

Measurement of the Short-Term Stability of Quartz Crystal Resonators and the Implications for Crystal Oscillator Design and Applications

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Abstract—A new technique is presented which makes it possible to measure the inherent short-term stability of quartz crystal resonators in a passive circuit. Comparisons with stability measurements made on crystal controlled oscillators indicate that noise in the electronics of the oscillators very seriously degrades the inherent stability of the quartz resonators for times less than 1 s. A simple model appears to describe the noise mechanism in crystal controlled oscillators and points the way to design changes which should improve their short-term stability by two orders of magnitude. Calculations are outlined which show that with this improved short-term stability it should be feasible to multiply a crystal controlled source to 1 THz and obtain a linewidth of less than 1 Hz. In many cases, this improved short-term stability should also permit a factor of 100 reduction in the length of time necessary to achieve a given level of accuracy in frequency measurements.

INTRODUCTION

CRYSTAL controlled oscillators play a key role in frequency metrology in that they are used in nearly all precision frequency measurement and generation devices. Measurements of the inherent frequency stability of the quartz crystal resonators used in the frequency control of oscillators are therefore of particular interest.

The technique described here permits one to measure the inherent frequency stability of quartz crystal resonators in a passive circuit without the noise usually associated with an active oscillator. Comparisons with stability measurements made on crystal controlled oscillators indicate that noise in the electronics of the active oscillators very seriously degrades the inherent stability of the quartz resonators for times less than 1 s. Hence, in addition to making possible the evaluation and modeling of the frequency stability of crystal resonators and the dependence on various manufacturing processes, these measurement techniques also make it possible to evaluate directly the degradation of the frequency stability due to different oscillator circuits. A simple model is developed which appears to explain this degradation in stability and which also points the way to design changes in crystal oscillators which should greatly improve their short-term stability.

Calculations indicate that with this improved short-term stability, it should be feasible to frequency multiply

PASSIVE CRYSTAL MEASUREMENT SYSTEM

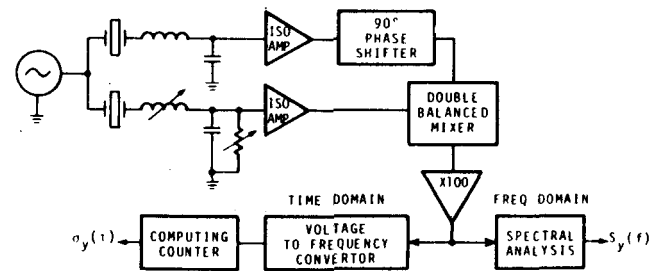


Fig. 1. Passive crystal measurement system.

a crystal controlled source to 1 THz and obtain a linewidth of less than 1 Hz. This would make possible a much more precise transfer of frequency information and stability between the infrared and RF regions of the spectrum than is possible now. In many cases, this improved short-term stability would also permit a substantial reduction in the length of time necessary to reach a given level of accuracy in frequency measurements.

MEASUREMENT TECHNIQUES

The frequency stability of the crystal resonators has been measured in the system schematically shown in Fig. 1. Two crystals which are as identical as possible are driven from the same low-noise source. By careful adjustment of crystal tuning and the balancing of the relative Q 's, the output from the mixer can be made first order insensitive to both residual amplitude and frequency modulation in the source. This reduction of noise from the source makes it possible to measure small time varying frequency deviations of the crystals. Special care must be taken to operate the mixer at the ac quadrature point, which is usually slightly different from the dc zero point, in order to realize the full potential of this method. The high sensitivity of this system for detecting frequency changes of the crystals can be found from analyzing the equivalent circuit shown in Fig. 2.

The complex amplitude of the voltage appearing at the mixer input from either crystal, labeled V_{out} , is propor-

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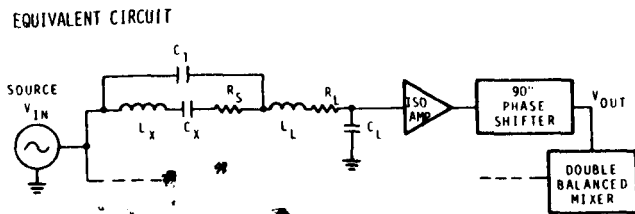


Fig. 2. Equivalent circuit.

tional to the following frequency dependent terms

$$\frac{V_{out}}{V_{in}} = \frac{1}{\omega_0 C_L (R_S + R_X)} \left(1 - i \frac{\Delta\omega}{\gamma_X} \frac{R_S}{R_S + R_L} \right) \approx -\frac{iK}{\omega_0} \left(1 - i \frac{\Delta\omega}{\gamma_X} \frac{R_S}{R_S + R_X} \right) \quad (1)$$

where it is assumed that

$$\frac{1}{L_X C_X} = \omega_0^2, \quad \frac{1}{L_L C_L} \doteq \omega_0^2$$

$$\omega_{source} = \omega_0 + \Delta\omega$$

$$\frac{\Delta\omega}{\gamma_X} \ll 1, \quad \text{where } \frac{\omega_0}{2\gamma_X} = \text{unloaded } Q \text{ of the crystal}$$

$$\gamma_L \gg \gamma_X, \quad \frac{\omega_0}{2\gamma_L} = \text{unloaded } Q \text{ of the } L_L, C_L, R_L \text{ circuit}$$

$$\frac{1}{\omega_0 C_1} > R_S > R_L. \quad (2)$$

One notes that there is a phase shift across the circuit which is proportional to the frequency difference between the source and the natural resonant frequency of the crystal. This phase shift under the preceding assumptions is just

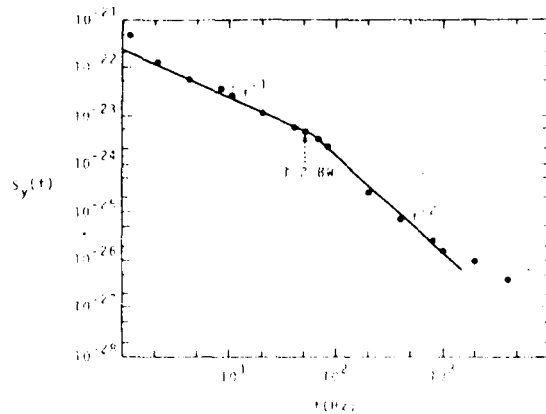
$$\Delta\theta = \frac{R_S}{R_S + R_L} \frac{\Delta\omega}{\gamma_X} \quad (3)$$

note that

$$\frac{R_S + R_L}{R_S} \gamma_X \equiv \gamma$$

is $\frac{1}{2}$ the loaded angular bandwidth which in most cases is very close to γ_X . Identical results are of course obtained for the other crystal. The 90° phase shifter was made from a variable length delay line. The double balance mixer is used as a phase detector and produces a dc voltage which is proportional to the instantaneous phase difference from 90° between the two signals. The mixer sensitivity was typically 3 mV per degree phase difference from 90° which, when used with (3) yields a mixer output (in volts) of

$$V = \frac{\Delta\omega}{\gamma} \frac{360}{2\pi} 0.003 \text{ V} \quad (4)$$

Fig. 3. Frequency domain data for a 10-MHz crystal pair with a bandwidth of 100 Hz, 10A-100, at 28°C .

for $\gamma/2\pi = 1$ Hz, the mixer output is, therefore, 170 mV per Hz frequency change in the natural resonant frequency of either quartz resonator, assuming they both have a bandwidth of 2 Hz.

The signal from the mixer is then processed for either frequency or time domain data (see Fig. 1). Spectral analysis of the mixer output yields $[S_v(f)]^{1/2}$, $S_v(f)$ being the spectral density of frequency fluctuations.

The time domain data is taken using a voltage to frequency converter and a computing counter. The average frequency measured by the computing counter during a sample time τ has a component which is proportional to the average frequency deviation between the two crystals during the sample time τ . By taking many consecutive measurements at each sample time τ , one can compute the Allan variance $\sigma_v^2(\tau)$ in the usual way [1].

The actual time domain data was taken using a second difference method which removed linear frequency drifts of the crystals. Since the object of this work was to investigate the short-term stability of crystal resonators and crystal controlled oscillators, this approach greatly reduced the requirements on the temperature stability of the enclosures without any sacrifice in data quality.

EXPERIMENTAL RESULTS

Fig. 3 shows an example of frequency domain data for a 10-MHz crystal pair. $S_v(f)$ is plotted versus Fourier frequency offset f , assuming equal contribution from each crystal. Note that the noise has a flicker of frequency character, i.e., $S_v(f) = h_{-1}f^{-1}$ for frequencies less than half-bandwidth of the crystals (50 Hz). For frequencies larger than the half-bandwidth, $S_v(f) = h_{-2}f^{-2}$ which is characteristic of a random walk frequency modulation process (the h_m 's are constants). Crystal drive was approximately 200 to 250 μW .

Fig. 4 shows the time domain data for this same 10-MHz crystal pair and also the frequency domain data converted to time domain. Conversion between frequency and time domain can be easily done whenever $S_v(f)$ follows a simple power law dependence of f over an extended range [1] as in Fig. 3. The time domain stability for the

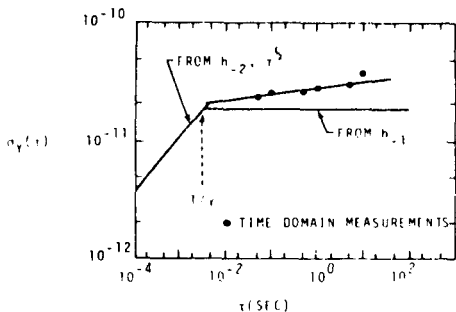


Fig. 4. Time domain data for crystal pair 10A-100 at 28°C. Also shown are the frequency domain data of Fig. 3 converted to time domain.

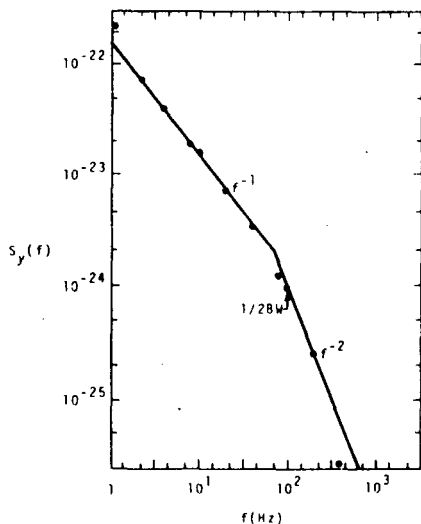


Fig. 5. Frequency domain data for a 5-MHz crystal pair with a bandwidth of 200 Hz, 5A-200, at 28°C.

flicker of frequency region is

$$\sigma_y(\tau) = (2 \ln 2h_{-1})^{1/2} \quad (5)$$

while for the random walk frequency modulation region

$$\sigma_y(\tau) = \left(\frac{4\pi^2\tau}{6} h_{-2} \right)^{1/2} \quad (6)$$

The most surprising result is the indication, from the frequency domain data, that the time domain stability improves as one goes to measurement times which are shorter than the inverse half angular bandwidth. The functional dependence on measurement time in this region is $\sigma_y(\tau) \sim \tau^{1/2}$.

Frequency domain measurements on a low-Q 5-MHz crystal pair are shown in Fig. 5. As in the case of the 10-MHz pair, $S_y(f)$ goes as $h_{-1}f^{-1}$ out to approximately the half-bandwidth point (100 Hz) and then goes as $h_{-2}f^{-2}$. Again, the frequency domain data indicate that $\sigma_y(\tau)$ goes as $\tau^{1/2}$ for $\tau < 1/\gamma$. Direct confirmation of this requires measurements of $\sigma_y(\tau)$ at sample times of order 10^{-4} s which could not be made with this apparatus due to dead time problems [1]. The solution to this problem is to examine the data for a high-Q crystal pair where $1/\gamma$ is of order 1 s.

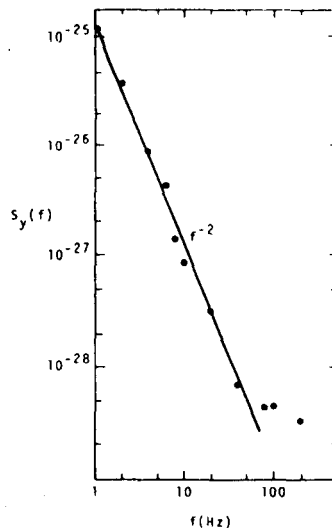


Fig. 6. Frequency domain data for a 5-MHz crystal pair with a bandwidth of 2 Hz, 5B-2, at 28°C.

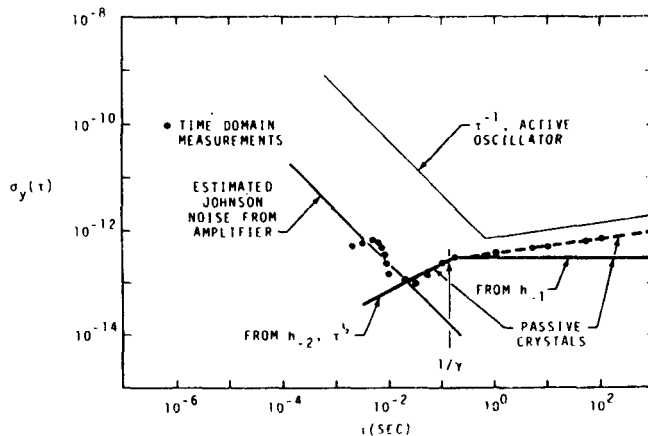


Fig. 7. Time domain data for crystal pair 5B-2 along with the frequency domain data of Fig. 6 transferred to time domain. Also shown are the time domain data for a high quality oscillator controlled by one 5B-2 crystal at 65°C.

Fig. 6 shows the frequency domain data from a 5-MHz crystal pair with a bandwidth of 2 Hz. Transition from the f^{-1} to f^{-2} behavior should occur at 1 Hz which is the lower limit of the spectrum analyzer used. Note that $S_y(f)$ goes as f^{-2} out to at least 50 Hz so that one should expect $\sigma_y(\tau)$ to go as the square root of τ from at least time of order $(2\pi 50)^{-1}$ or 3 ms or smaller. Fig. 7 shows the time domain data for this crystal pair along with the frequency domain data transferred to time domain.

The increase in stability for times less than the inverse bandwidth is confirmed by these measurements. As one goes to shorter and shorter times the stability becomes worse again due to noise in the isolation amplifier and the measurement system. This noise causes an uncertainty in the measurement of the position of the zero crossing. If the noise is white then the frequency fluctuations will have a white phase modulation character and $\sigma_y(\tau)$ will go as τ^{-1} for times larger than the inverse bandwidth of the measurement system.

The influence of the additive Johnson noise in the electronics on the measured stability can be understood using the following simplified model referred to Fig. 2. Multiplicative phase modulation in the isolation amplifiers has been reduced by the use of local negative feedback as originally suggested by [2]. Assume that the source voltage is $A_0 \cos \omega_0 t$ and that there is a white noise voltage $a(t)$ with effective bandwidth b referenced to the isolation amplifier input. Then the effective amplifier input is approximately given by

$$v(t) \simeq \frac{1}{\omega_0 C_L (R_S + R_L)} A_0 \cos \omega_0 t + a_c(t) \cos \omega_0 t + a_s(t) \sin \omega_0 t \quad (7)$$

where $a_c(t) \cos \omega_0 t$ and $a_s(t) \sin \omega_0 t$ are the in-phase and quadrature components of the noise, respectively, and have zero-mean value and bandwidth $b/2$. The rms values of $a_c(t)$ and $a_s(t)$ equal the rms value of $a(t)$ and there is no correlation between them, i.e.,

$$\overline{a_c^2(t)} = \overline{a_s^2(t)} = \overline{a^2(t)} \quad (8)$$

$$\overline{a_s(t)a_c(t)} = 0. \quad (9)$$

The average slope at zero crossing is

$$A' \omega_0 \equiv \omega_0 A_0 \frac{1}{\omega_0 C_L (R_S + R_L)}. \quad (10)$$

The quadrature component of the noise leads to an rms uncertainty in the zero crossing of approximately

$$\Delta t_{\text{rms}} \simeq \frac{a_s(t)_{\text{rms}}}{A' \omega_0} = \frac{a(t)_{\text{rms}}}{A' \omega_0} \quad (11)$$

for times greater than $1/2b$ since it takes about that much time for $a_s(t)$ to assume a new value. Therefore,

$$\sigma_v(\tau) \simeq \frac{\Delta t}{t} = \frac{a(t)_{\text{rms}}}{A' \omega_0 \tau}, \quad \text{for } \tau > \frac{1}{2b}. \quad (12)$$

For times shorter than $1/2b$ it can be shown [3], [4]

$$\sigma_v(\tau) = \frac{a(t)_{\text{rms}}}{A' \omega_0} \left(\frac{b}{\tau} \right)^{1/2}, \quad \tau < \frac{1}{2b}. \quad (13)$$

The line labeled "Johnson noise from amplifier" indicates the estimated contribution of our measurement system to $\sigma_v(\tau)$. Calculations of the Johnson noise in the series loss resistance of the crystal R_s show that this contribution to $\sigma_v(\tau)$ is very small compared to that of most active circuits.

For comparison, the stability of these same two crystals in a high performance crystal oscillator at 65°C is also indicated in Fig. 7. Note the dramatic difference in stability for times less than 1 s. This difference is just the noise contribution due to the electronics in the crystal controlled oscillators. The difference for measurement times greater than 1 s is primarily due to the decrease in the stability of this crystal pair at 65°C. Fig. 7 vividly illustrates the power of this technique for evaluating crystal controlled oscillator circuits.

The character of the oscillator stability for times less than 1 s is very similar to that of the crystal measurements below 10 ms only the level is higher. Using (12) and assuming a crystal drive of order $1 \mu\text{W}$, $R_s = 100 \Omega$, $a(t)_{\text{rms}} = (2 \text{ nV/Hz}) (4 \times 10^4 \text{ Hz})^{1/2} = 400 \text{ nV}$, yields $\sigma_v(\tau) = 10^{-12} \tau^{-1}$ as observed.

This suggests (in accordance with widely held beliefs—see, e.g., [4], [5]) that Johnson noise sources in the amplifier and oscillator stages that are not filtered by the crystal are the cause of the observed noise in this and most low drive crystal oscillators. Short-term stability could be greatly improved merely by increasing crystal drive. A factor of 100 increase in crystal drive should produce a factor of 10 improvement in the short-term stability of the oscillator. Recent measurements by J. Grosslambert, G. Marianneau, M. Oliver, and J. Uebersfeld on a crystal controlled oscillator with $50 \mu\text{W}$ of crystal drive, yield a factor of 10 improvement in the short-term stability over results for the oscillator with approximately $1 \mu\text{W}$ of drive shown in Fig. 7 [6]. In both oscillators multiplicative phase modulation was reduced by local negative feedback [2]. Additional improvement can be obtained by using a high input impedance buffer amplifier and driving it with a series tuned tank from a point in the oscillator where the noise is bandwidth limited by the crystal, as in Fig. 2. This increases A without changing $a(t)$, since $a(t)$ is primarily determined by the Johnson noise in the stages following the basic oscillator.

Stability measurements on 5 crystal pairs suggest that the flicker of frequency level is roughly proportional to Q^{-1} . That is high- Q crystals generally have a correspondingly lower flicker of frequency level, however, crystals with the same Q can vary as much as a factor of 10 in stability in the flicker region. This indicates that fabrication techniques have a considerable influence on stability beyond just considerations of the obtainable Q . The measurement system described here could easily be used to examine the influence of various manufacturing processes on crystal stability. Some measurements versus temperature were also made. The 5B-2 crystal pair demonstrated a factor of 2 higher stability at 25°C than at either -75°C or $+65^\circ\text{C}$. Another 5-MHz crystal pair with the same Q was a factor of 2 more stable at 65° than at 25° achieving a level about 2 times more stable than even the 5B-2 crystal pair illustrated in Fig. 7.

Since both crystal pairs were high-quality 5th overtone AT cuts, these measurements are not conclusive. It appears necessary to make stability measurement versus temperature over a very wide temperature range with many intermediate points in order to understand the temperature dependence.

APPLICATIONS FOR CRYSTAL OSCILLATORS WITH IMPROVED SHORT-TERM STABILITY

There are a number of important applications for crystal controlled oscillators with greatly improved short-term stability.

1) Use as a source for multiplication to the infrared. The linewidth could be nearly 1500 times narrower than for any previous system and nearly 10^6 times narrower than previous crystal controlled oscillator systems [7], [8].

2) Use as a short-term base in counting systems to reduce the time necessary to achieve a given level of accuracy. In some cases acquisition times could be reduced by more than a factor of 100.

3) Use as a short-term local oscillator for precision frequency standards. One of our present standards, NBS-5, is partly limited now by the stability of its local oscillator [9].

Future standards will need even better short-term stability.

MULTIPLICATION TO THE INFRARED AND THE CALCULATION OF LINEWIDTHS

There is today a growing need for frequency measurements and frequency control in the infrared to optical region of the electromagnetic spectrum. One of the problems associated with multiplying the frequency of a crystal controlled oscillator to these high frequencies is that the fast linewidth or instantaneous 3-dB linewidth of the signals becomes very large (another consideration is of course signal-to-noise). For example, a state-of-the-art commercially available quartz crystal oscillator would yield a fast linewidth W_f of nearly 1 MHz when multiplied to 1 THz—even using a perfect multiplier chain. This is to be compared with its fast linewidth of approximately 10^{-5} Hz at 5 MHz and 10^{-2} Hz at 10 GHz. This will be discussed in detail below. Analysis shows that a factor of 5 improvement in the short term stability would reduce the linewidth at 1 THz to only a few hertz. The reduction in linewidth would allow the detector bandwidth to be greatly reduced which should also improve the detected signal to noise ratio. This would improve one's ability to measure or control frequencies in the infrared and provide a platform for multiplication to the visible.

The fast linewidth W_f can be estimated from the following approximate expression

$$\int_{W_f/\pi}^{\infty} S_v(f) \frac{\nu^2}{f^2} df = 1 \text{ rad}^2 \quad (14)$$

where ν is the frequency at which W_f is being calculated. Note that $S_v(f)\nu^2/f^2$ is just the spectral density of phase fluctuations at frequency ν .

Equation (14) integrates these phase fluctuations as a function of Fourier frequency from ∞ to W_f/π such that the total rms phase fluctuation is equal to 1 rad. One radian is approximately the phase fluctuation necessary to make the signal out of phase with itself.

The integral in (14) is easily broken up into parts to handle the various dependences of $S_v(f)$ on f , found in laboratory devices, e.g., see Fig. 8. For a single power law dependence of $S_v(f)$ on f , equation (14) yields values for W_f which are within a few percent of those derived by Halford in his pioneering paper [10]. The comparison

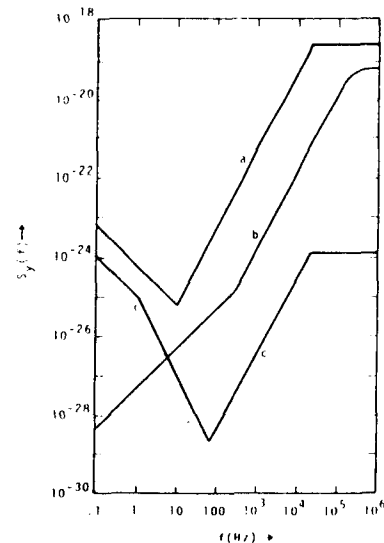


Fig. 8. Frequency domain data for: (a) high quality oscillator controlled by one 5B-2 crystal, (b) 5-MHz to 9-GHz multiplier chain, (c) 5B-2 crystal plus Johnson noise of measurement system.

TABLE I

Type of Noise $S_v(f)$	Allan Variance	W_f Halford [7]	W_f equation (12)
White FM $S_v(f) = h_0 f_0$	$h_0/2\tau$	$\pi h_0 \nu^2$	$\pi h_0 \nu^2$
Flicker FM $S_v(f) = h_{-1} f^{-1}$	$2 \ln 2 h_{-1}$	$\left(\frac{8\pi}{3\sqrt{3}} h_{-1} \nu^2\right)^{1/2}$	$\left(\frac{\pi^2}{2} h_{-1} \nu^2\right)^{1/2}$
Random Walk FM $S_v(f) = h_{-2} f^{-2}$	$2/3 \pi^2 h_{-2}$	$(2\sqrt{2} \pi h_{-2} \nu^2)^{1/2}$	$\left(\frac{\pi^3}{3} h_{-2} \nu^2\right)^{1/2}$

* Comparison of several estimates of the fast linewidth W_f as a function of the type of noise present. $S_v(f)$ is assumed to follow a single power law dependence on Fourier frequency offset from the carrier f over the entire region of interest.

for several types of commonly encountered types of noise are given in Table I. Curve *a* of Fig. 8 shows $S_v(f)$ calculated from the measured oscillator stability curves of Fig. 7. This was done using (5) in the flicker of frequency region and $\sigma_v(\tau) = 3bh_{-2}/(4\pi^2\tau^2)$ in the white phase modulation region. In addition, a single pole filter centered at 5 MHz with a bandwidth of ± 20 kHz was used to keep $S_v(f)$ finite at high frequencies. This value is also consistent with measurements of the bandwidth of the amplifier section of the oscillator. Curve *b* of Fig. 8 shows the measured $S_v(f)$ for our low noise 5-MHz to 9.2-GHz multiplier chains. Curve *c* shows $S_v(f)$ for crystal pair 5B-2 from Fig. 6 plus the estimated Johnson noise shown in Fig. 7. Again a ± 20 -kHz single-pole pass-band filter has been assumed.

Curve *a* of Fig. 9 shows the linewidth calculated from curve *a* of Fig. 8 using (14) as a function of final output frequency ν . Although this result is very surprising and totally unanticipated from earlier calculations where $S_v(f)$ was assumed to follow a single power law dependence on f [10] it is consistent with measurements using similar oscillators and multiplier chains [7], [8], [10], [11],

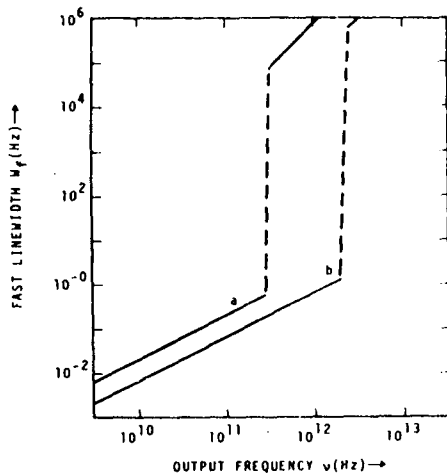


Fig. 9. Dependence of the fast linewidth on the synthesized output frequency for an input spectrum given by: (a) curve *a* in Fig. 8, (b) the sum of curves *b* and *c* in Fig. 8.

where the linewidth increased from less than 1 Hz at 10 GHz to nearly 1 MHz at 0.9 THz. The actual change in the dependence of $S_\nu(f)$ on f is of course more gradual than assumed in the model for the calculations. This would tend to increase the frequency span over which the linewidth step shown by curve *a* occurs. Nevertheless, the basic result of nearly 5 orders of magnitude increase in linewidth when ν is increased from approximately 130 to 500 GHz should be valid.

Curve *b* of Fig. 9 shows the resulting linewidth calculated from (14) as a function of ν , assuming that $S_\nu(f)$ is given by the sum of curves *b* and *c* in Fig. 8. We thus derive the fast linewidth appropriate to driving our present X-band multiplier chains with a crystal controlled oscillator which realizes the inherent stability of the crystal resonators shown in Fig. 7. The linewidth for frequencies above the step at approximately 2 THz is then dominated by the X-band multiplier chain. Further improvements in these multiplier chains would permit the linewidth step to be moved to even higher frequencies. The effect of noise on W_f in the frequency multiplication from 10 GHz to the THz region is of course reduced by 4×10^6 over the initial stages.

CONCLUSION

A new technique which allows the measurement of the inherent frequency stability of quartz crystal resonators was introduced. Measurements using this technique demonstrated that the resonators are much more stable than the corresponding crystal controlled oscillators, especially for times less than 1 s. The utility of these techniques for measuring the effects of different oscillator designs on stability, as well as the possibility of evaluation and opti-

mizing manufacturing processes to produce crystal resonators with better short-term stability was demonstrated. A simple model of noise degradation in crystal controlled oscillators at short measurement times and suggested design changes to minimize this effect were also given. A method for estimating the fast linewidth (instantaneous 3-dB linewidth) of a signal as a function of frequency was presented. It was shown that crystal oscillators, if featuring short-term stabilities equal to those of the present crystals, would yield fast linewidths of less than 1 Hz after multiplication to 1 THz.

This is nearly a factor of 10^6 less than present crystal-controlled oscillators and a factor of 1500 less than previously obtained with any system at 1 THz [7]. Such performance would therefore greatly aid frequency metrology in this region of the spectrum as well as provide a platform for multiplication to higher frequencies. We are continuing work towards realizing this performance potential.

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