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A HIGH-Γ, STRONGLY-COUPLED, NON-NEUTRAL ION PLASMA†

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INTRODUCTION

We have produced a strongly coupled non-neutral $^9Be^+$ ion plasma with a coupling parameter of approximately 100 or greater. The ions were spatially confined by a Penning trap [Penning 1936, Dehmelt 1967, 1969, Wineland et al. 1983] and cooled and compressed using laser cooling [Itano and Wineland 1982]. Measurements were made of the plasma shape, rotation frequency, density and temperature. In this paper we describe the experimental confinement geometry, the laser cooling of ions and the experimental data which are compared with theoretical predictions. Future experiments to measure the plasma static structure function, measure ion diffusion, and improve the temperature measurement are discussed.

CONFINEMENT GEOMETRY

The Penning trap, shown in Fig. 1, is composed of two "endcap" electrodes and a "ring" electrode which are biased with respect to each other by a d.c. electric potential. The electrode surfaces are approximate hyperboloids of revolution. The symmetry axis of the trap is parallel with a static magnetic field. The configuration is similar to that used by the group at the University of California at San Diego (UCSD) [Malmberg and deGrassie 1975]. The hyperboloidal shaped electrodes give rise to an applied trap potential

$$\varphi_T = \frac{m\omega_z^2}{4q}(2z^2 - r^2) , \qquad (1)$$

where m is the ion mass and the axial frequency ω_z is defined by the equation

$$\omega_z^2 = \frac{4qV_0}{m(r_0{}^2 + 2z_0{}^2)} . \qquad (2)$$

V_o is the electric potential applied between the ring and endcaps and r_o and z_o are the characteristic trap dimensions as shown in Fig. 1. There are three principal motions of a single ion in this trap. The potential φ_T is quadratic in z, and this gives rise to simple harmonic motion of the ion at frequency ω_z along the z axis of the trap. In the radial direction the ions are confined by a magnetic field, and the ion motion is a superposition of two circular motions, the cyclotron and magnetron motions. The cyclotron motion is shifted somewhat in frequency from its value in a pure magnetic field by the radial electric field [Dehmelt 1967, 1969, Wineland et al. 1983]. The magnetron or rotation motion is a circular drift of the guiding center of the cyclotron motion due to the **E** x **B** forces of the trap. These motions are shown in Fig. 2.

For the work discussed in this paper, typical trap parameters are an electric potential V_0 of 2 volts, a B field of 1.4 tesla and trap dimensions of r_0 = 0.417 cm and z_0 = 0.254 cm. For these parameters the magnetron frequency for a single $^9Be^+$ ion is 15.1 kHz, the cyclotron frequency is 2.38 MHz and the axial frequency is 267 kHz.

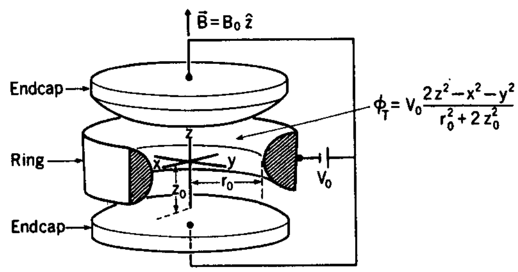

Fig. 1. The Penning trap consists of two endcaps and one ring electrode which are biased with respect to each other by a potential V_0. The hyperbolic surfaces of the electrodes produce a quadrupole potential which confines the ions in the z direction. The static B field confines the ions in the radial direction.

THERMAL EQUILIBRIUM STATE

For a collection of ions in the trap, the resulting single species plasma is assumed to be in thermal equilibrium because of Coulomb collisions. If the plasma is in thermal equilibrium there is no shear in the plasma and the plasma rotates as a rigid body. The density is constant up to the edge of the plasma where it drops off in a distance that is characterized by the Debye length [Prasad and O'Neil 1979].

The single particle distribution function for a magnetically confined non-neutral ion plasma has been given by Davidson [Davidson 1974] and by Malmberg and O'Neil [Malmberg and O'Neil 1977]. For positive ions we have

$$f(r,z,\mathbf{v}) = n(r,z)[m/(2\pi k_B T)]^{3/2}\exp\{-(m/2k_B T)(\mathbf{v} + \omega r\hat{\theta})^2\} \qquad (3)$$

$$n(r,z) = n_0\exp\{-(1/k_B T)[q\varphi(r,z) + (m\omega/2)(\Omega - \omega)r^2]\}. \qquad (4)$$

Here φ is the total electrostatic potential, ω can be identified as the rotation frequency of the plasma, $\Omega = qB_0/mc$ is the cyclotron frequency, and n_0 is the density of the ions at the center of the trap.

In the T = 0 limit, in order that f and n remain finite we find that

$$q\varphi(r,z) + (m\omega/2)(\Omega - \omega)r^2 \to 0 \tag{5}$$

for r,z inside the plasma. This equation tells us that the electrostatic potential is independent of z and along with Poisson's equation, that the density must be constant throughout the plasma and equal to n_0. From Poisson's equation and Eq. 5, n_0 is given by

$$n_0 = \frac{m\omega(\Omega-\omega)}{2\pi q^2} \quad . \tag{6}$$

The distribution function predicts simple shapes for the plasma in the limits that T = 0 and the trap dimensions are large compared to the plasma dimensions. The potential of the plasma is given by the expression

$$\varphi = \varphi_I + \varphi_T + \varphi_{ind}, \tag{7}$$

where φ_I is the Coulomb potential of the ions in the absence of the trap walls and φ_{ind} is the potential due to the charges induced on the trap electrodes. If the electrode spacing is large compared to the dimensions of the plasma we can neglect φ_{ind} and solve for the ion potential. From Eqs. 1, 5, and 7 we find that

$$\varphi_I = \frac{-m}{2q}\{\omega(\Omega - \omega) - \omega_z^2/2\}r^2 - \frac{m\omega_z^2 z^2}{2q} \tag{8}$$

$$= -2/3\ \pi q n_0(\alpha r^2 + \beta z^2) \tag{9}$$

where Eq. 9 is used to define α and β. In general Eq. 9 represents the potential inside a uniformly charged spheroid. For example for $\alpha=\beta=1$, φ_I is the potential inside a uniformly charged sphere.

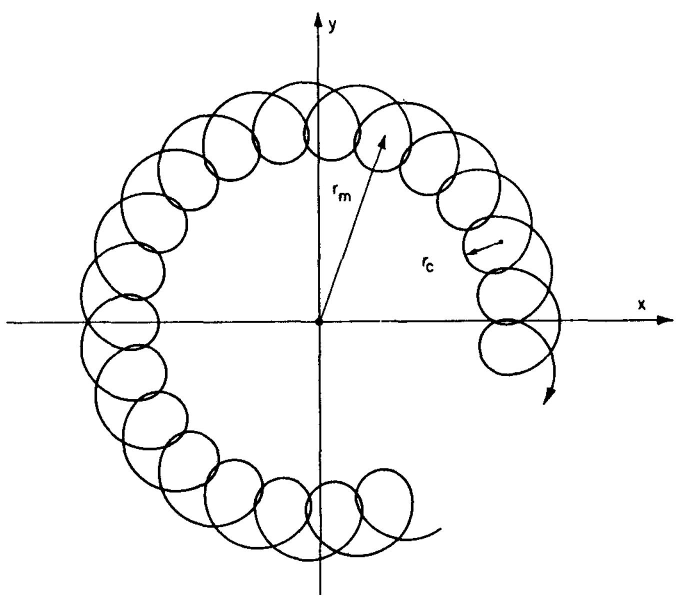

Fig. 2. The orbit of a single ion in the x-y plane consists of two circular motions. r_c is the radius of the cyclotron motion and r_m is the radius of the magnetron motion. The figure sketches the ion orbit for the case $r_c < r_m$.

ONE COMPONENT PLASMA

In the frame of reference rotating about the trap axis with frequency ω, the ion plasma behaves like a neutral one component plasma. That is, the positively charged ions behave as if thay are moving in a uniform negatively charged background. In particular, Malmberg and O'Neil [Malmberg and O'Neil 1977] have shown that the static properties of magnetically confined non-neutral plasma are the same as those for a one component plasma.

A one component plasma [Ichimaru 1982] is composed of a single species of charge imbedded in a uniform background of neutralizing charge. The one component plasma is characterized by a coupling constant

$$\Gamma = q^2/ak_BT \tag{10}$$

which is the ratio of the nearest neighbor Coulomb energy to the thermal energy of a particle. The Wigner-Seitz radius a is defined by

$$a^3 = 3/(4\pi n) \tag{11}$$

where n is the particle density. When $\Gamma > 1$ the plasma is said to be strongly coupled. When $\Gamma > 2$ the plasma should exhibit liquid like behavior and the particles should exhibit short range order. Slatterly, Doolen,and DeWitt [Slatterly et al. 1980] have predicted that at $\Gamma = 178$ the plasma undergoes a phase transition to become a crystal like structure.

Because the transition is of the first order, the plasma may remain in a metastable fluid like state when it is supercooled below the transition temperature. Ichimaru and Tanaka have investigated the supercooled one component plasma and presented evidence for a possible dynamic glass transition at a value of $\Gamma \approx 1{,}000$ [Ichimaru and Tanaka 1986]. (The correspondence between the magnetically confined non-neutral plasma and the one component plasma rigorously exists only for static properties. The possibility of a dynamic glass transition in a one component plasma is therefore a suggestion of what might happen in the magnetically confined non-neutral plasma.)

LASER COOLING

The technique of laser cooling utilizes the resonant scattering of laser light by atomic particles. By directing a laser beam at the plasma one can decrease the thermal velocity of the particle in a direction opposite to the laser beam. The laser is tuned in frequency to the red, or low frequency side of the atomic "cooling transition" (typically an electric dipole transition). Some of the ions moving toward the laser will be Doppler shifted into resonance and absorb a photon. Ions moving away from the laser will be Doppler shifted away from resonance and the absorption rate will correspondingly decrease. When an ion absorbs a photon its velocity is decreased by an amount

$$\Delta v = \hbar k/m \tag{12}$$

due to momentum conservation. Here Δv is the change of the ion's velocity in the direction of the laser beam, $k = 2\pi/\lambda$ where λ is the wavelength of the cooling radiation, and m is the mass of the ion. The ion spontaneously re-emits the photon in a random direction (for low laser intensities where the stimulated emission rate is small), which when averaged over many scattering events does not change the momentum of the ion. The net effect then is that for each photon scattering event the ion's average velocity is reduced by the amount shown in Eq. 12. To cool an atom from 300 K to mK temperatures takes typically 10^4 scattering events. For this experiment the cooling limit, due to photon recoil effects [Wineland and Itano 1979], for a given transition is given by a temperature equal to $\hbar\gamma/2k_B$, where γ is the radiative linewidth of the atomic transition. For a linewidth γ of $2\pi\cdot 19.4$ MHz, which was the natural linewidth of the cooling transition in our experiment, the minimum obtainable temperature is 0.5 mK.

It is interesting to see how the laser affects the angular momentum of the plasma [Wineland et al. 1985]. The z component of the canonical angular momentum for a single particle is

$$l_z = mv_\theta r + qBr^2/2c \tag{13}$$

where v_θ is the ion's azimuthal component of velocity and r is the ion's radial cylindrical coordinate. The two terms represent the mechanical angular momentum and the field angular momentum. The total z component of the angular momentum is

$$L_z = \int dz \int 2\pi r dr \int d^3\mathbf{v} f(r,z,\mathbf{v}) l_z \tag{14}$$

$$= m(\Omega/2 - \omega)N\langle r^2\rangle \ . \tag{15}$$

Eq. 15 tells us that the total angular momentum about the z axis is proportional to the mean of the square of the radius of the plasma. Here N is the number of ions and f is the distribution function. For our experimental conditions Ω is usually much larger than ω [O'Neil 1980] so that

$$L_z \approx m(\Omega/2)N\langle r^2\rangle \geq 0 \ . \tag{16}$$

If the cooling laser is directed at the side of the plasma which is receding from the laser (due to the plasma rotation), angular momentum is removed from the plasma and the radius of the plasma must decrease. As the radius decreases, the density of the plasma increases. The limiting density, known as the Brillouin density [Davidson 1974], occurs when the rotation frequency $\omega = \Omega/2$. The Brillouin density is given by

$$n = \frac{m\Omega^2}{8\pi q^2} \ . \tag{17}$$

Collisions with background gas particles increase the angular momentum of the plasma. This is one of the effects that could limit the compression of the plasma. Axial asymmetry of the trap is also a limiting effect. The plasma group at UCSD has observed that the axial asymmetry of their cylindrical traps plays an important role in the electron confinement time [Driscoll et al. 1986]. It is also expected to be a limiting effect in the experiments reported here [Wineland et al. 1985].

At a magnetic field of 1.4 T, the Brillouin density for $^9Be^+$ is $5.9\cdot 10^8$ ions/cm^3. This density and the 0.5 mK temperature limit gives a theoretical limit on the coupling of $\Gamma \approx 4{,}500$. Consequently the possibility of obtaining couplings large enough to observe a liquid-solid phase transition looks promising. If the cooling or quench can be done rapidly enough, it appears possible to investigate the existence of a dynamic glass transition at $\Gamma \approx 1000$. Currently the laser cooling technique is capable of reducing the temperature of a cloud of ions from room temperature to less than 10 mK ($\Gamma \approx 100$) in a few seconds. This cooling rate, if continued to lower temperatures, compares favorably with the minimum nucleation times ranging from 80 to $8\cdot 10^5$ seconds as estimated by Ichimaru and Tanaka [Ichimaru and Tanaka 1986] for a $^9Be^+$ plasma with the above density.

The experimental configuration reported in this paper was similar to that reported by Bollinger and Wineland [Bollinger and Wineland 1984] where plasma coupling parameters as high as 10 were achieved. In their paper they suggested that by cooling the plasma in directions both perpendicular and parallel to the trap B field one could achieve lower temperatures and higher coupling. The results of this experiment are reported below.

Excitation Scheme

The excitation scheme is shown in Fig. 3. The cooling laser drives $^9Be^+$ ions between the $2s^2S_{1/2}$, $m_j = +1/2$ and the $2p^2P_{3/2}$, $m_j = +3/2$ states. Ions are optically pumped into the $2s^2S_{1/2}$ $m_j = +1/2$ state with 94 % efficiency [Wineland et al. 1984]. That is, for laser intensity below saturation, 94 % of the ions reside in the $2s^2S_{1/2}$ $m_j = 1/2$ state. The cooling laser has a wavelength of 313 nm and a power of approximately 50 μW. Resonance fluorescence (i.e. the scattered light) from this transition is detected in a photomultiplier tube. A second laser drives ion population from the $2s^2S_{1/2}$, $m_j = +1/2$ state to the $2p^2P_{3/2}$, $m_j = -1/2$ state where the ions decay with 2/3 probability to the $2s^2S_{1/2}$, $m_j = -1/2$ state causing a decrease in the observed fluorescence from the ions. The power of this "probe laser" is « 1 μW. Fig. 4 shows the resonance line shape when the probe laser is scanned through the transition. The probe laser is used to measure the shape of the plasma, its rotation frequency, density, number of ions, and temperature as described below.

Experimental Apparatus

The experimental apparatus is shown in Fig. 5. The cooling laser passes through a 50% beam splitter. Upon exiting the splitter one beam enters the plasma perpendicular to B and the other beam (diagonal cooling beam) enters between the ring and one endcap at a angle of 55 degrees with respect to B. The probe beam passes through a telescope which is used to precisely translate the beam spatially. Because the diagonal cooling beam scatters so much light from the steering mirrors it is chopped at 1 kHz and resonance fluorescence from the perpendicular cooling beam is detected only when the diagonal beam is off. The B field strength is 1.4 tesla. The vacuum in the Penning trap is approximately 10^{-8} Pa (133 Pa = 1 torr) allowing the ions to be trapped for many hours.

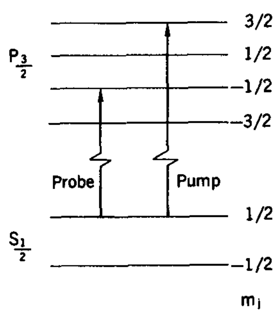

Fig. 3. The excitation scheme for the n=2 level of the $^9Be^+$ ion showing the laser cooling (pump) and depopulation (probe) transitions. Hyperfine structure has been neglected.

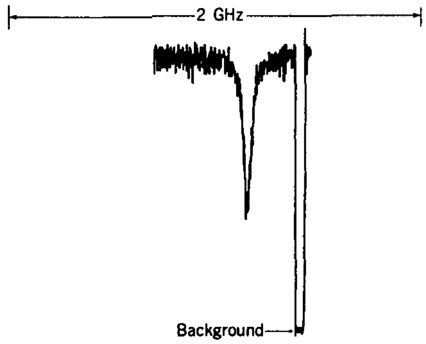

Fig. 4. The ion resonance fluorescence as a function of probe laser frequency. The bottom of the right most feature is the background signal.

Diagnostics

The depopulation signal is observable only when the probe beam intersects the plasma. This is used to determine the shape of the plasma. Spheroidal plasma shapes (with symmetry axis along z (Fig. 1)) with dimensions ranging from 200 μm to 500 μm were measured. A spheroid is the volume obtained when an ellipse is rotated about one of its axes.

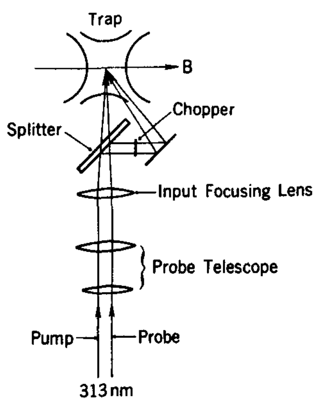

Fig. 5. The experimental apparatus for probing strongly coupled plasmas. The plasma is cooled and probed by lasers both perpendicular and at a 55 degree angle with respect to the B field.

The probe depopulation signal shifts in frequency as a function of the radial distance from the trap axis due to the Doppler shift caused by the rotation of the plasma. The rotation frequency

$$\omega/2\pi = (\Delta\nu/\Delta y)(\lambda/2\pi) \qquad (18)$$

is calculated from the frequency shift $\Delta\nu$ when the probe laser position is moved by an amount Δy. The density was determined from the zero temperature formula Eq. 6. The number of ions is given by the volume of the spheroidal plasma times the density.

The temperature of the plasma can be measured in directions both perpendicular and parallel to the magnetic field. The temperature in the perpendicular direction is measured by pointing the probe laser at the plasma in the direction perpendicular to the B field. This laser is scanned in frequency and the full width half maximum of the unsaturated depopulation transition is measured. The lineshape is a Voigt profile whose width is composed of the natural linewidth, the Doppler width, and a width due to the convolution of the laser spot size with the plasma rotation. The Doppler width can be deconvoluted from the Voigt width giving the temperature of the plasma. The probe laser can also be pointed at the plasma at an angle of 55 degrees with respect to the B field. The full width at half maximum of this resonance contains Doppler widths from temperatures in both perpendicular and parallel directions to the magnetic field. With the perpendicular temperature from the previous measurement the parallel temperature can be determined. Table 1 summarizes the measurements on seven ion clouds.

Table 1. The experimental data. The error convention is as follows - 1.8(10) = 1.8 ± 1.0. V_0 is the trap voltage in V, 2Z and 2R are the axial and radial extent of the plasma in μm, n is the density of the plasma in units of $10^7/cm^3$, $\omega/2\pi$ is the plasma rotation in kHz, $T_\parallel$ and $T_\perp$ are the parallel and perpendicular temperatures of the plasma in mK, and Γ is the coupling parameter for the plasma based on $T_\parallel$.

V_0	2Z	2R	n	#IONS	$\omega/2\pi$	$T_\perp$	$T_\parallel$	Γ
2	130(30)	450(30)	2.4(6)	330(170)	25(6)	2.3(5)	10(5)	80
2	150 .	450 .	1.9(5)	300(150)	20(5)	1.8(10)	7.4(70)	100
2	130 .	480 .	2.2(6)	350(170)	23(6)	.9(15)	8.9(60)	90
2	160 .	450 .	2.6(7)	450(230)	27(7)	2.4(10)	6(6)	130
2	160 .	260 .	2.8(7)	150(80)	30(7)	2.7(30)	2.4(60)	340
4	80 .	390 .	3.9(10)	250(120)	40(10)	2.9(20)	20(12)	50
1	190 .	360 .	1.2(3)	150(80)	13(3)	2.9(10)	4.7(60)	130

DISCUSSION OF THE DATA AND CONCLUSIONS

The plasmas are oblate spheroids of revolution. The Brillouin density at a field strength of 1.4 tesla is 5.9×10^8 ions/cm^3. The measured densities were typically 2×10^7 ions/cm^3. This is a factor of 30 less than the theoretical limit. The mechanism which limits the density is not well understood but it may have to do with axial asymmetries in the Penning trap [O'Neil 1980, Wineland et al. 1985].

The theoretical cooling limit discussed earlier is 0.5 mK. While the uncertainty in the temperature measurement is large enough to include this limit in some cases, the temperatures measured were consistently higher by about an order of magnitude. The 0.5 mK limit was derived for the case of a single ion in a trap. Recently it has been shown [Itano 1986] that for a cloud of ions, the temperature limit depends on the distance the cooling laser is from the center of the cloud, the rotation frequency, and the saturation of the cooling transition. These factors could account for the temperatures we measured. With some small changes in the way the laser cooling is done we should be able to reach the 0.5 mK limit.

The largest coupling parameter measured was $\Gamma \approx 340$. The uncertainty in this measurement was large due to the uncertainty in the temperature measurement, which in this case was 2.4(60) mK. This temperature uncertainty results in a range of values for Γ of 100 to a maximum of 2,000 due to the theoretical cooling limit. This coupling may be in the range where we would expect the plasma to be crystalline.

The lowest temperatures were measured on plasmas of several hundred ions. This can not be truly called a three dimensional plasma. Since surface effects in the ion clouds may be important in our experiment the results are probably best compared to a theory which is somewhere between a plasma theory and a theory for ion clusters.

BRAGG SCATTERING FROM A STRONGLY COUPLED PLASMA

Slatterly, Doolen, and DeWitt [Slatterly et al. 1980] have derived expressions for the pair distribution function and the static structure function. The static structure function is the spatial Fourier transform of the pair distribution function and is what one expects to see in the diffraction pattern resulting from the scattering of coherent light from the ions. For low coupling parameter Γ the function is fairly flat but for $\Gamma \sim 100$ one sees sharp peaks in the amplitude of the structure function due to short range order. It should be possible to measure $S(q)$ directly and compare this result to the calculations of Slatterly, Doolen and DeWitt.

An experimental apparatus to observe Bragg scattering which is currently under construction is shown in Fig. 6. Light from the cooling laser is scattered by the plasma and produces an interference pattern. This pattern is detected by a photon counting imaging tube.

For the densities we have measured the first interference fringe should occur at an angle of 0.6 degrees. We expect that the total count rate into the detector should be on the order of 100 counts/s. Therefore the suppression of background and scattered light into the detector will be of primary importance.

TAGGED ION DIFFUSION

A measurement of the ion diffusion may tell us whether the plasma is solid or liquid. Some experiments have observed crystallization in two dimensional [Grimes and Adams 1979] and solid state systems [Rosenbaum et al. 1985]. Wuerker et al. [Wuerker et al. 1959] observed crystallization of aluminum particles in a Paul trap. A possible technique for measuring ion diffusion in our experiment is as follows. The probe beam will be tuned on resonance and then pulsed on for a short period of time thereby "tagging" a group of ions. If the plasma is liquid, the tagged ions will diffuse between the spatially separated probe and cooling lasers and a depopulation signal will be observed at some time after the probe laser pulse. If the plasma is solid no tagged ion diffusion occurs and no signal will be observed.

IMPROVED TEMPERATURE MEASUREMENT

One difficulty with the present temperature measurement technique is that the natural linewidth of the probing transition ultimately limits the sensitivity of the measurement. Stimulated resonant Raman transitions, as for example studied by Thomas et al. [Thomas et al. 1982], avoid these difficulties. The natural linewidth of these nonlinear transitions is equal to the natural linewidth of the ground states which can be extremely small. If the angle between the two Raman beams is appropriately chosen the spectrum contains information about the velocity distribution of the plasma and is not affected by the upper state linewidth [Wineland 1984].

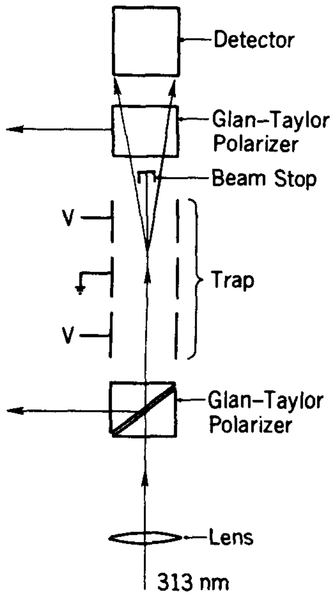

Fig. 6. The proposed apparatus for detecting the Bragg interference pattern. The probe beam is collinear with the B field along the symmetry axis of the trap. The Glan-Taylor polarizers are crossed to suppress light which does not come from the ions.

CONCLUSION

In this paper we have discussed the measurement of the temperature, density, rotation frequency, and shape of a $^{9}Be^{+}$ ion plasma. Temperatures as low as 2 mK were measured. This, along with a measured density of $3\cdot10^{7}$ cm^{-3} corresponds to a coupling parameter of $\Gamma = 340$. With an improved, highly axially symmetric trap operating at high magnetic fields we hope to be able to reach even lower temperatures and higher densities. This should result in values of the coupling parameter Γ that are much higher than the value predicted to observe crystallization in a one component plasma.

The technique of Bragg scattering resonant coherent laser light from the plasma makes possible a measurement of the static structure function. This measurement can be compared to the theoretically predicted value for a one component plasma. A measurement of the ion diffusion in the plasma may allow us to determine if the plasma is a liquid or a solid. Finally a measurement of the plasma velocity distribution using a stimulated Raman transition should improve the temperature measurement.

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