PICOSECOND TIME DIFFERENCE MEASUREMENT SYSTEM

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Abstract

Recently a time difference measurement system was developed in the Frequency & Time Standards Section of the National Bureau of Standards and independently at the National Research Council which shows more than three orders of magnitude improvement over that which is currently available commercially. Current state-of-the-art time difference measurement devices have specified accuracies of about 1 nanosecond. Measurement precision and potential accuracy of better than 1 picosecond has been demonstrated in this new time difference measuring device. This has significant implications in frequency and time metrology using: state-of-the-art frequency standards and clocks. A brief report was given on this measurement system at the PTTI Planning Meeting at the Naval Research Laboratory on the 5th of December 1974. This paper will give more detailed circuit diagrams necessary to build up such a measurement system.

This particular measurement system has the advantage that it can measure time differences with accuracies of a few picoseconds and with repetition rates ranging from a few milliseconds to as slow a repetition rate as would be desirable, thus expanding convenient measurement of time domain stabilities of frequency and time standards over several decades with only one measurement system. The system is also very amenable to self-calibration and self-noise analysis. Specifically, a fractional frequency stability of about 10^{-16} was measured for the noise of this measurement system at a sample time of 10^3 s.

Introduction & Perspective

When making measurements between a pair of frequency standards or clocks, it is often desirable to have less noise in the measurement system than the composite noise in the pair of standards being measured. This places stringent requirements on measurement systems as the state-of-the-art of precision frequency & time standards has advanced to its current level. [1] As will be shown, perhaps one of the greatest areas of disparity between measurement system noise and the noise in current standards is in the area of time difference measurements. Commercial equipment can measure time differences to about 10^{-10}s, but the time fluctuations - second to second - of state-of-the-art standards is as good as 10^{-13}s.

The disparity is unfortunate because if time differences between two standards could be measured with adequate precision then one may also know the time fluctuations, the frequency differences, and the frequency fluctuations. In fact, one can set up an interesting hierarchy of kinds of measurement systems: 1) those that can measure time; $x(t)$; 2) those that can measure changes in time or time fluctuations $\delta x(t)$; 3) those that can measure frequency, ν ($y \equiv (\nu-\nu_0)/\nu_0$); and 4) those that can measure changes in frequency or frequency fluctuations, $\delta\nu$ ($\delta y \equiv \delta\nu/\nu_0$). As depicted in Table 1 if a measurement system is of status 1 in this hierarchy, i.e., it can measure time, then time fluctuations, frequency and frequency fluctuations can be deduced. However, if a measurement system is only capable of measuring time fluctuations (Status 2 - Table 1), then time cannot be deduced, but frequency and frequency fluctuations can. If frequency is being measured (Status 3 - Table 1), then neither time nor time fluctuations may be deduced with fidelity because essentially all commercial frequency measuring devices have "dead time" (technology is at a point where that may soon change with data processing speeds that are now available). Dead time in a frequency measurement destroys the opportunity of integrating the fractional frequency to get to "true" time fluctuations. Of course, if frequency can be measured, then trivially one may deduce the frequency fluctuations. Finally, if a system can only measure frequency fluctuations (Status 4 - Table 1), then neither time, nor time fluctuations, nor frequency can be deduced from the data. If the frequency stability is the primary concern then one may be perfectly happy to employ such a measurement system, and similarly for the other statuses in this measurement hierarchy. Obviously, if a measurement method of Status 1 could be employed with state-of-the-art precision, this would provide the greatest flexibility in data processing. The dual mixer time difference system set forth in this paper is purported to be such a method. Before discussing this method in detail, we will briefly review the other measurement method examples of Table 1.

The measurement method example of Status 2 in Table 1 is shown in block diagram in Figure 1. This method - as described elsewhere [2-6] - has proved very useful in analyzing the spectral density, $S_\phi(f)$, of the phase (time) fluctuations in precision oscillators; ϕ is the phase in radians and f is the Fourier frequency. We use "phase (time)" because they are linearly and simply related by:

$$x(t) = \frac{\phi(t)}{2\pi\nu_0}, \qquad (1)$$

where ν_0 is the oscillator's nominal carrier frequency. The "keystone" in all the state-of-art measurement systems reviewed in this paper is the low-noise Schottky barrier diode mixer. The signal from an oscillator under test is fed into one port of a mixer. The signal from a reference oscillator is fed into the other port of this mixer. The signals are in quadrature, that is, they are 90 degrees out of phase so that the average voltage out of the mixer is nominally zero, and the instantaneous voltage corresponds to phase fluctuations rather than to the amplitude fluctuations between the two signals. The output of this mixer is fed through a low pass filter and them amplified in a feedback loop, causing the voltage controlled oscillator (reference) to be phase locked to the test oscillator. The gain is adjusted to obtain a very loose phase lock condition. For times shorter than the attack time of the loop a voltage fluctuation will be proportional to a phase or time fluctuation which is equivalent to the condition where the oscillators are "free running" (the attack time is the inverse of 2π times the unity gain bandwidth of the loop). In turn the output of the low noise amplifier may be fed to a spectrum analyzer, for example, to measure the Fourier components of the phase fluctuations. Alternatively, the output of the low-noise amplifier may be fed to a voltage to frequency converter which in turn is fed to a counter. A frequency fluctuation, $\delta\nu_c$, as read on the counter is proportional to a phase (time) fluctuation, $\delta\nu_c \sim \delta x(t,\tau)$ allowing one to also analyze the fluctuations in the time-domain. With typical attack times this measurement method is useful for measuring the noise for Fourier frequencies equal to or greater than 1 Hz and for sample times of the order of 2π seconds and shorter. It is also specifically very useful if discrete sidebands such as 60 Hz or detailed structure exist in the spectrum. The measurement method example of Status 3 in the hierarchy of Table 1 is shown in block diagram in Figure 2a. As illustrated, the signals from two independent oscillators are fed into the two ports of a double balanced mixer. The difference frequency or the beat frequency out, ν_b, is obtained as the output of a low pass filter which follows the mixer. This beat frequency is then amplified and fed to a frequency counter and printer or some recording device. The fractional frequency can simply be obtained by dividing ν_b, by the nominal carrier frequency ν_0, $|y(t,\tau)| = \nu_b/\nu_0$. This heterodyne or beat frequency method and slight variations of it are commonly used for measuring precision oscillators. This method has the advantage of being simple and inexpensive; and state-of-the-art measurement precision is achievable. It has the disadvantages that only the magnitude of the frequency difference can be measured; i.e., without additional information one cannot ascertain which of the two oscillators is higher or lower than the other in frequency. Also, they must be at different frequencies in order to get a beat frequency; and typically, for common oscillators it is difficult without special synthesis to make this beat frequency even as high as 1 Hz, hence, frequency sample times are usually limited to about 1 second and longer. Commercial frequency counters cause a dead time in a data sequence equal to or greater than the period of the beat frequency. Dead time causes three fundamental problems: 1) One cannot precisely deduce the time fluctuations by integrating the frequency; 2) Increased complications often occur in the stability analysis of the data causing in some instances misleading conclusions; and 3) The time required to take the data in order to do a frequency stability characterization of a pair of oscillators is often significantly increased over that necessary with no dead time. Some have used a method of dating the zero-crossings of the beat frequency to successfully avoid the dead time problem.

The measurement method example of Status 4 in the hierarchy of Table 1 is shown in block diagram in Figure 2b. It is essentially the same as in Figure 1, except in this case the loop is in a tight phase lock condition; i.e., the attack time of the loop should be of the order of a few milliseconds. In such a case, the phase fluctuations are being integrated so that the voltage output is proportional to the frequency fluctuations relative to the "unlocked" reference oscillator and is no longer proportional to the "free running" phase fluctuations for sample times longer than the attack time of the loop. The bias box is used to adjust the voltage on a voltage variable frequency tuning capacitor (varicap) to a convenient tuning point. The voltage fluctuations prior to the bias box (biased slightly away from zero) are fed to a voltage to frequency converter which in turn is fed to a frequency counter where the frequency fluctuations can be read with great amplification of the instabilities between this pair of oscillators. The frequency counter data are logged with a printer or some other data logging device. The coefficient of the varicap and the coefficient of the voltage to frequency converter are used to determine the fractional frequency fluctuations, y_i, between the oscillators, where i denotes the i^{th} measurement as shown in Figure 2b. A system sensitivity of a part in 10^{14} per Hz resolution of the frequency counter is easily achievable. This method is useful for making time-domain measurements of the frequency fluctuations for sample times of the order of one second and longer (usually limited in long-term by the maximum gate time of the frequency counter employed).

Dual Mixer Time Difference System

A block diagram of the dual mixer time difference system is shown in Figure 3. Let us restate that if the time or the time fluctuations can be measured directly an advantage is obtained over just measuring the frequency, because one can calculate the frequency from the time without dead time as well as know the time behavior. Special laboratory test-sets similar to that shown in Figure 3 have been developed in the past with equivalent time-difference precisions as good as 0.6 picosecond. [7, 8]

The system described in this section demonstrated a precision of 0.1 picosecond and with the potential of achieving 0.01 picoseconds (10^{-14}s). Such precisions open the door to making time measurements as well as frequency and frequency stability measurements for sample times as short as a few milliseconds as well as for longer sample times and all without dead time. In Figure 3, Oscillator 1 could be considered under test and Oscillator 2 could be considered the reference oscillator. These signals go to the ports of a pair of double balanced mixers. Another oscillator with separate symmetric buffered outputs is fed to the remaining other two ports of the pair of double balanced mixers. This common oscillator's frequency is offset by a desired amount from the other two oscillators producing approximately the same beat frequencies with Oscillator 1 and with Oscillator 2 as illustrated in Figure 3. These beat frequencies will be out of phase by an amount proportional to the time difference between Oscillator 1 and 2 when running as clocks - excluding the differential phase shift that my be inserted; and will differ in frequency by an amount equal to the frequency difference between Oscillators 1 and 2. If Oscillator 1 and Oscillator 2 are on the same frequency the time difference remains constant. In contrast the heterodyne or beat frequency method is not useful if the oscillators are near zero beat frequency which is often the case with atomic standards (cesium, rubidium and hydrogen frequency standards).

A phase shifter may be inserted as illustrated in Figure 3 to adjust the phase so that the two beat frequencies are nominally in phase; this adjustment sets up the nice condition that the noise of the common oscillator tends to cancel when the time difference is determined in the next step - depending on the level and the type of noise as well as the sample time involved as described below. After amplifying these beat signals, the start port of a time interval counter is triggered with the zero crossing of one beat and the stop port with the zero crossing of the other beat. If the phase fluctuations of the common oscillator are small during this time interval as compared to the phase fluctuations between Oscillators 1 and 2 over a full period of the nominal beat frequencies, the noise of the common oscillator is insignificant in the measurement noise error budget, which means the noise of the common oscillator can be worse than that of either Oscillator 1 or 2 and still not contribute significantly. The above condition will exist if the following equation is satisfied:

$$\Delta t \cdot \left\langle \sigma^2_{y_{common}} (2, \tau, \Delta t, f_h) \right\rangle^{1/2} << \tau \cdot \sigma_{y_{12}}(\tau) \qquad (2)$$

where the variances (or their square roots) are as defined by the IEEE subcommittee on frequency stability [9, 10]. The left side of Eq. (2) is representative of the phase (time) noise of the common oscillator over Δt and the right side is of the combined phase (time) noise of Oscillators 1 and 2 over τ.

By taking the time difference between the zero crossings of these beat frequencies, what effectively is being measured is the time difference between Oscillator 1 and Oscillator 2, but with a precision which has been amplified by the ratio of the carrier frequency to the beat frequency over that normally achievable with this same time interval counter. The time difference $x(i)$, for the i^{th} measurement between Oscillators 1 and 2 is given by Eq. (3):

$$x(i) = \frac{\Delta t(i)}{\tau\nu} - \frac{\phi}{2\pi\nu} + \frac{n}{\nu} \tag{3}$$

where $\Delta t(i)$ is the i^{th} time difference as read on the counter, τ is the beat period, ν is the nominal carrier frequency, ϕ is the phase delay in radians added to the signal of Oscillator 1, and n is an integer to be determined in order to remove the cycle ambiguity. It is only important to know n if the absolute time difference is desired; for frequency and frequency stability measurements and for time fluctuation measurements, n may be assumed zero unless a cycle or more elapses during a set of measurements. If the frequency difference between Oscillators 1 and 2 is large enough to cause n to accumulate significantly over the duration of an experiment, one could easily build a divider (counter) to determine n as a function of time.

The fractional frequency can be derived in the normal way from the time fluctuations.

$$Y_{1,2}(i,\tau) = \begin{cases} \dfrac{\nu_1(i,\tau) - \nu_2(i,\tau)}{} \\[2ex] \dfrac{x(i+1) - x(i)}{\tau} \\[2ex] \dfrac{\Delta t(i+1) - \Delta\tau(i)}{\tau^2 \nu} \end{cases} \tag{4}$$

The last equality in Eq. (4) is convenient for using the direct readings of the time interval counter but is not valid if n changes between the i and i + 1 readings. In Eqs. (3) and (4), the assumptions are made that the transfer or common oscillator is set at a lower frequency than Oscillators 1 and 2, and that the beat $\nu_1 - \nu_0$ starts and $\nu_2 - \nu_0$ stops the time interval counter. The sample time by appropriate calculation can be any integer multiple of τ:

$$Y_{1,2}(i, m\tau) = \frac{x(i+m) - x(i)}{m\,\tau} \tag{5}$$

where m is any positive integer. If needed, τ can be made to be very small by having very large beat frequencies. One may replace the common or transfer oscillator with a low phase noise synthesizer, which derives its basic reference frequency from Oscillator 2, for example. In such a set up the nominal beat frequencies are simply given by the amount the output frequency of the synthesizer is offset from ν_2. Sample times as short as a few milliseconds were easily obtained. Logging the data at such a rate can be a problem without special equipment, e.g., magnetic tape. The system developed at the National Research Council (NRC) differs from that of the National Bureau of Standards (NBS) in this respect: that the transfer oscillator is always offset 500 Hz. To obtain, e.g., a 1 second averaging time, pulses from a 10 MHz clock are gated by the 500 time intervals between leading edges of the 500 Hz beats before being fed into a counter preceded by a divide by 5 stage. The counter has two counting registers which are used on alternate seconds at the end of each second the accumulated count is transferred to the storage and display register which is so arranged that it always displays 200,000.0 picosecond full scale. The use of the dual counter eliminates dead time between measurements while the use of 500 Hz although it widens the bandwidth eliminates problems with stabilization of dc levels in stages after the first squarer. At NBS a computing counter was used to achieve sample time stabilities as short as a few milliseconds and with no dead time (see appendix for computing counter program possibilities). A circuit diagram for this dual mixer time difference system is shown in Figure 4.

A review of some of the advantages of the dual mixer time difference system are: If the oscillators, including the transfer oscillator, and a time interval counter are available, the component cost is fairly inexpensive ~ \$500, most of which is the cost of the phase shifter (the phase shifter is not necessary if the noise of the common oscillator is sufficiently low). The measurement system bandwidth is easily controlled (note that this should be done in tandem with both low pass filters being symmetrical). The measurement precision is such that one can measure essentially all state-of-the-art oscillators. For example, if the oscillators are at 5 MHz, the beat frequencies are 0.5 Hz and the time interval counter employed has a precision of 0.1 μs, then the potential measurement precision is 10^{-14}s (10 femto seconds) for τ = 2s; other things may limit the precision such as noise in the amplifiers. This precision does not imply that absolute time differences can be measured this well; calibrating all the relevant phase delays limits the time difference accuracy to the order of a picosecond. As has been stated above, there is no dead time in the frequency averages (see ref. 10), which is a distinct advantage for sample times shorter than a second - a region of sample times where dead time is difficult to avoid in most other measurement methods. The sample time is conveniently selectable by adjusting the frequency of the common oscillator and is given by the nominal beat period or multiple of the same. If one replaces the common oscillator by a synthesizer then the beat period may be selected very conveniently. The synthesizer should have fairly low phase noise to obtain the maximum precision from the system. If discrete sidebands exist in the spectrum, having a synthesizer as the common oscillator is also very convenient because by setting the beat frequency equal to the sideband one can estimate the stability as if the sideband weren't present (the sideband has been filtered out). The system measures time difference putting it in Status 1 of the measurement hierarchy. One may calculate from the data both the magnitude and the sign of the frequency difference. This system, therefore, allows the measurement of time fluctuations as well as time difference, and the calculation of frequency fluctuations as well as frequency differences between the two oscillators in question. The system may be calibrated and the system noise be measured by simply feeding the same signal symmetrically split from one oscillator in place of Oscillators 1 and 2.

A review of the disadvantages are: The system is somewhat more complex than the others. Because of the low frequency beats involved, precautions must be taken to avoid ground loop problems; there are some straightforward solutions; e.g., a saturated amplifier followed by a differentiator and isolation transformer worked very well in avoiding ground loops as shown in Figure 4. Buffering is needed on the split output signals from the common oscillator because the mixers present a dynamic load to the oscillator - allowing the possibility of cross-talk. The time difference reading is modulo the beat period. For example, at 5 MHz there is a 200 nanosecond per cycle ambiguity that must be resolved if the absolute time difference is desired: this ambiguity is usually a minor problem to resolve for precision oscillators.

It is instructive to compare the measurement methods discussed to this point in terms of the nominal region of sample times over which eacy typically finds usefulness. This is illustrated in Figure 5. One sees again a significant advantage using the dual mixer time difference approach because of the significant number of decades of sample time covered. To estimate $\sigma_y(\tau)$ using the beat frequency method it is necessary to use a set of bias functions as explained in Ref. [2 and 10], since this method as described in the text generates data having dead time. However, it should be noted that by precisely logging the dates of zero crossings of a beat frequency, the problem of dead time may be avoided. This method has proven useful in the Frequency & Time Standards Section of the NBS. An estimate of $\sigma_y(\tau)$ can be obtained from the frequency-domain method suggested in Figure 1 by using Table 2 or Ref. [1, 2, 8 or 9]. Table 2 is restricted to integer power-law spectral densities for the noise models.

Some Measurements Using the Dual Mixer Time Difference (DMTD) System

By using a low-noise synthesizer as the local or common oscillator and by feeding the signal from a quartz crystal oscillator symmetrically split into the ports for Oscillators 1 and 2 of the DMTD system we estimated the measurement noise, and this is shown in Figure 6 as a $\sigma_y(\tau)$ vs τ plot. The curve plotted may be reasonably modeled as: $\sigma_y(\tau) \simeq 10^{-13}\ \tau^{-1}$ (τ in seconds) with slight exceptions at sample times of milliseconds, where the measurement system bandwidth was opened from 1.8 Hz up to 300 Hz, and at sample times of $\tau > 10^3$s where $\sigma_y(\tau)$ flattened at several parts in 10^{17}; apparently some diurnal effects were perturbing the system. Figure 7a is a plot using the DMTD system of a strip chart recording of a digital to analog output of the significant digits from the time interval counter between a quartz oscillator and a high performance commercial cesium oscillator. In other words this is a plot of the time difference between these two oscillators as a function of time. The high frequency fluctuations (over fractions of a second) would most probably be those between the quartz oscillator and the quartz oscillator in the cesium servo system. The low frequency fluctuations (over seconds) would most probably be those induced by the cesium servo in its effort to move the frequency of its quartz oscillator to the natural resonance of the cesium atom - causing a random walk of the time fluctuations for sample times longer than the servo attack time. Figure 7b shows a similar plot obtained with the NRC system for the phase difference between the NRC standard Cs V and a similar cesium oscillator as in Figure 7a. The averaging times of 1 second causes, of course, much coarses steps although the general shape of the curves is similar.

Figure 8 is a $\sigma_y(\tau)$ vs τ plot of the same pair of oscillators as for Figure 7a using the DMTD system. The plot contains a lot of information: The measurement noise of the dual mixer system is indicated. One can see the short term stability performance of the quartz oscillators for $\tau \leq 1$ s. One can see a little bit of 60 Hz present as indicated by slightly increased values of $\sigma_y(\tau)$ at 1/2 and 3/2 of τ = 1/60 Hz. One observes the attack time of the servo in the cesium electronics perturbing the short term stability of the quartz oscillator and degrading it to the level of the shot noise of the cesium resonance for $10\ \text{s} \leq \tau \leq 10^2$s. The white noise frequency modulation characteristic then becomes the predominant power law causing $\sigma_y(\tau)$ to improve as $\tau^{-1/2}$ until the flicker floor of the quartz crystal oscillator, in this case 6 parts in 10^{13}, becomes the predominant noise source for $\tau \geq 10^2$ s. Thus, using this particular measurement system on this pair of precision frequency standards, their stabilities were well characterized for sample times of a few milliseconds all the way out to 1000 seconds. Longer sample times are of course easily achievable - patience is the main ingredient!

Figure 9 is an example of where the common or transfer oscillator was known to be about 5 times worse in frequency instability than the two oscillators being measured. The common oscillator was a high performance commercial cesium beam frequency standard and the oscillators being measured were primary frequency standards at NBS (NBS-4 and NBS-5). Using the DMTD method and having the phase relationship between the beat frequencies near enough coincident to satisfy Eq. (2), one may deduce that the long term fractional frequency stability of NBS-4 or NBS-5 was better than 1×10^{-14} for $\sigma_y(\tau)$, τ = 1 day.

Conclusions

Using low noise techniques one can measure the time and/or frequency performance characteristics of state-of-the-art frequency standards and clocks. The low level of measurement noise is achieved using Schottky diode mixers with low noise amplifiers following.

It has been shown that in practice when making measurements between a pair of clocks or oscillators, the greatest degree of flexibility is achieved in data analysis if the time-difference can be measured with adequate precision. A symmetric dual mixer time difference system was demonstrated which fulfilled this maximum flexibility. A significant variety of alternatives exist which could fruitfully employ this system. For example, if the common oscillator involved could be synthesized internal to the system, and the time difference device could be also constructed internal to the system, these coupled with a micro-processor and appropriate peripherals could make a very powerful state-of-the-art frequency and time standards data acquisition and analysis system.

Also a common transfer oscillator followed by a suitable distribution amplifier could drive several balanced mixers simultaneously. The beat signals then could be multiplexed into a time interval counter or into a precise dating device and the data thus obtained at regular intervals would be useful in a timekeeping system with several contributing clocks.

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Appendix: Computing Counter Program

The following program may be used in a computing counter, is useful in determining the fractional frequency stability, $\sigma_y(\tau)$, and is unique as compared with other similar types of programs to determine stability in that it does so with no dead time. The following program actually determines the root-mean square second difference, $(\Delta^2\ (\Delta t))$ rms, of the time difference readings between a pair of clocks or oscillators, and therefore complements very nicely the dual mixer time difference measurement system described in the text. The fractional frequency stability may be calculated from computer program results as follows:

$$\sigma_y(\tau) = \frac{1}{\sqrt{2}\,\tau^2\nu}\,(\Delta^2(\Delta t))_{rms} \qquad \text{(A1)}$$

If additional programming steps were available, of course one could program the computing counter to calculate an estimate of $\sigma_y(\tau)$ directly. Following is the program procedure to generate $(\Delta^2(\Delta t))_{rms}$:

1. clear x
2. c ⇄ x
3. Plug-in
4. a ⇄ x
5. Plug-in
6. a ⇄ x
7. $\overrightarrow{a\ x\ y}$
8. - (subtract)
9. b ⇄ x
10. Xfer Program
11. Plug-in
12. a ⇄ x
13. $\overrightarrow{a\ x\ y}$
14. - (subtract)
15. b ⇄ x
16. $\overrightarrow{b\ x\ y}$
17. - (subtract)
18. $\overrightarrow{xy}$
19. x (multiply)
20. $\overrightarrow{c\ x\ y}$
21. + (add)
22. c ⇄ x
23. Repeat
24. Xfer Program
25. c ⇄ x
26. $\overrightarrow{N\ x\ y}$
27. ÷ (divide)
28. $\sqrt{x}$
29. Display x
30. Pause

The confidence of the estimate will improve approximately as the square root of the number of times (N) the sub-loop is repeated as preset by the programmer [11].

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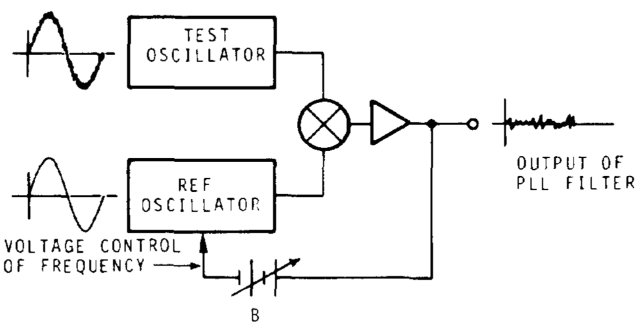

FIGURE 1.

A phase (or time) fluctuation measurement system. The reference oscillator is loosely phase-locked to the test oscillator - attack time is about 1 second. The reference and test oscillators are fed into the two ports of a Schottky barrier diode double balanced mixer whose output is fed through a low pass filter and low noise amplifier, a battery bias box and to the varicap of the reference oscillator. The instantaneous output voltage of the phase locked loop (PLL) following the low noise amplifier will be proportional to the phase or time fluctuations between the two oscillators which is useful for times short compared to the attack time of the servo.

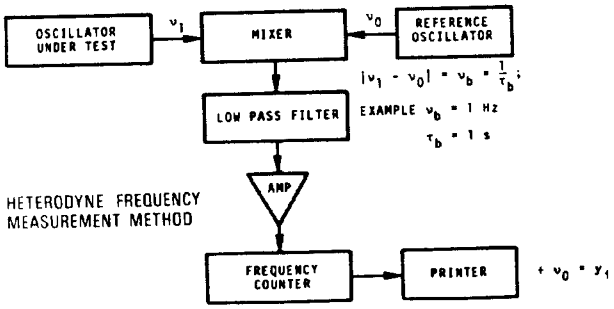

FIGURE 2a.

A frequency and frequency fluctuation measurement system. The difference frequency, $|\nu_1 - \nu_0|$ is measured with a frequency counter. A counter measuring the period (or multiple period) of the beat (difference) frequency could equivalently be used.

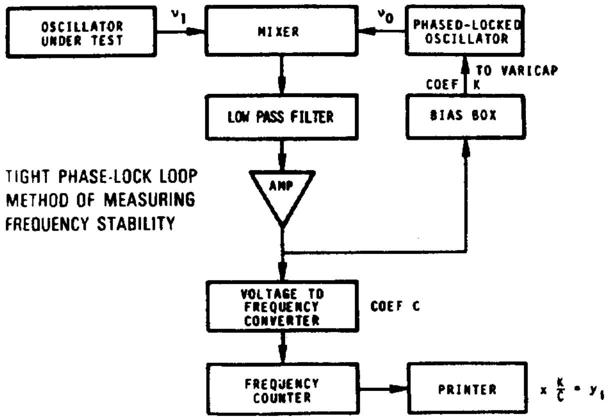

FIGURE 2b.

A frequency fluctuation measurement system. The attack time of the phase lock loop in this case is much less than a second. The amplifier (AMP) output voltage fluctuations for sample times significantly longer than the servo loop attack time will be proportional to the frequency fluctuations which would have existed between the "free running" oscillators.

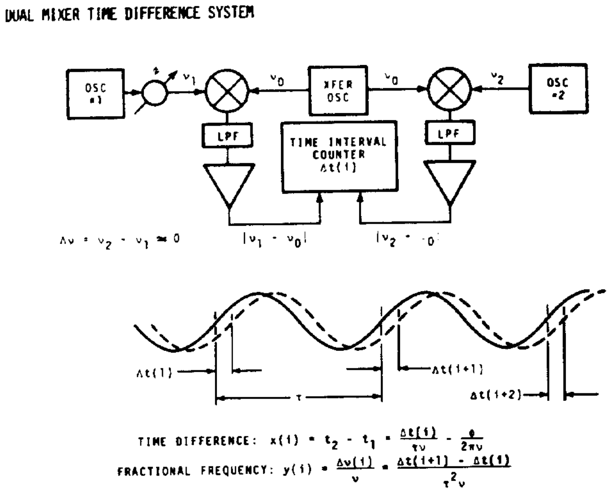

FIGURE 3.

A time difference and time fluctuation measurement system. The low pass filters (LPF) determine the measurement system bandwidth and must pass the difference frequencies which are depicted by the solid-line and dashed-line sinusoids at the bottom of the figure. The positive going zero volts crossing of these difference (beat) frequencies are used to start and stop a time interval counter after suitable low noise amplification. The i[th] time difference between Oscillator 1 and 2 is the $\Delta t(i)$ reading of the counter divided by $\tau\nu$ and plus any phase shift added, ϕ, where $\nu \approx \nu_1 \approx \nu_2$ is the nominal carrier frequency. The frequency difference is straight forwardly calculated from the time difference values.

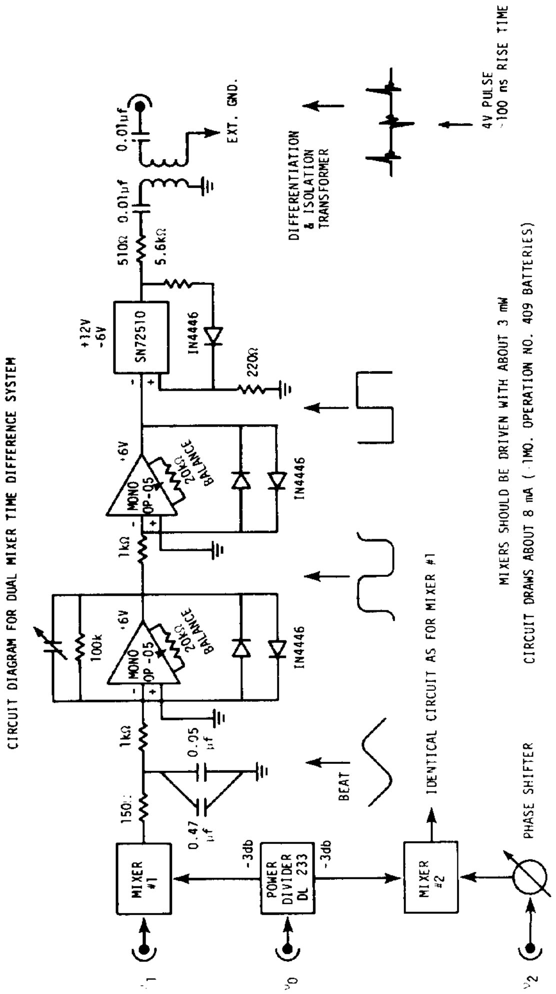

FIGURE 4. An example circuit diagram for a Dual-Mixer-Time-Difference System

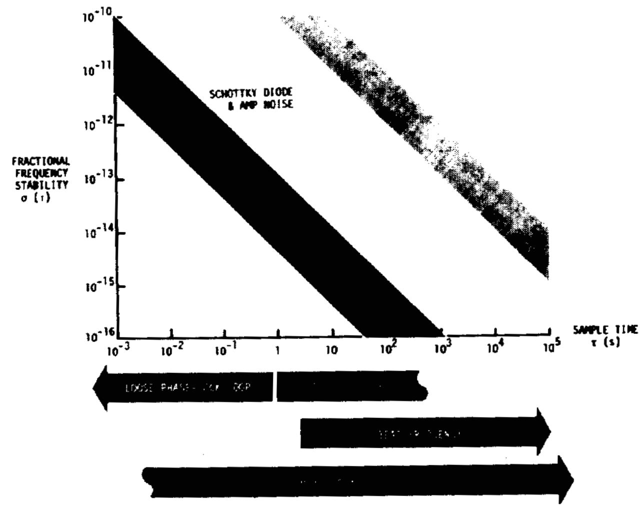

Figure 5. Comparison of the nominal sample time range of some state-of-the-art methods of measuring frequency stability.

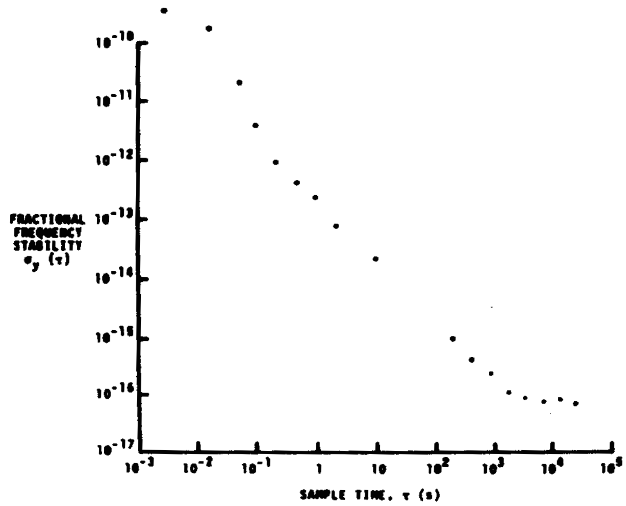

Fig. 6 The fractional frequency stability plot as a function of sample time, τ of the measurement noise of the dual mixer time difference system. The first 3 points are with system bandwidth of 300 Hz, then 2 at 18 Hz, 1 at 8 Hz, 1 at 3 Hz, and the rest of the stability points are at 1.8 Hz. The flatening as the stability reaches a few parts in 10^{17} is believed due to diurnal temperature effects.

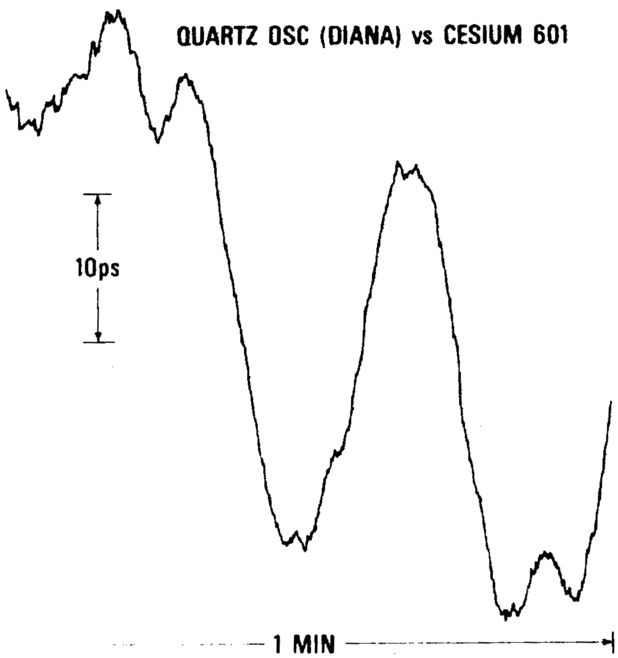

FIGURE 7a.

A copy of a strip chart recording of the time fluctuations versus running time using the dual mixer time difference measurement system. The oscillators involved were a high performance commercial cesium standard and a high quality quartz crystal oscillator. The common oscillator employed was a low noise synthesizer. The measurement system noise was about 0.1 ps.

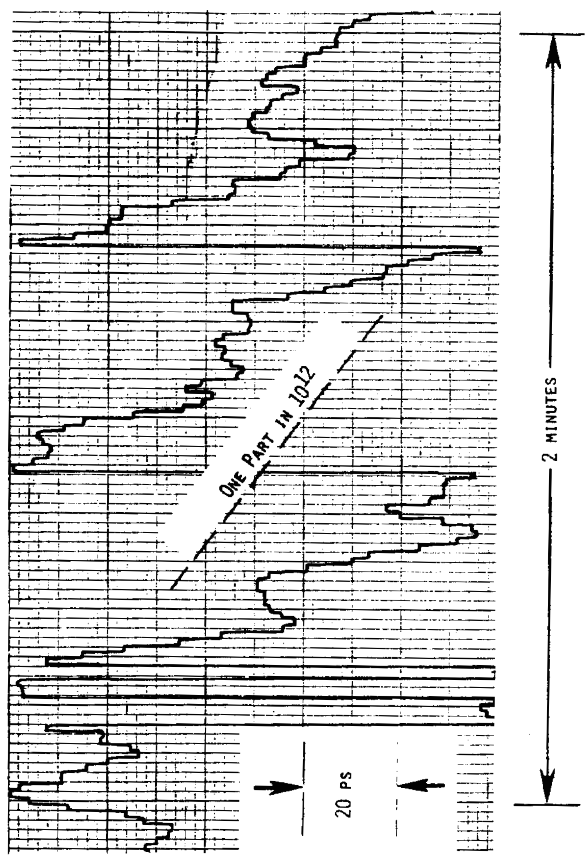

FIGURE 7b. Time Difference between NRC CsV and High Performance Commercial Cesium using NRC Dual Mixer Time Difference System.

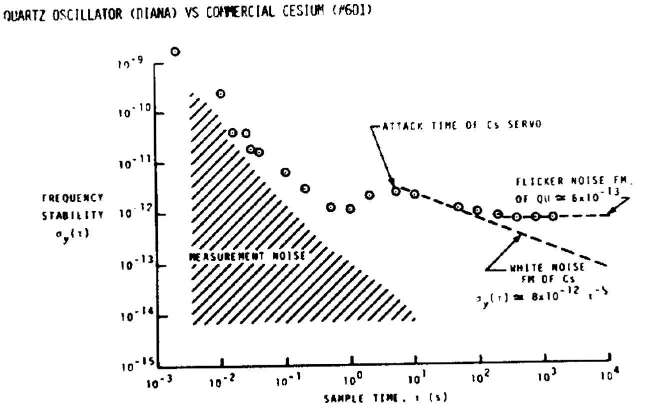

FIGURE 8.

A $\sigma_y(\tau)$ versus τ plot of the fractional frequency fluctuations, $y(t)$ between a high performance commercial cesium beam frequency standard and a commercial quartz crystal oscillator.

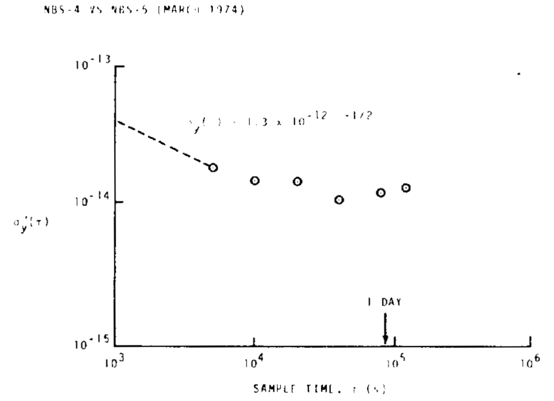

Figure 9. A measurement of the fractional frequency stability $\sigma_y(\tau)$ of the two primary frequency standards NBS-4 & NBS-5 versus sampling time τ using the dual mixer time difference method of measurement. The frequency instability of the common oscillator employed was about a factor of 5 greater than that of either NBS-4 or NBS-5.

Hierarchy Status	Example of Measurement Method	Data deducible from the measurement			
		Time	δ Time	Freq.	δ Freq.
1	Dual Mixer Time Diff.	$x(t) = \frac{\Delta t}{\tau\nu}$	$\delta x(t,\tau) = x(t+\tau) - x(t)$	$y(t,\tau) = \frac{\delta x(t,\tau)}{\tau}$	$\delta y(t,\tau) = y(t+\tau,\tau) - y(t,\tau)$
2	loose phase-locked reference oscillator	can't measure	$\delta x(t,\tau) = \frac{\delta \phi(t,\tau)}{2\pi\nu}$	"	"
3	Heterodyne or beat frequency	can't measure	can't measure	$\lvert y(t,\tau) \rvert = \frac{\nu_{beat}}{\nu_o}$	"
4	tight phase-locked reference oscillator	can't measure	can't measure	can't measure	$\delta y(t,\tau) = c \cdot \delta V$

TABLE 1. Hierarchy of measurement methods: showing that the measurement of time differences between a pair of oscillators (clocks) gives the greatest data processing capability.

$S_y(f) = h_\alpha f^\alpha$ $\alpha =$	$S_y(f) = a\,\sigma_y^2(\tau)$ $a =$	$\sigma_y^2(\tau) = b\,S_\phi(f)$ $b =$
2 (white phase)	$\frac{(2\pi)^2 \tau^2 f^2}{3 f_h}$	$\frac{3 f_h}{(2\pi)^2 \tau^2 \nu_o^2}$
1 (flicker phase)	$\frac{(2\pi)^2 \tau^2 f}{3.81 + 3 \ln(\omega_h \tau)}$	$\frac{[3.81 + 3 \ln(\omega_h \tau)] f}{(2\pi)^2 \tau^2 \nu_o^2}$
0 (white frequency)	2τ	$\frac{f^2}{2 \tau \nu_o^2}$
-1 (flicker frequency)	$\frac{1}{2 \ln(2) \cdot f}$	$\frac{2 \ln(2) \cdot f^3}{\nu_o}$
-2 (random walk frequency)	$\frac{6}{(2\pi)^2 \tau f^2}$	$\frac{(2\pi)^2 \tau f^4}{6 \nu_o^2}$

TABLE 2. Conversion table from time-domain to frequency-domain and from frequency-domain to time-domain for common kinds of integer power law spectral densities; $f_h (= \omega_h / 2\pi)$ is the measurement system bandwidth.

$$S_\phi(f) = \frac{\nu_o^2}{f^2} S_y(f)$$