Frequency Stabilization of X-Band Sources for Use in Frequency Synthesis into the Infrared

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Abstract-An X-band source of excellent frequency stability is needed in infrared frequency multiplication of high order. Such a source has been used in frequency multiplication by a factor of 401 using a point-contact Josephson junction as a frequency multiplier and mixer. Noise data on three X-band systems are reported. Two of these systems use klystrons as the source of X-band power; the other uses a Gunn oscillator. Each of these three systems employs both cavity and injection stabilization. Injection stabilization, using a quartz-oscillator-driven multiplier chain, provides the second-tosecond and minute-to-minute stability needed for the Josephson junction experiment. To our knowledge, this is the first published noise data where cavity and injection stabilization are simultaneously employed. The quality of the best system reported here is much better-both around 1 Hz from the carrier and around 50 kHz from the carrier-than the source used to multiply by a factor of 401 to 3.8 THz.

I. BACKGROUND

EVIDENCE of the excellent frequency stability of the methane-stabilized He–Ne laser has been reported [1], [2]. With this laser stabilized in this way, very accurate wavelength measurements have been made [3], [4]. A frequency measurement of this same laser then permits a much more precise determination of the speed of light than was previously possible [5]. For this and other reasons, such as the possibility of an improved primary frequency standard, there is an increasing interest in the accurate measurement of infrared and visible-light frequencies.¹

The ratio of a laser frequency with respect to that of the cesium frequency standard is always large. For example, the frequency ratio of methane to cesium is nearly 10 000. This fact has necessitated the use of several lasers and microwave sources interposed between the two end points [5]. The practicalities of this procedure have required that all of these intermediate sources have good frequency stability.

The use of a source at X band as one of the intermediate frequencies offers several advantages. 1) Such a source can be conveniently cavity stabilized and/or phase locked to a quartz oscillator. 2) A fair amount of stabilized power

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can be obtained for use in frequency multiplication of high order [6]. 3) If only low-order harmonic generation is being used (say, less than times 20) then only one millimeter-wave klystron need be used to get to the HCN laser—a standard source in this type of experiment [5]. 4) Since the frequency of cesium is also in X band (≈ 9.192 GHz) the comparison with the primary frequency standard is facilitated.

In an attempt to reduce the number of intermediate sources needed, a point-contact Josephson junction has been used as a frequency multiplier and mixer. A major aim of this effort is multiplication in one step from X band to the 28- μ m line of the H₂O laser [7], a multiplication of approximately 1100. In frequency multiplication experiments with the Josephson junction beyond the 200th order, it became apparent that random FM of the X-band source was significantly deteriorating the observed beat between the laser and the high-order X-band harmonic [8]. The source used in the experiment of [8] was an X-band reflex klystron phase locked to a quartz crystal oscillator. Further measurements have shown that this source has a broad and intense peak in FM noise located about 50 kHz from the carrier.

The results referred to in [8] made it clear that higher order harmonic generation would, at least, require an X-band source with greater frequency stability. To this end, considerable effort has been expended, and it is the results of this effort which we report here.

II. NOISE MEASUREMENT METHODS

Before discussing the noise data taken on several systems, it is necessary to describe the methods used to obtain this data. First of all, it is important to realize that it is random FM of the carrier which is the practical problem. For any X-band source whose stability is good enough for use in the Josephson junction work, the random AM of the carrier causes negligible sidebands compared to those caused by FM noise. This is true at least for modulation frequency values f in the range 2.5 Hz $\leq f \leq 100$ kHz. Our major effort has, therefore, been concentrated upon FM noise measurements.

Measurements have been made in both the frequency and time domains. The frequency-domain technique which we have used most extensively is that described by Ashley *et al.* [9] and by Ondria [10]. We refer to this method as the cavity discriminator method [13]. This method works quite well for f values from at least as small as 1 kHz to at least as large as 100 kHz. The upper frequency limit

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¹ At the 1972 Conference on Precision Electromagnetic Measurements, Boulder, Colo., it was indicated that, at least, the National Bureau of Standards, National Research Council, National Physical Laboratory, and Massachusetts Institute of Technology were actively working in this area.

is, in part, determined by the quality factor Q of the discriminator cavity in the measurement apparatus. (The discriminator cavity we used has an unloaded Q of about 20 700.) The frequency-domain data will be given in terms of $S_{\delta\nu}$, the spectral density of the frequency fluctuations [11]. The quantity $S_{\delta\nu}$, which, in general, is a function of the modulation frequency f, is given by $S_{\delta\nu} = (\delta\nu_{\rm rms})^2/B$ where $\delta\nu_{\rm rms}$ is the root-mean-square frequency deviation (caused by noise centered at some specified frequency f) as measured in a bandwidth B. The data are given in decibels with respect to 1 Hz²/Hz.

To get data corresponding to f values of 1 Hz and smaller, it is necessary to make measurements in the time domain. This was done with a high resolution frequency counter which is interfaced with a small, programmable computer. The quantity measured is $\sigma_y(2,T,\tau,f_h)$, the square of which is known as an Allan variance [11]-[13]. Here, the number 2 specifies the number of data points used in obtaining a sample variance. The total time, including dead time of the counter, to obtain one frequency measurement is T; and τ is the corresponding time over which the frequency measurement is averaged. The highfrequency cutoff of the measuring system is given by f_h .

Frequency stability measurements using both of these methods offer two advantages. First, a larger range of spectrum (in f or in τ , depending on your point of view) can be covered. Work with the Josephson junction has shown that it is necessary to consider this larger range. Second, since the ranges of the two methods have a region of overlap, there is a possibility for an accuracy check. This check is made by converting the frequency-domain data into equivalent time-domain data [11], [13]. In order to get additional information for the range 1 Hz \leq $f \leq 10^3$ Hz, we have used yet a third method. This is a frequency-domain measurement which we call the comparison oscillator method [13]. This measurement is based on the spectral analysis of the correction voltage which phase locks one source to the other. The comparison oscillator method overlaps with both the cavity discriminator method and the time-domain method. Both the comparison oscillator and time-domain methods have, in principle, the advantage that they are insensitive to amplitude noise which may also be present on the source under test. The comparison oscillator method has the additional advantage that it can more easily be extended to larger values of f (i.e., to f values above 1 kHz).

III. SIGNAL SOURCES

We are reporting here, primarily, the measurements made upon three sources. Two of the sources are reflex klystrons and the third is a transferred electron oscillator (Gunn diode). Each of them is stabilized by an external cavity. Much of our data are taken with the source additionally stabilized by injection locking to a system driven by a quartz-crystal oscillator.

In the earliest work with the Josephson junction it was found that, even for multiplication factors as low as 84, it was necessary to electronically lock a klystron to a quartz oscillator to get sufficient second-to-second carrier stability. But, as previously mentioned, this source had an unacceptably large FM noise peak at about 50 kHz.

Based on some earlier work on cavity stabilization and injection locking [14], [15], a klystron system simultaneously employing *both* methods of stabilization was assembled. To our knowledge, [6] was the first reported use of such a system, and these are the first reported noise data on such a system.

The klystron which was used in [6] has a reflector voltage-modulation sensitivity of about 3 MHz/V and an unstabilized output power of about 120 mW. This klystron is stabilized by a TE₀₁₁ right-circular cylindrical cavity whose unloaded Q is about 20 000. The cavity has an insertion loss of about 6 dB and is coupled as described in [14]. This klystron coupled to this cavity (it was only coupled to the TE₀₁₁ cavity) will henceforth be called the low-power klystron (LPK) system.

The other klystron has a reflector voltage-modulation sensitivity of 400 kHz/V and an unstabilized output power of 1.4 W. This klystron is stabilized by a TE₀₁₅ cavity whose unloaded Q is 50 000. The cavity has an insertion loss of about 11 dB and is coupled as described in [14]. This klystron coupled to this cavity (it was only coupled to the TE₀₁₅ cavity) will henceforth be called the high-power klystron (HPK) system.

The Gunn oscillator has a voltage-modulation sensitivity of 170 kHz/V and an output power of 50 mW. For the Gunn oscillator, FM noise data were taken both free running and with the oscillator stabilized by the TE_{015} cavity.

Each of the three preceding systems has additionally been stabilized by the injection-locking technique [15]. That is, the source was simultaneously cavity stabilized and injection locked. Two signals were used for injection locking. The first, which was used only with the LPK, consisted of a reflex klystron stabilized by a quartzcrystal-driven commercial oscillator synchronizer. This "lock box" is designed to operate with a crystal whose frequency is about 100 MHz. This injection signal came, in fact, from the source used for the original frequency synthesis work with the Josephson junction $\lceil 16 \rceil$. The noise data for this signal are given in Figs. 1 and 2. Hereafter we refer to this injection-locking signal as IS1. The second injection signal, which was used for both the HPK and the Gunn, is obtained from a quartz-crystaldriven multiplier chain. Hereafter we refer to this injectionlocking signal as IS2. The quartz crystal oscillator used in IS2 is a very high quality commercial model and oscillates at 5 MHz. The multiplier chain, which is driven by this quartz oscillator, was designed and developed for use with the cesium primary frequency standard at the National Bureau of Standards, Boulder, Colo. This multiplier chain has been extensively tested for FM noise in the range 2.5 Hz $\leq f \leq 5$ kHz [17]. It has been shown that it is the quartz oscillator, not the multiplier chain, that makes the major contribution to the FM noise of the

X-band output over this range of f values. The data for this system are given in Figs. 5-7.

IV. RESULTS

In initiating the search for a better X-band source for the Josephson junction work, the requirements were only vaguely definable. They were that the fractional frequency stability from second-to-second should be better than 10^{-8} , and that S_{δ_7} should be considerably less than ± 17 dB for f values of the order of 50 kHz. Because of the difficulty in controlling various parameters of the Josephson junction experiment, the maximum tolerable noise at the higher frequencies is still not known. It is known, however, that the FM noise behavior of the LPK, with its injection locking, is acceptable for a frequency multiplication factor as high as 401 [6]. At f = 50 kHz, S_{δ_7} of that system is about -10 dB.

The latest system, the HPK, without injection locking, is more stable for τ between 0.01 and 1 s and for f greater than 10³ Hz than is the LPK with its injection locking. Nevertheless, the requirements of the Josephson junction work for further improvements in long-term stability $[\sigma_y(2, T = \tau, \tau = 1 \text{ min}) \leq 10^{-10}]$ continue to call for quartz oscillator stabilization.

The use of injection locking by a quartz-driven source to improve the long-term stability involves a sacrifice. In our experience, there has always been a deterioration of the stability for f values greater than about 1 kHz when injection locking is being used. That is, each klystron stabilized by its cavity and the Gunn oscillator stabilized by its cavity has a smaller $S_{\delta r}$, for $f \geq 1$ kHz, than does the complete system.

The use of cavity stabilization has two virtues. First, it reduces $S_{\delta \nu}$ and σ_{ν} over the entire range which we have examined. This can be seen, for example, in comparing the free-running and cavity-stabilized curves of both Figs. 3 and 4. Second, because it does "prestabilize" the source, less injection power is needed for a constant duration of the injection-locking time. This second point means that the injection-locking bandwidth can be narrower if a cavity is used than if it is not [18]. Thus the degradation of $S_{\delta \nu}$ at high values of f is less severe.

Another important point is the place where the injection signal is inserted. With both of the klystrons and the Gunn oscillator, the signal is inserted in that short length of waveguide between the cavity and the source. Inserting the signal at the output iris of the cavity causes a much larger degradation of $S_{\delta r}$ for $f \geq 1$ kHz.

For practical operation of the Josephson junction experiment it is necessary that the injection lock held for of the order of 5 min or longer before the system needs to be relocked. In the system discussed here, this requires a power ratio $R_P \equiv P_{\text{source}}/P_{\text{inj}} \approx 50$ to 60 dB. (P_{source} is the power output of the source without cavity stabilization and P_{inj} is the power injected in the space between the cavity and the source.) This ratio determines the amount of degradation of the high-frequency stability that occurs.



Fig. 1. Time-domain data $\sigma_y(2,T = \tau,\tau,f_h)$, for injection-locking source, IS1 and LPK. See Section II for discussion of quantity $\sigma_y(2,T = \tau,\tau,f_h)$ and of method by which it was measured.

Fig. 1 displays the time-domain data for the LPK (no injection locking) and for IS1. The locking bandwidth when LPK is locked to IS1 ($R_P \approx 56$ dB) is roughly 2 kHz. Therefore, for f values much below 500 Hz, the stability of the composite system should be that of IS1. The better stability of IS1 for 0.01 s $\leq \tau \leq 10$ s (in the frequency domain this is approximately from 100 Hz to 0.1 Hz) is obvious from the figure. Although frequency-domain data were taken with LPK locked to IS1, no time-domain data were taken for this system.

Fig. 2 shows the cavity discriminator data for the LPK, the IS1, and for LPK locked to IS1 (again, with $R_P \approx 56$ dB). The intense noise peak in IS1 and its effect on the high-frequency noise of the composite system is obvious.

Fig. 3 shows $\sigma_y(2,T = \tau,\tau,f_h)$ for the free-running Gunn; the Gunn stabilized by the cavity; and the Gunn with both forms of stabilization. The tremendous improvement in σ_y , with injection locking, for $\tau > 0.1$ s is apparent. The injection locking is effectively suppressing the excessive 1/f noise in the Gunn diode.

Fig. 4 shows $S_{\delta \nu}$ (by the cavity discriminator method) for the Gunn in each of all three conditions. The degrading effect of IS2 on the noise for 1 kHz $\leq f \leq$ 100 kHz is apparent. Data taken, in the free-running condition, by the comparison oscillator method is also shown.

Fig. 5 gives $\sigma_y(2,T = \tau,\tau,f_h)$ for HPK, IS2, and for HPK injection locked to IS2. The same beneficial effect of injection locking here, as with the Gunn, is to be noted. The data for this figure were obtained by analyzing the



Fig. 2. S_{sr} data, by cavity discriminator technique, for LPK, IS1, and LPK-IS1. See Section II for discussion of quantity S_{sr} and of techniques by which it was measured.



Fig. 3. $\sigma_y(2,T = \tau,\tau,f_h)$ for Gunn oscillator under several conditions: free-running, stabilized by the TE₀₁₅ cavity, cavity-stabilized, and injection-locked to IS2.

beat (whose frequency was about 461 MHz) between each of the three sources (HPK, IS2, and HPK-IS2) and another source which is known to have been less noisy, in this range of τ , than any of those three. (Although 461 MHz is higher than the nominal f_h value (\approx 320 MHz), the signal level was still large enough to be usable.)

Fig. 6 gives $S_{\delta r}$ for HPK, IS2, and HPK locked to IS2. The degrading effect of IS2 on the noise for 640 Hz $\leq f \leq 100$ kHz is again obvious.

Of the three systems: LPK-IS1; Gunn-IS2; and HPK-IS2; the HPK-IS2 has the best noise performance and the largest output power. Therefore, in the frequency synthesis experiments planned in the near future with the Josephson junction, this source will be used. Because of this we have gone to further effort to determine its FM noise in the region $f < 10^3$ Hz. To do this we have made $S_{\delta r}$ measurements over this range by the comparison oscillator method.

Fig. 7 shows data taken on IS2 by the comparison oscillator method. The noise of the second source (a second multiplier chain and quartz oscillator) used in this measurement is known relative to that of IS2, and, thus, it was possible to determine the noise of IS2 alone.

In Fig. 8, we show the equivalent time-domain values based on Fig. 7 and on some data taken by the comparison oscillator method for a *carrier* frequency of 5 MHz [19]. One of the two oscillators in the 5-MHz experiment was that used to drive the multiplier chain of IS2. For comparison, the actual time-domain data for IS2 is repeated. The disagreement is discussed below.

Fig. 9 compares the data of Fig. 7 with that of the IS2 curve of Fig. 6. Unfortunately, the second of the two sources used in the comparison oscillator measurements



Fig. 4. $S_{b\nu}$ for Gunn oscillator under several conditions: free-running, cavity-stabilized, cavity-stabilized, and injection-locked to IS2. These three curves were obtained by cavity discriminator method. Fourth curve is free-running Gunn as measured by comparison oscillator method.

was so much more noisy than IS2, for $f > 10^3$ Hz, that we cannot compare the cavity discriminator and comparison methods for $f > 10^3$ Hz.

The agreement between the two methods at 640 Hz and at 1 kHz is quite good. For lower values of f the results diverge. Our primary experience in measuring FM noise at X band has been with the cavity discriminator method: and we have known for some time that the method can give erratic results for f values much below 500 Hz. For this particular data we have not determined the cause of the erratic behavior. In previous work, however, we have identified two sources of trouble (at low values of f) with the cavity discriminator method. A parametric or 1/fnoise in the detector diodes causes the threshold to increase at lower modulation frequencies. Even more important, the acoustic vibration of the discriminator cavity and other waveguide components is not accounted for by the threshold measurement. For f values above 500 Hz, however, we have a rather general confidence in the cavity discriminator method. This is based on the agreement between the two methods (Fig. 9 and the free-running data of Fig. 4); on the agreement between actual timedomain data and equivalent time-domain data calculated from cavity discriminator data at 1 kHz; and on the very general internal consistency of our cavity discriminator data for f > 500 Hz.

Our measurements below f = 500 Hz are on less firm ground. The σ_y data and the comparison oscillator $S_{\delta\nu}$ data, on IS2, agree (qualitatively) on the improvement in stability for f < 100 Hz. This can be seen by comparing Figs. 5 and 7. But Fig. 8 shows that the two methods are in increasing quantitative disagreement below f = 100 Hz.



Fig. 5. $\sigma_y(2,T = \tau,\tau,f_h)$ for HPK, IS2, and HPK-IS2.



Fig. 6. So for HPK, IS2, and HPK-IS2, by cavity discriminator method.



Fig. 7. $\hat{S_{br}}$ data taken by comparison oscillator method on IS2 system. See Section III for discussion of this system.

Because of our extensive experience with time-domain measurements on quartz-crystal oscillators, we tend to believe our time-domain measurements. Our experience with the comparison oscillator method, at X band, is very limited and we are inclined to discount those results when they disagree with the time-domain method.

Using data from Figs. 5–7 we have estimated $S_{\delta\nu}$ for HPK-IS2 over the range 1 Hz $\leq f \leq 10^5$ Hz. The result is given in Fig. 10. The rationale behind this figure is as follows: $S_{\delta\nu}$ in the range 640 Hz $\leq f \leq 10^5$ Hz is directly

measurable by the cavity discriminator method. For the points at 1, 10, and 100 Hz, $S_{\delta\nu}$ is calculated from the $\sigma_y(2,T = \tau,\tau,f_h)$ data. The intermediate points in this range are estimated by the need to connect these points and by the fact that the σ_y data exhibits essentially no dependence on the bandwidth of the filter that was used between the mixer and the counter. Since the region around $f = 10^2$ Hz is obviously a maximum in $S_{\delta\nu}$, we assumed a region going as f^0 (white frequency noise), which would blend in with σ_y -based data on the low-fre-



Fig. 8. $\sigma_y(2,T = \tau,\tau,f_h)$ information for IS2. Crosses are timedomain values calculated from data of Fig. 7 and some data taken by comparison oscillator method for carrier frequency of 5 MHz. The points enclosed by squares are repeat of $\sigma_y(2,T = \tau,\tau,f_h)$ data for IS2 from Fig. 5.



Fig. 9. $S_{\delta r}$ information on IS2 obtained by cavity discriminator and comparison oscillator methods in range 10 Hz $\leq f \leq 1$ kHz. See text for further information.



Fig. 10. Estimate of St for HPK-IS2 based on portions of data from Figs. 5-7.

quency side and a straight-line extrapolation of the cavity discriminator data on the high-frequency side. We think that this estimate is good to $\pm 5 \text{ dB}$ below f = 640 Hz and to ± 3 dB above that value.

Using Fig. 10 and the fact that σ_y continues to improve slightly from $\tau = 1$ s to $\tau = 10^2$ s, we can estimate a quantity which is very important for frequency synthesis from X band into the infrared whether the frequency multiplier and mixer be the Josephson junction or not. This quantity is the linewidth of the X-band signal after frequency multiplication. Assuming a noiseless multiplication by a factor of 1112 we get a value of 6800 Hz as an estimated linewidth $\lceil 21 \rceil$. In this case, a nearly equal value can be obtained by a method described by Halford $\lceil 22 \rceil$.

V. CONCLUSIONS

An X-band source has been developed whose minute-tominute frequency stability is more than adequate for the Josephson junction infrared frequency multiplication experiment of times 1112. The adequacy of the stability in the range of 1 Hz < f < 100 kHz will actually be tested in a frequency multiplication experiment using the Josephson junction.

A knowledge of frequency stability of X-band sources in the range 1 Hz $< f < 10^3$ Hz is important for work in atomic frequency standards as well as for frequency synthesis into the infrared. With this need, further work to improve the measurement of frequency stability in this region seems appropriate.

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