ANALYSIS OF HIGH PERFORMANCE COMPENSATED THERMAL ENCLOSURES

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Abstract

Approximate analysis of the conventional thermal enclosures such as ovens and cryostats reveals that the limitation to achievable thermal regulation is in many cases not the gain of the thermal servo loop, but rather the fact that the experiment under observation within the thermal enclosure is still coupled to the outside temperature. So, even if the thermal enclosure is perfectly stable in temperature, the experiment is not. A new configuration is suggested which uses an additional sensor to measure changes in the outside temperature and compensate the temperature set point of the thermal enclosure in order to just correct for the temperature error induced by the coupling to the outside.

Approximate Analysis of Conventional Thermal Enclosures

The following approximate analysis is intended to illustrate how additional thermal sensors can be used to compensate a high performance thermal enclosure such as an oven or cryostat in order to substantially improve the effective thermal regulation of the enclosed experiment. Figure 1 shows the basic elements of the problem for a conventional single layer oven. The temperature of the outer shell T_O, the temperature of the heater/cooler T_H, and the temperature of the experiment T_E are initially assumed to be uniform. R_{HS} represents the thermal impedance between the oven heater/cooler and the sensor; R_{OS} the thermal impedance between the sensor and the outside; R_{HE} the thermal impedance between the heater/cooler and the experiment; R_{OE} the thermal impedance between the outside and the experiment, τ_{HS} the thermal lag between the sensor and the heater/cooler due to R_{HS} and the heat capacity of the sensor; τ_{HE} the thermal time constant of the experiment due to its heat capacity and the effective thermal impedance which is the parallel combination of R_{HE} and R_{OE}. The effects of small time dependent thermal gradients are also included in the model of the compensated thermal enclosure discussed later and schematically illustrated in figure 5 by the term $\delta_T(\tau)$.

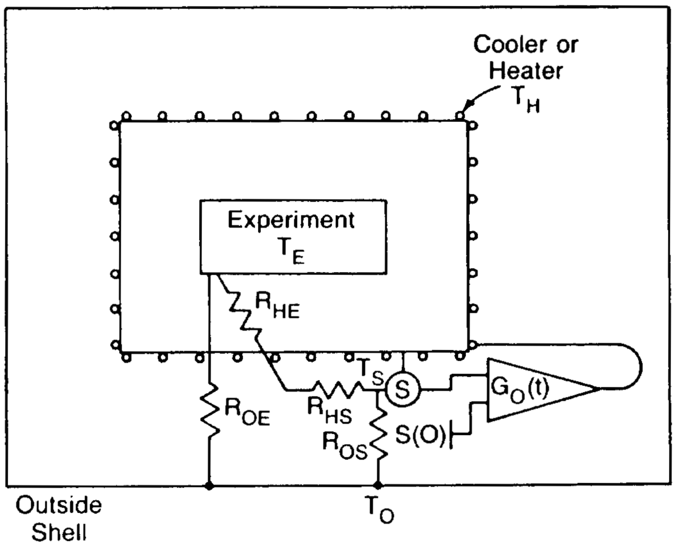

Fig. 1 Schematic representation of an experiment contained within an oven shell. The various temperatures are represented by T's and the thermal impedances by R's. $T_S(0)$ is the set point of the oven, and S is the thermal sensor.

The thermal performance of the enclosure is often characterized by thermal gain, $G_O(\tau)$, which is loosely defined by

$$T_s(\tau) - T_s(o) = \frac{T_o(\tau) - T_o(o)}{1 + G_0(\tau)} \qquad (1)$$

where the left side of the equations is the change in the sensor temperature, T_S, at a time delay "τ" from the initial value due to a change in the outside temperature, T_O, from its initial value at time t=0. Note that $G_O(\tau)$ is a function of the averaging time and of the general form

$$G_0(\tau) = \frac{\tau + \tau_0}{\tau} \quad G_E(\tau) \qquad (2)$$

where τ_0 is the thermal delay time from the outside shell to the oven and $G_E(\tau)$ is the step response function of the electronic servo gain. From servo theory [1-3] one can show that the heater servo is stable when the response time (unity gain time) of $G_E(\tau)$ is about 4 times slower than the heater sensor delay time, τ_{HS}. $G_E(\tau)$ typically crosses the unity gain point with a slope of 6 dB per octave, although any slope below 12 dB per octave is stable. Given this limitation on the gain slope and the need to reduce the effects of high frequency noise in the servo, $G_E(\tau)$ generally has a functional form

$$G_E(\tau) = \frac{\left(\frac{(\tau_0/\tau_{HS})\tau}{4\tau + 4\tau_0}\right) \left(\frac{\beta\tau}{\beta t + \tau_{HS}}\right)}{\left(1 + \frac{\tau}{\beta\tau_{HS}}\right) \left(1 + \frac{\tau}{\beta^2\tau_{HS}}\right)} \qquad (3)$$

where the first bracket comes from using linear feedback with the gain set to cross unity at 6 dB per octave and $\tau=4\tau_{HS}$, the second bracket contains the terms yielding additional electronic filtering of the high frequency fluctuations for times shorter than τ_{HS}/β, and the following terms show the effects of adding integraters to increase the gain slope at longer and longer times. β is typically chosen to between 4 and 10. This is illustrated in figure 2. By using a carefully adjusted balance between the proportional part of the gain and that of the integrator one can make the loop optimally fast with imperceptible overshoot [2]. From this representation of $G_E(\tau)$ it is clear that the maximum value of $G_E(\tau)$ that can be obtained at long averaging times or delay times is scaled as $(\tau/\tau_{HS})^{n+1}\ \beta^{-n}!$, where n is the number of integraters in the $G_E(\tau)$. The smaller the delay time τ_{HS}, the easier it is to make $G_E(\tau)$ and hence $G_O(\tau)$ large. This analysis explicitly assumes that the sensor temperature error produces a linear correction in the applied heater/cooler temperature.

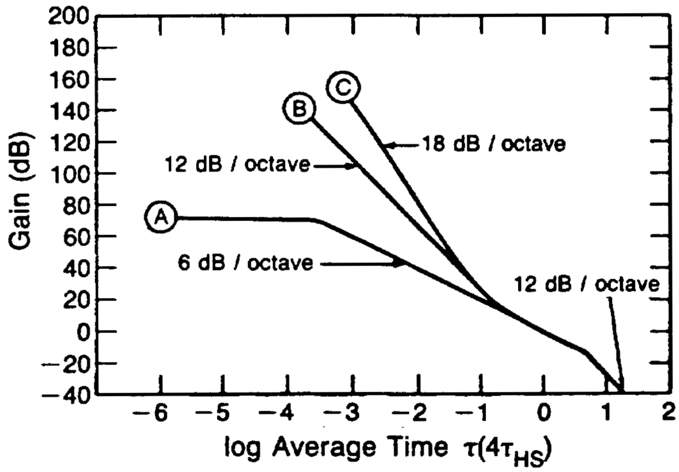

Fig. 2 Oven servo gain $G_O(\tau)$ in dB versus averaging time (measurement time) in units of $4\tau_{HS}$ for proportional gain in curve A, for proportional gain plus one integrator with β~4 in curve B, and proportional gain plus 2 integrators with β~4 in curve c.

It should also be noted that the sensor temperature is not exactly that of the heater/cooler, T_H, even for infinite $G_O(\tau)$, because the sensor has finite thermal resistance to the oven heater/cooler characterized here by R_{HS} and coupling to the outside characterized here by R_{OS}. The thermal error of the heaters/coolers in steady state is then of order

$$T_H - T_S \sim (T_O - T_H)\ \frac{R_{HS}}{R_{OS} + R_{HS}} \quad . \tag{4}$$

The difference between the temperature T_E of the experiment and that of the sensor versus $G_O(\tau)$ is often characterized by a graph similar to that shown in figure 3. The asymptotic value of T_E as $G_O(\tau) \to \infty$ is approximately given by

$$T_E(\tau) - T_S(0) \sim \left[T_O - T_H\right] \left[1 + \frac{R_{HS}}{R_{OS}+R_{HS}}\right] \left[\frac{R_{HE}}{R_{HE}+R_{OE}}\right] \left[\frac{\tau}{\tau + \tau_{HE}}\right] . \tag{5}$$

Eq. 5 shows that even in the limit of $G_O0(\tau) \to \infty$, there can still be a significant thermal error. The first term of the second bracket comes from the thermal error between the heater/cooler and the sensor (eq. 4) and the second, and often more important term, caused by the thermal coupling of the experiment to the outside. The role of τ_{HE} in the 4th bracket is seen to be a filtering of the thermal transients in the thermal control servo at very short times (high frequencies). In steady state with $G_O(\tau) \to \infty$, $R_{HS}/R_{OS} << 1$, (i.e., $T_H = T_S(0)$, $R_{HE} << R_{OE}$), this reduces to

$$T_E(\tau) - T_S(0) = [(T_O(\tau) - T_S(0)]\ \frac{R_{HE}}{R_{OE}} =$$

$$[\underbrace{(T_O(\tau) - T_O(0))}_{6a} + \underbrace{(T_O(0) - T_S(0))}_{6b}]\ \frac{R_{HE}}{R_{OE}} \tag{6}$$

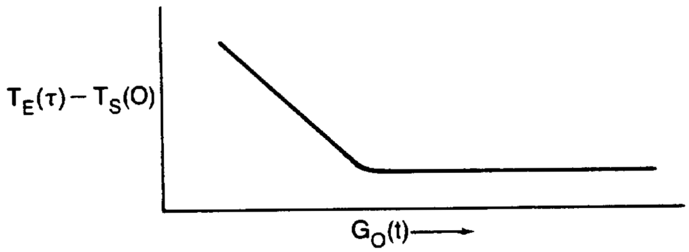

Fig. 3 Temperature error of the experiment versus thermal gain of the oven, $G_O(\tau)$.

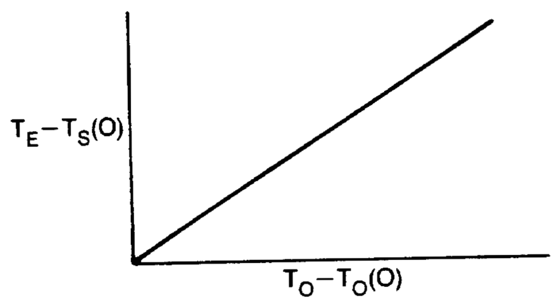

Fig. 4 Steady state temperature error of the experiment, T_E versus changes in the outside temperature, T_O, from its nominal value $T_O(0)$. The slope of this line yields a good estimate of R_{HE}/R_{OE}.

The confirmation that this is the correct model of a conventional oven is demonstrated by the fact that, for most ovens, the temperature change of the experiment due to changes in the outside temperature is basically linear.

A Compensated Thermal Enclosure

The formulation of the problem, as given above, for a conventional thermal enclosure illustrates that achieving large thermal gains with an enclosure is not necessarily sufficient to keep the enclosed experiment at the correct or even at a constant temperature. If R_{OE}/R_{HE} is larger than $G_0(\tau)$, then the effects outlined above will dominate the temperature performance of the enclosure that surrounds the experiment. Clearly R_{HE} should be chosen as small as possible consistent with attenuating the high frequency thermal noise in the thermal regulation servo and reducing the thermal gradients in the shell of the thermal enclosure. These thermal gradients scale roughly as R_{OVOV}'/R_{HOV} (R_{EE}'/R_{HE}) where R_{OVOV}' is the nominal thermal impedance from one endcap of the oven to the other, R_{HOV} is the thermal impedance from the heater to the oven shell, and R_{EE}' is the thermal impedance from one end of the experiment to the other. In some cases it is possible to decrease R_{HE} and increase the thermal heat capacity of the experiment in order to preserve the same value of τ_{HE}, and still decrease the relative temperature error which is proportional to R_{HE}/R_{OE}. There is also another solution to the problem which preserves the thermal filtering of R_{HE}; that is to measure $T_0(\tau)$ - T_H and use the result to change the thermal sensor set point by an amount equal to the right hand side of equation 6 [4]. This is shown in figure 5. In practice this is easily accomplished since one need only measure the relative temperature changes of the outside shell versus a nominal setpoint $T_0(0)$ (term 6a). Term 6b only contributes an offset which, for many applications, can be ignored. The effects of a time varying thermal gradient between the position of measurement of the outside temperature and the effective position of the thermal connection between the experiment and the outside are included in Eq. 7 via $\delta_T(\tau)$. The temperature error of the experiment using the scheme shown in Figure 5 is given by equation 7.

$$T_E(\tau)-T_S(0) = [(T_0(\tau)-T_0(0)]\frac{R_{HE}}{R_{OE}} - [(T_0(\tau)-T_0(0) + \delta_T(\tau)]G_{FB} + [(T_0(0)-T_S(0)]\frac{R_{HE}}{R_{OE}} \qquad (7)$$

The temperature coefficient of the experiment due to changes in the temperature of the outside is approximately given by

$$T_C = \frac{dT_E}{dT_0} = \frac{R_{HE}}{R_{OS}} - \left(1 + \frac{d\delta_T(\tau)}{dT_0}\right)G_{FB} \qquad (8)$$

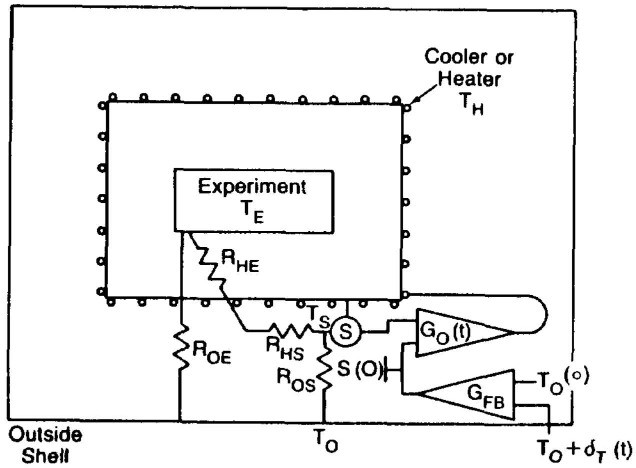

Fig. 5 Schematic representation of an experiment contained within an oven shell using feedback to compensate $T_S(0)$, the set point of the oven. The various temperatures are represented by T's and the thermal impedances by R's. S(0) is the set point of the oven, $G_0(\tau)$ is the oven gain, $T_0(0)$ is the nominal temperature of the outside shell, S2 is the second sensor, G_{FB} is the gain of the second feedback circuit which modifies the temperature of the primary enclosure set point S(0), and $\delta_T(\tau)$ is the time varying thermal gradient along the outside shell between the second sensor and the effective point of thermal coupling from the outside to the experiment.

One notes that T_C can be adjusted from plus to minus or zero by changing the feedback gain, G_{FB}, to within a precision limited only by the time variations of $\delta_T(\tau)$. Additionally, variations of $\delta_T(\tau)$ with T_0 are unimportant. Improvements in T_C are always possible if the open loop performance (i.e., G_{FB} = 0) conforms with Figure 4. Adjusting T_C to approximately zero is equivalent to increasing the thermal gain at the experiment to infinity. In practice it has proven possible to achieve thermal gains in excess of 10^5 using a single oven shell fabricated from 1.6 mm thick aluminum cylinder about 25 cm long and 18 cm in diameter. The ends were about 0.63 cm thick and carried the heaters. The experiment under control was an alumina cylinder of ~ 4 kg weight, R_{HE} was of order 1 hour. Particular care was taken to reduce thermal gradients across the experiment and to heat sink the cables connecting the experiment to the oven to increase R_{OE}.

The question which naturally arises when compensating the oven temperature in this manner is: what happens to the thermal transient response of the experiment? The reason that τ_{HE} is finite (hours in some cases) is that a long time constant is perceived to be necessary in order to reduce the thermal transients due to noise in the oven servo and thermal gradients in the experiment. The transient response can easily be understood by modeling it as an equivalent electrical circuit in (figure 6) where $T_E \rightarrow V_E$, $T_H \rightarrow V_H$ etc.

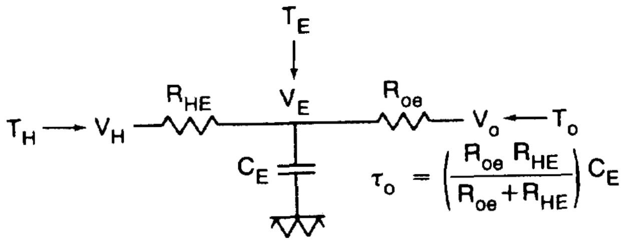

Fig. 6 Electrical analog for the thermal circuit of the heater/cooler, thermal sensor, experiment showing the various couplings.

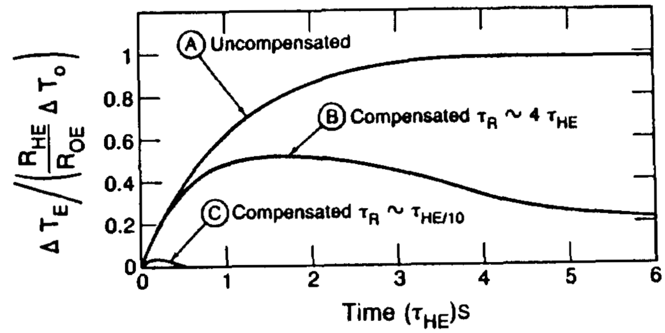

Fig. 7 Curve A shows the calculated thermal transient response of the experiment in an uncompensated thermal enclosure for a step change on the temperature of the outside shell normalized by R_{HE}/R_{OE}. Curve B shows the calculated response using a perfect thermometer on the experiment with the fastest stable feedback response in the compensated network, namely, $\tau_R = 4\tau_{HE}$. Curve C shows the calculated response for the compensated servo described in the text assuming that the second thermal sensor on the outer shell has a time constant of $\tau_{HS}/10$. In general the peak height for $\tau_R \ll \tau_{HE}$ is given in eq. 9.

If V_H or V_O changes, the value of V_E moves towards the new equilibrium value of $V_H + (V_O - V_H) R_{HE}/(R_{HE} + R_{OE})$ with a time constant of $\tau_{HE} = (C_E R_{HE} R_{OE})/(R_{HE}+R_{OE})$. If however V_H were to mirror V_O as $V_S - (V_O - V_S)R_{HE}/(R_{HE}+R_{OE})$, V_E would stay perfectly constant. This can be approximated in the real oven by having $T_H(\tau) \sim T_S(\tau)$ track the compensated value of $T_S(0) + [T_O(\tau) - T_O(0)]G_{FB}$ with time constant τ_R. The magnitude of the thermal transient at the experiment due to a change in T_O will be of order

$$\Delta T_E = \left(\frac{1}{e}\right)\left(\frac{R_{HE}}{R_{OE}}\right)\left(\frac{\tau_R}{\tau_{HE}}\right)\left((T_0(\tau) - T_0(0)\right) . \qquad (9)$$

Clearly this is reduced by making τ_R as small as possible. The lower limit of τ_R is $\sim 4(\tau_{HE} + \tau_{S2})$, where τ_{S2} is the thermal response of the second sensor measuring the relative temperature of the outside.

Therefore, if instead of using the oven compensation outlined here, one uses a perfect thermometer on the experiment to measure the temperature error due to T_O changing, the smallest value that could be obtained for τ_R is about $4\tau_{HE}$. Since τ_{S2} can be orders of magnitude less than the thermal lag of the experiment, τ_{HE}, this compensated oven approach will produce much better thermal transient reduction at the experiment than having a perfect thermometer on the experiment. This is illustrated in Figure 7, and qualitatively verified on the single oven system mentioned above. The sensor on the experiment can also be used in conjunction with the method outlined here to obtain superior long term performance.

Discussion

Based on the above analysis one can formulate guidelines for achieving very high thermal gains in single enclosures. These guidelines are:

1) Make τ_{HS}, the time constant between the heater/cooler and the thermal sensor as small as possible and use at least one integrator in order to make $G_O(t)$, the electronic gain, large.

2) Make the thermal resistance between the heater (or cooler) and experiment, R_{HE}, as small as possible and still achieve the desired reduction of high frequency noise and thermal gradients in the experiment.

3) Make R_{OE}, the thermal resistance from the outside to the experiment, as large as possible,

4) Make the thermal gradient, $\delta_T(\tau)$, between the measurement point of T_O for the compensation network and the effective point of thermal connection for R_{OE}, as small and as constant in time as possible.

5) Adjust the servo gain G_{FB} (of eg. 8) so as to minimize the thermal coefficient of the "experiment".

Following these guidelines generally, it is possible to achieve thermal regulation (gain) at the experiment which is 1 to 2 orders of magnitude better than that achieved in the same enclosure using the conventional approach. The thermal transient response can be similarly improved. In several experiments using a large single oven approximately 25 cm long and 18 cm in diameter, thermal gains in excess of 10^5 were obtained. Multiple enclosures can be treated as successive single enclosures.

Acknowledgements

I am indebted to many colleagues for fruitful discussions on this concept, especially Dr. L.L. Lewis who first suggested adding a sensor on the external shell in addition to the one on the experiment in order to compensate for external temperature changes of the primary thermal enclosure.

References

[1] F. M. Gardner, Phase Lock Technique, (John Wiley, New York, New York, 1966).

[2] Private communication with John Hall, Joint Institute for Laboratory Astrophysics, University of Colorado, Boulder, CO 80302.

[3] U.L. Rohde, Digital PLL Frequency Synthesizers, Prentice-Hall, Inc., Englewood Cliffs, NJ, 07632.

[4] It has been brought to our attention that a similar scheme (patent # 3970818) was developed by Jerry Friedricks of Motorola, 2553 N. Edgington, Franklin Park, IL 60131.