "overlapping" estimates are used, as outlined above, then the confidence interval of the estimate can be shown to be less than or equal to I_{\pm} as given above (Howe, Allan, Barnes, 1981).

5. Recommendations for Characterizing or Reporting Measurements of Frequency and Phase Instabilities

- a. Nonrandom phenomena should be recognized, for example:
 - o any observed time dependency of the statistical measures should be stated;
 - o the method of modeling systematic behavior should be specified (for example, an estimate of the linear frequency drift was obtained from the coefficients of a linear least-squares regression to M frequency measurements, each with a specified averaging or sample time τ and measurement bandwidth f_h);
 - o the environmental sensitivities should be stated (for example, the dependence of frequency and/or phase on temperature, magnetic field, barometric pressure, vibration, etc.);
- b. Relevant measurement or specification parameters should be given:
 - o the method of measurements;
 - o the characteristics of the reference signal;
 - o the nominal signal frequency ν_0 ;
 - o the measurement system bandwidth f_h and the corresponding low pass filter response;
 - o the total measurement time and the number of measurements M ;
 - o the calculation techniques (for example, details of the window function when estimating power spectral densities from time domain data, or the assumptions about effects of dead-time when estimating the two-sample deviation $\sigma_y(r)$);
 - o the confidence of the estimate (or error bar) and its statistical probability (e.g. "three-sigma");

- o the environment during measurement;
- o if a passive element, such as a crystal filter, is being measured in contrast to a frequency and/or time generator.

6. References

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APPENDIX I

1. Power-Law Spectral Densities

Power-law spectral densities are often employed as reasonable and accurate models of the random fluctuations in precision oscillators. In practice, these random fluctuations can often be represented by the sum of five independent noise processes, and hence:

$$S_y(f) = \begin{cases} \sum_{\alpha=-2}^{+2} h_{\alpha} f^{\alpha} & \text{for } 0 < f < f_h \\ 0 & \text{for } f \geq f_h \end{cases}$$

where h_{α} 's are constants, α 's are integers, and f_h is the high frequency cut-off of a low pass filter. High frequency divergence is eliminated by the restrictions on f in this equation. The identification and characterization of the five noise processes are given in Table 1, and shown in Fig. 1.

2. Conversion Between Frequency and Time Domain

The operation of the counter, averaging the frequency for a time τ , may be thought of as a filtering operation. The transfer function, $H(f)$, of this equivalent filter is then the Fourier transform of the impulse response of the filter. The time domain frequency instability is then given by

$$\sigma^2 (M.T., \tau) = \int_0^{\infty} S_y(f) |H(f)|^2 df.$$

where $S_y(f)$ is the spectral density of frequency fluctuations. $1/T$ is the measurement rate ($T-\tau$ is the dead time between measurements). In the case of the two-sample variance $|H(f)|^2$ is $2(\sin^2 \pi r f) / (\pi r f)^2$. The two-sample variance can thus be computed from

$$\sigma_y^2(\tau) = 2 \int_0^{f_h} S_y(f) \frac{\sin^4 \pi r f}{(\pi r f)^2} df.$$

Specifically, for the power law model given, the time domain measure also follows a power law.

TABLE 1 - The functional characteristics of the independent noise processes used in modeling frequency instability of oscillators

Description of Noise Process	Slope characteristics of log log plot				
	Frequency domain		Time-domain		
	$S_y(f)$ or $S_\beta(f)$	$S_\beta(f)$ or $S_x(f)$	$\sigma_y^2(\tau)$	$\sigma_y(\tau)$	Mod. $\sigma_y(\tau)$
	α	β	μ	$\mu/2$	μ'
Random Walk Freq Modulation	-2	-4	1	1/2	1
Flicker Frequency Modulation	-1	-3	0	0	0
White Frequency Modulation	0	-2	-1	-1/2	-1
Flicker Phase Modulation	1	-1	-2	-1	-2
White Phase Modulation	2	0	-2	-1	-3

$$S_y(f) = \frac{(2\pi f)^2}{(2\pi\nu_0)^2} S_\beta(f) = h_n f^\alpha$$

$$\sigma_y^2(\tau) = |\tau|^\mu$$

$$S_\beta(f) = \nu_0^2 h_n f^{\alpha-2} = \nu_0^2 h_n f^\beta \quad (\beta = \alpha-2)$$

$$\sigma_y(\tau) = |\tau|^{\mu/2}$$

$$S_x(f) = \frac{1}{4\pi^2} h_n f^{\alpha-2} = \frac{1}{4\pi^2} h_n f^\beta$$

$$\text{Mod. } \sigma_y(\tau) = |\tau|^{\mu'}$$

TABLE 2 - Translation of frequency instability measures from spectral densities in frequency domain to variances in time domain and vice versa (For $2\pi f_h \tau \gg 1$)

Description of noise process	$\sigma_y^2(\tau) =$	$S_y(f) =$	$S_\beta(f) =$
Random Walk Frequency Modulation	$A f ^2 S_y(f) \tau ^1$	$\frac{1}{A} [r^{-1} \sigma_y^2(\tau)] f^{-2}$	$\frac{\nu_0^2}{A} [r^{-1} \sigma_y^2(\tau)] f^{-4}$
Flicker Frequency Modulation	$B f S_y(f) \tau ^0$	$\frac{1}{B} [r^0 \sigma_y^2(\tau)] f^{-1}$	$\frac{\nu_0^2}{B} [r^0 \sigma_y^2(\tau)] f^{-3}$
White Frequency Modulation	$C f^0 S_y(f) \tau ^{-1}$	$\frac{1}{C} [r^1 \sigma_y^2(\tau)] f^0$	$\frac{\nu_0^2}{C} [r^1 \sigma_y^2(\tau)] f^{-2}$
Flicker Phase Modulation	$D f^{-1} S_y(f) \tau ^{-2}$	$\frac{1}{D} [r^2 \sigma_y^2(\tau)] f^1$	$\frac{\nu_0^2}{D} [r^2 \sigma_y^2(\tau)] f^{-1}$
White Phase Modulation	$E f^{-2} S_y(f) \tau ^{-2}$	$\frac{1}{E} [r^2 \sigma_y^2(\tau)] f^2$	$\frac{\nu_0^2}{E} [r^2 \sigma_y^2(\tau)] f^0$

$$A = \frac{4\pi^2}{6}$$

$$C = 1/2$$

$$E = \frac{3f_h}{4\pi^2}$$

$$B = 2 \log_e 2$$

$$D = \frac{1.038 + 3 \log_e (2\pi f_h \tau)}{4\pi^2}$$

APPENDIX II

ADDITIONAL VARIANCES THAT MAY BE USED TO DESCRIBE FREQUENCY INSTABILITIES IN THE TIME DOMAIN

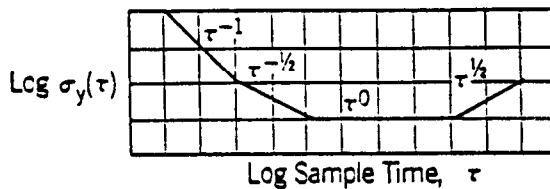
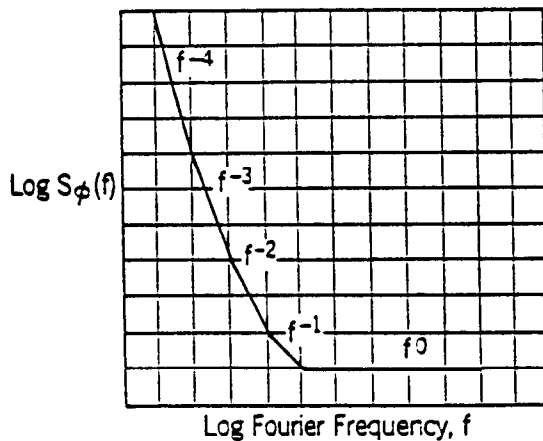
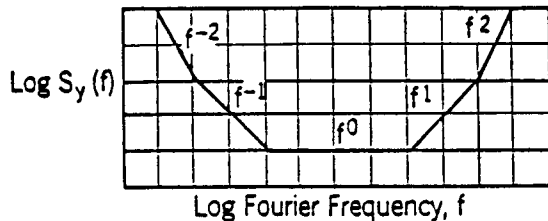
$$\sigma_y^2(\tau) = h_{-2} \frac{(2\pi)^2}{6} \tau + h_{-1} 2 \log_e 2 + h_0 \frac{1}{2\tau} + h_1 \frac{1.038 + 3 \log_e(2\pi f_h \tau)}{(2\pi)^2 \tau^2} + h_2 \frac{3f_h}{(2\pi)^2 \tau^2}$$

This implicitly assumes that the random driving mechanism for each term is independent of the others. In addition, there is the implicit assumption that the mechanism is valid over all Fourier frequencies, which may not always be true.

The values of h_α are characteristic models of oscillator frequency noise. For integer values (as often seems to be the case for reasonable models), $\mu = -\alpha - 1$, for $-3 \leq \alpha \leq 1$, and $\mu = -2$ for $\alpha \geq 1$, where $\sigma_y^2(\tau) \sim \tau^\mu$.

Table 2 gives the coefficients of the translation among the frequency stability measures from time domain to frequency domain and from frequency domain to time domain.

The slope characteristics of the five independent noise processes are plotted in the frequency and time domains in Fig. 1 (log-log scale).



Slope characteristics of the five independent noise processes.

FIGURE 1

1. Modified Allan, or Modified Two-Sample, Variance, Mod $\sigma_y^2(\tau)$

Instead of the use of $\sigma_y^2(\tau)$, a "Modified Variance" Mod $\sigma_y^2(\tau)$ may be used to characterize frequency instabilities (Stein, 1985; Allan, 1987). It has the property of yielding different dependence on τ for white phase noise and flicker phase noise. The dependence for Mod $\sigma_y^2(\tau)$ is $\tau^{-3/2}$ and τ^{-1} respectively. Mod $\sigma_y^2(\tau)$ is defined as:

$$\text{Mod } \sigma_y^2(\tau) = \frac{1}{2r^2 m^2 (N-3m+1)} \sum_{j=1}^{N-3m+1} \left[\sum_{i=j}^{m+j-1} (x_{i+2m} - 2x_{i+m} + x_i) \right]^2$$

where N is the original number of time measurements spaced by r_0 , and $r = m r_0$, the sample time of choice ($N=M+1$). A device or signal shall be characterized by a plot of $\sigma_y(\tau)$ or $\sigma_y^2(\tau)$ or Mod $\sigma_y(\tau)$ or Mod $\sigma_y^2(\tau)$ vs. sampling time τ , or by tabulating discrete values or by equivalent means such as a statement of power laws (Appendix I).

2. Other Variances

Several other variances have been introduced by workers in this field. In particular, before the introduction of the two-sample variance, it was standard practice to use the sample variance, s^2 , defined as

$$s^2 = \int_0^{f_h} S_y(f) \left(\frac{\sin \pi f \tau}{\pi f \tau} \right)^2 df$$

In practice it may be obtained from a set of measurements of the frequency of the oscillator as

$$s^2 = \frac{1}{N} \sum_{i=1}^N (y_i - \bar{y})^2$$

The sample variance diverges for some types of noise and, therefore, is not generally useful.

Other variances based on the structure function approach can also be defined (Lindsey and Chi, 1976). For example, there are the Hadamard variance, the three-sample variance and the high pass variance (Rutman 1978). They are occasionally used in research and scientific works for specific purposes, such as differentiating between different types of noise and for dealing with systematics and sidebands in the spectrum.

APPENDIX III

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