POSITION LOCATION USING SEQUENTIAL GPS MEASUREMENTS

by
Marc Weiss
National Bureau of Standards
Boulder, Colorado 80303

Abstract

This paper reports the development of a program to derive a first order correction to initial estimates of local coordinates and local clock bias from GPS time using a single channel GPS receiver of the C/A code. The program measures sequentially the local minus GPS time via four different satellites based on an initial esti mate of local coordinates. Then using these measurements along with known locations of the satellites the first order corrections to the X, Y, and Z coordinates and the local time bias from GPS time are obtained. With reasonable geometry the first order correction is theoretically good to one meter if the initial estimate of coordinates is within 3 km of the correct values. Over a very short 15 m baseline we found a 2.7 m differential position location error. Over a long baseline using a poor geometric satellite configuration the differential navigation solution was apparently within 20 m of the true values. Absolute position location relative to a first order survey point was off by only 7.2 m, and this without including ionospheric corrections to transmission delays to the satellites.

Program

The purpose of this program is to provide coordinates as support for GPS common view time transfer. Thus, differential position location, i.e., relative positioning, is more important than absolute position location. Local coordinates should be known to 3 km and we want to solve accurate to 5 m. So, in our solution we do only a single first order correction. We do not iterate. In solving for position we solve for both spatial coordinates and time bias. Thus, the navigation program can itself be used for time transfer.

The program uses measurements, via four different satellites, of the local clock bias from the GPS time scale to derive a first order correction to local coordinates and mean clock bias. Measurements are made sequentially on the four satellites. These four values are used to solve to first order for the user's space-time position as discussed in the appendix. The first order correction itself is accurate to 1m if the initial estimate is within 3 km.

After the first solution for position location, local minus GPS measurements can still be continued. After each measurement another first order correction is applied to the same initial position estimate using the four most recent time bias measurements. This can be continued as long as the four satellites are up. These are not entirely independent in that each measurement is used in four solutions.

Results

The most important results are in terms of the accuracy for differential navigation. The navigation program was run during different periods for two different antenna locations of 15.8m apart. Each solution was run using the same satellite during about the same geometric configurations each night. The GDOP for this data was never more than 3.0. The data using the first antenna location is from 6 separate days in a 16 day period. For the second location we have data from 10 separate days in a 30 day period. The overall RMS of the data for the first location is 3.5 m. For the second location the overall RMS of the data is 6.3 m. The distance between the two locations from the navigation solution is 18.5 m. This differs from the measured distance of 15.8 m by 2.7 m.

Differential navigation was done on a longer baseline of the order of 1500 km, from NBS in Boulder, Colorado to a tracking station of JPL at Goldstone, California. It was not possible to do simultaneous navigation at the distant locations using four satellites in a good geometric configuration. Also we did not have a consistent coordinate system to compare against at both locations. In terms of absolute position location the values found were poor; a result of GDOPs from 10 to over 100. The peak to peak variation in location values vary over 200 m. But the RMS of the differential values was only 22.66 m. The results were checked by viewing local clock minus GPS time measurements of both locations for a variety of satellite elevations and azimuths to see if the measured clock bias, based on the position given by differential navigation, depended on satellite position. This analysis verified that the differential solution was accurate to about 20 m. The figure shows graphs of the last three digits of the X, Y, and Z coordinate values versus solution number for the Boulder and Goldstone receivers from 18 solutions performed simultaneously at the two locations over a 36 minute period. The variation in height of the graphs with solution number shows the effect of a poor GDOP. Thus the actual values given for the coordinates from the navigation solutions are poor. The correlation between the values given for Boulder and Goldstone for the X, Y, and Z coordinates suggests the accuracy of the differential navigation between the two position solutions, whose RMS was 22.66 m.

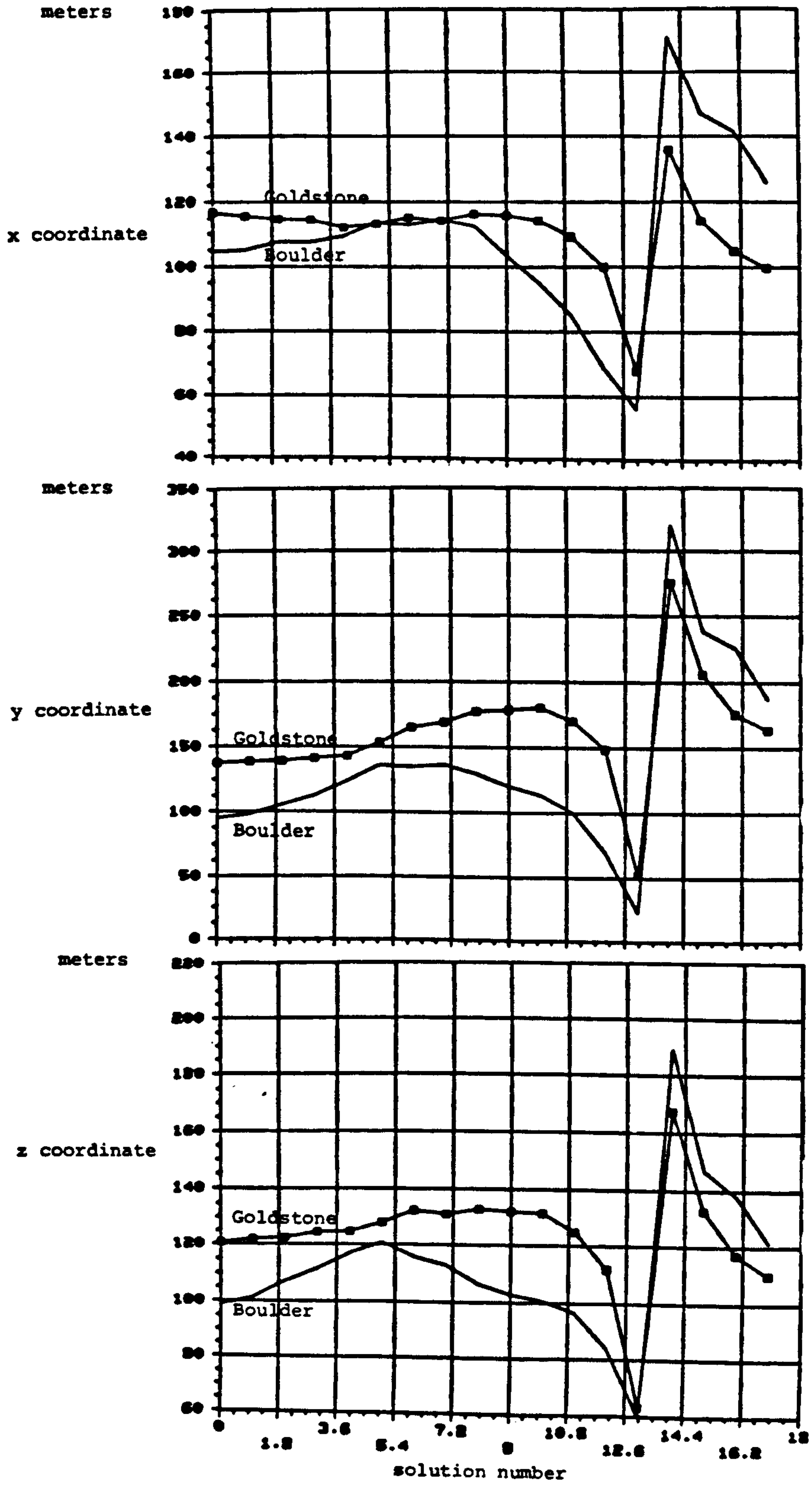


Figure:

The last three digits of coordinate values from consecutive navigation solutions done simultaneously at Boulder, Colorado and Goldstone, California. Absolute position location varies greatly, due to poor satellite geometry, but differential positioning is more highly correlated.

The accuracy of the navigation solution for absolute position location is hard to determine so far since we have used no corrections for the ionosphere yet. These will be included in the next version to appear about January 1983. For now we can say that the coordinates from the first order survey for Boulder differ from the mean of the 10 solutions by:

	Survey-Solution	RMS for solutions
X	+0.4 m	2.2 m
Y	+6.6 m	4.2 m
Z	-2.4 m	4.1 m

These results indicate that the error is largely down from the zenith, as we would expect due to ionospheric effects. In any case, the solution is within 10 m of the first order survey point.

Appendix: The Accuracy of the First Order Correction

The navigation solution itself has also been derived elsewhere.(1) The basic equation is the simple relation that if $\vec{X}$ is user's position vector, and $\vec{S}_i$ is the i^{th} satellite's position vector then the range vector $\vec{R}_i$ satisfies

$$(1) \quad \vec{R}_i = \vec{S}_i - \vec{X}.$$

We assume $\vec{S}_i$ is correct and known, and try to find $\vec{X}$ given an initial estimate $\vec{X}' = \vec{X} + \Delta\vec{X}$. Using this we estimate the range vector:

$$\vec{R}'_i = \vec{S}_i - \vec{X}' = \vec{S}_i - \vec{X} - \Delta\vec{X}.$$

If we write $\vec{R}'_i = \vec{R}_i + \Delta\vec{R}_i$, so $\Delta\vec{R}_i$ is the range error vector then

$$\vec{R}_i + \Delta\vec{R}_i = \vec{S}_i - \vec{X} \quad \Delta\vec{X} \quad .$$

If we subtract equation (1) from this we have

$$\Delta\vec{R}_i = -\Delta\vec{X} \; .$$

So $\Delta\vec{R}_i$ is independent of i and we may omit the index. We therefore have

$$(2) \quad \Delta\vec{R} = -\Delta\vec{X} \; .$$

We assume the satellite clock correction is exact. So, when we measure the time interval from the local clock 1 pps to the received 1 pps we measure exactly the transmission time of the signal plus the local clock bias. With our third and final assumption that the propagation velocity of the signal is known, we have that our measurement is exactly the range scalar to the satellite, ρ_i, plus the local clock bias b. Thus we measure:

$$m_i = \rho_i + b.$$

In the solution, we correct our measurement by computing the slant range based on our estimated coordinates, and estimating our clock bias. We have estimates:

$$\rho'_i = \rho_i + \Delta\rho_i$$

$$b' = b + \Delta b \; .$$

Then the number we actually use in the navigation solution is

$$(3) \quad m'_i = m_i - \rho'_i - b' = - \quad \Delta\rho_i - \Delta b$$

We solve for $\Delta\vec{X}$ and Δb by solving for $\Delta\vec{R} = -\Delta\vec{X}$ to first order. We do this as follows. Define the two unit vectors in the directions of the true and estimated range vectors:

$$e_i = \vec{R}_i/\rho_i \qquad e'_i = \vec{R}'_i/\rho'_i \; .$$

So $\quad e'_i \cdot \vec{R}'_i = \rho'_i$

and $\quad (4) \quad e'_i \cdot (\vec{R}_i + \Delta\vec{R}) = \rho_i + \Delta\rho_i \; .$

Now $e'_i \cdot \vec{R}_i$ equals the magnitude of $\vec{R}_i$ times the cosine of the angle, θ_i, between $\vec{R}'_i$ and $\vec{R}_i$:

$$e'_i \cdot \vec{R}_i = \rho_i \cos(\theta_i).$$

Substituting in (4) we find

$$e'_i \cdot \Delta\vec{R} = \rho_i(1-\cos(\theta_i)) + \Delta\rho_i.$$

With (2)

$$e'_i \cdot (\Delta\vec{X}) = -\Delta\rho_i \; - \rho_i(1-\cos(\theta_i))$$

Our approximation is

$$\cos(\theta_i) = 1 - \frac{\theta^2}{2} + \ldots \doteq 1.$$

We use

$$e'_i \cdot \Delta\vec{X} = -\Delta\rho_i.$$

Since our corrected measurement is of the form (3) $m'_i = -\Delta\rho_i - \Delta b$

we use

$$(5) \quad e'_i \cdot \vec{\Delta}X - \Delta b = m'_i$$

Equation (5) consists of the corrected measurement, m'_i, on the right hand side, the unknowns, $\Delta\vec{X}$ and Δb on the left hand side and the unit vector e'_i on the left hand side based on the known satellite position, $\vec{S}_i$, and the estimated user position, $\vec{X}'$. We have one such equation for each of four different satellites. We may write these four in matrix form

$$A \, \underline{\Delta X} = \underline{m}'$$

where:

$$A = \begin{bmatrix} e'_{1x} & e'_{1y} & e'_{1z} & -1 \\ e'_{2x} & e'_{2y} & e'_{2z} & -1 \\ e'_{3x} & e'_{3y} & e'_{3z} & -1 \\ e'_{4x} & e'_{4y} & e'_{4z} & -1 \end{bmatrix}$$

$$\underline{\Delta X} = \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta b \end{bmatrix} \qquad \underline{m}' = \begin{bmatrix} m'_1 \\ m'_2 \\ m'_3 \\ m'_4 \end{bmatrix} .$$

We then solve

$$\underline{\Delta X} = A^{-1} \underline{m'} .$$

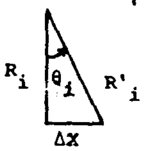
How good is this? The exact solution is

$$\underline{\Delta X} = A^{-1} \underline{m'} - A^{-1} \underline{\varepsilon}$$

where

$$\underline{\varepsilon} = \begin{bmatrix} \rho_1(1-\cos(\theta_1)) \\ \rho_2(1-\cos(\theta_2)) \\ \rho_3(1-\cos(\theta_3)) \\ \rho_4(1-\cos(\theta_4)) \end{bmatrix} .$$

So the error depends on A^{-1} and $\cos \theta_i$. The rows of A^{-1} form row vectors whose magnitudes equal the geometric dilution of precision factor (GDOP); the top row giving the XDOP, the second giving the YDOP, the third giving the ZDOP and the last the TDOP.(1) The dilution of precision (DOP) factors are the amounts by which errors in satellite ephemerides and clocks are magnified in the solution for the user's X, Y, Z, and T due to the geometric configuration of the satellites relative to the user. Thus we can get an idea of the size of $A^{-1}\varepsilon$ by using GDOP $\cdot \rho_i(1-\cos(\theta_i))$. The range to a satellite, ρ_i, is of the order of $2 \cdot 10^7$ m. The angle θ_i is the largest when ΔX and R_i are perpendicular. Then



$$\sin(\theta_i) = \frac{\|\Delta X\|}{\|R_i\|}, \quad \cos(\theta_i) = \left(1 - \left(\frac{\|\Delta X\|}{\|R_i\|}\right)^2\right)^{1/2} .$$

Using the binominal theorem, which is accurate to first order since

$$\left(\frac{\|\Delta X\|}{\|R_i\|}\right)^2 << 1:$$

$$\cos(\theta_i) = \left(1 - \frac{\|\Delta X\|^2}{\|R_i\|^2}\right)^{1/2} \doteq 1 - \tfrac{1}{2} \frac{\|\Delta X\|^2}{\|R_i\|^2}$$

So $\rho_i(1-\cos\theta_i) \doteq \|\Delta X\|^2/2\rho_i \doteq \|\Delta X\|^2 / 4\cdot 10^7$.

We see the size of the error is about

$$(6) \qquad \underset{\text{(meters)}}{\text{error}} = \frac{\text{GDOP}}{4\cdot 10^7} \cdot \|\Delta X\|^2$$

The resultant error is proportional to the GDOP and to the square of the error in the initial estimate. The GDOP should be under 4. To get an error under 1 meter, then, we need

$$1 = \frac{1}{10^7} \|\Delta X\|^2$$

or

$$\|\Delta X\| \doteq 3.2 \cdot 10^3 \text{ m} .$$

Thus to get an error of 1 meter with a reasonable geometry for the satellites we need to be within 3 km for the initial estimate.

Some small deviation from this analysis also results from the lack of accuracy of our assumptions. We have assumed the satellite position data and clock correction are exact as transmitted, and that the exact propagation velocity from the satellite is known. Errors in satellite ephemeris and clock bias will be introduced due to limitations in the GPS control segment. The use of propagation velocity to convert a time measurement to a pseudo-range measurement is limited by the correctness of ionospheric and tropospheric models. In that we are concerned mainly about common view of satellites much of these errors will cancel as discussed elsewhere.(2)

References

1. P. S. Jorgensen, Navstar/Global Positioning System 18-Satellite Constellations, Navigation 27, 2 (Summer 1980).
2. D. Allan and M. Weiss, Accurate Time and Frequency Transfer During Common-View of a GPS Satellite, Proc. 34th Annual Symp. on Frequency Control (SFC), 334 (1980).