GLOBAL POSITIONING SYSTEM FOR ACCURATE TIME AND FREQUENCY TRANSFER AND FOR COST-EFFECTIVE CIVILIAN NAVIGATION

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ABSTRACT

This paper described some alternative applications of Global Positioning System (GPS) including a method for very accurate time transfer and for civilian position location much less expensively than the designed Department of Defense method. The first part of the paper discusses several time transfer techniques with emphasis on what we call the "common-view" approach, and the second part considers the system for position location. Both applications depend on the fact that accurate ephemerides are available for GPS and that GPS time is based on atomic clocks. It is assumed that the civilian or clear-access code (C/A) is available during the observation periods. The accuracy of time transfer over a few thousand km using the "common-view" approach is estimated at about 10 ns, and the accuracy of position location at about 100 m.

INTRODUCTION

This paper describes some alternative applications of GPS including a method for very accurate time transfer and for civilian position location which could, in principle, be done much less expensively less expensively than the designed DOD method. The first part of the paper discusses several time transfer techniques with emphasis on what we call the "common-view" approach, and the second part considers the system for position location. The primary difference between the two applications is that the time transfer technique requires the user to know his location, while the position location technique allows the user to determine his position (and clock offset from GPS system time, if he desires) by making observations of signals from several satellites; the usual scheme for determining position from signals emanating from several known locations. Both applications depend on the fact that accurate ephemerides are available for GPS and that GPS time is based on atomic clocks. It is assumed that the civilian or clear-access code (C/A) is available during the observation periods.

FOUR METHODS FOR ACCURATE TIME TRANSFER

There are four interesting methods to employ GPS for accurate time transfer or for accurate time and frequency comparisons (see Fig. 1).

First, Clock A and a GPS receiver are used to deduce from a GPS satellite's ephemeris, from clock A's location, and from received GPS time decoded from the same satellite, the time difference (Clock A - GPS time) (1). This method is the simplest and least accurate (estimated to be better than about 100 ns with respect to GPS time) (2), but has global coverage, is in the receive-only mode, requires no other data, yields receiver prices that could be competitive on a mass production basis, and could service an unlimited audience. Also, GPS time will be referred to UTC(USNO) and will be known with respect to UTC(BIH), UTC(NBS), and other major timing centers.

Second, Clock A and Clock B at different locations anywhere on earth can be compared by making successive observations of the same GPS satellite clock, at least one of which will appear above their horizons with delayed view times of less than 12 hours. This is analogous to the clock flyover mode reported by J. Besson (3) and others. The time prediction error for the satellite cesium clocks to be used in the GPS satellites will be about 5 ns over 12 hours. Since the same GPS satellite clock will be viewed by both A and B, biases in the satellite ephemeris may tend to cancel depending upon geometry, etc. Accuracies of from 10 ns to 50 ns are anticipated. This method requires communication of the data between A and B, and hence the logistics may limit the customers.

Third (see figures 1 & 2), two users with Clock A and Clock B at different locations, but in simultaneous common-view of a single GPS satellite clock, can take advantage of common mode cancellation of ephemeris errors in determining the time difference ($t_A - t_B$). The satellite clock error contributes nothing. Since the GPS satellites are at about 4.2 earth radii (12 hour orbits), for continental distances between A and B ($\lesssim$ 3000 km) the angle $\angle$ (A-Satellite-B) will be $\lesssim 10°$, and the effects of satellite ephemeris errors will be reduced by a factor of more than 10 over the first method. Using a fairly straightforward receiver system, an accuracy of about 10 ns in measuring the time difference ($t_A - t_B$) appears probable. This again requires data communication between A and B. With improved ephemerides and propagation delay characterization, the potential accuracy limit for this method appears to be about 1 ns. The receiver should be relatively inexpensive, and given the reasonable costs of data modems and the potential accuracies achievable via this method, it makes it very attractive and cost effective for national, and in some instances, for international time comparisons.

Fourth, a method being developed for Geodesy by JPL (Jet Propulsion Laboratory) (4) has baseline accuracy goals of about 2 cm over baselines of the order of 100 km. This method can be inverted to do time comparisons with subnanosecond accuracies. The two clocks A and B separated by about 100 km have two broadband receivers with tunable tracking antennae such that sequentially, 4 satellites can be tracked concurrently at A and B. The data are cross-correlated after the fact, the same as in long baseline interferometry, to determine location and time difference ($t_A - t_B$). The data density is higher than with the other approaches and the baselines are relatively short, but the accuracy is excellent.

It appears that as GPS becomes fully developed, GPS time may become operational world time. Methods 1, 2, or 3 above would yield significant improvements in national and international time comparisons. If commercial vendors take advantage of some of these methods, receiver costs could be made reasonable. The same basic receiver could be used in methods 1, 2, or 3; the main difference would be in the software support, modems, and local clocks. This method has the most attractive accuracy/cost ratio and is being pursued by NBS. The theoretical advantages and disadvantages are reported herein.

SYSTEM ERROR ANALYSIS

Errors Resulting from Satellite Ephemeris Location Uncertainty

The time transfer error is dependent upon the ephemeris or position error of a satellite. Common-view time transfer yields a great reduction in the effect of these errors between two stations, A and B, as compared to transfer of time from the satellite to the ground. Common-view time transfer is accomplished as follows:

1) Stations A and B receive a common signal from a satellite and each records the local time of arrival, t_A and t_B respectively.
2) From a knowledge of station and satellite position in a common coordinate system, the range between the satellite and each of the stations is computed, r_A and r_B respectively.
3) The time of transmission of the common signal according to each station, A and B, is computed by subtracting from the times of arrival, the times of propagation from the satellite to each of the respective stations, i.e., the time to travel the distances, r_A and r_B, are τ_A and τ_B (the range delays) and are given by $\tau_A = r_A/c$ and $\tau_B = r_B/c$ where c is the speed of light. This speed is subject to other corrections as are treated later.

4) Finally, the time difference, τ_{AB}, of station A's clock minus station B's clock at the times the signals arrived is:

$$\tau_{AB} = (t_A - \tau_A) - (t_B - \tau_B) = (t_A - t_B) - (\tau_A - \tau_B).$$

If the ephemeris of the satellite is off, the computed ranges from the stations to the satellite will be off an amount dependent on the way the ephemeris is wrong and the geometrical configuration of the satellite-station systems. The advantage of common-view time transfer is that the computed bias is affected not by range errors to individual stations, but by the difference of the two range errors. Thus, much of the ephemeris error cancels out. To see how this works in detail, suppose the ephemeris data implies range delays of τ_A' and τ_B', but the actual position of the satellite, if known correctly, would give range delays of $\tau_A = \tau_A' - \Delta\tau_A$ and $\tau_B = \tau_B' - \Delta\tau_B$. Then the error in time transfer would be $\Delta\tau_{AB} = \Delta\tau_B - \Delta\tau_A$, where $\tau_{AB} = \tau_{AB}' - \Delta\tau_{AB}$ is the true time difference (clock A - clock B) and where τ_{AB}' is the computed time difference from the actual time of arrival measurements and ephemeris data. Thus, $\Delta\tau_{AB}$, the time transfer error due to ephemeris error, depends not on the magnitude of the range errors, but on how much they differ.

The error in time transfer, $\Delta\tau_{AB}$, as mentioned above, depends on the locations of the two stations and of the satellite, as well as the orientation of the actual position error of the satellite. Figures 3 through 18 at the end of the paper give $\Delta\tau_{AB}$ for some ground stations of interest with different discrete levels of error shown as contour graphs dependent on where the satellite is. For each pair of ground stations, we have selected contour graphs from a possible set of four contour graphs for current and future typical ephemeris errors and for whether the satellite is going north or south in its orbital plane. Within a particular graph, the contour level at a point corresponds to the root-mean-square value of $\Delta\tau_{AB}$ when the common view satellite is directly above that location. The current values of ephemeris error for the GPS satellites are estimated at about 10 meters in-track, i.e., in the satellite's direction of motion; 7 meters cross-track, and 2 meters radial (5). This corresponds to 41.23 ns rms error (square root of the sum of squares/c). The projected values for 1985 are 7 m in-track, 3 m cross-track, and 0.6 m radial, corresponding to 25.46 ns rms error (5).

Notice that the rms errors make an elongated ellipsoid and are dependent on satellite direction. Thus, to compute the range errors to a given pair of stations for a given satellite location, one needs to know the satellite direction at that location. The satellite moves in a fixed plane in space with the earth rotating under it.

The program which computed the figures used an orbital plane making an angle of 63° with the ecliptic with the satellite moving west to east in the plane. As an approximation, the orbit was assumed circular at 4.2 earth radii (12 hour period). At a given latitude, the satellite direction in degrees east of north is determined by the orbital plane and whether the direction is northerly or southerly. Corrections for the earth's rotation need to be included. Thus, each figure was created by: 1) choosing a given pair of ground stations, a set of values for ephemeris error, and whether the satellite was moving north or south in its orbital plane; 2) for a given location on a map containing the ground stations, finding the satellite direction (a function of latitude only) and three independent position error vectors from the three different types of ephemeris error; and 3) approximating $\Delta\tau_{AB}$ for each of the independent position error vectors, then finding the square root of the sum of their squares for the total $\Delta\tau_{AB}$ at that location. In this way a chart of values of $\Delta\tau_{AB}$ was computed, which were then plotted in contour plots superimposed on a world map in cylindrical projection. Clearly, there are regions shown where the satellite will be below the horizon for one or both stations, so the maps are over-inclusive in this regard.

The $\Delta\tau_{AB}$ were approximated in the following way. Let us fix a coordinate system at the earth's center to define basis vectors. Then let $\underline{A}$ and $\underline{B}$ be the position vectors of stations A and B, repectively, and $\underline{S}$ the position vector of the satellite. Then the range vectors, pointing to the satellite from the ground stations, are:

$$\underline{R}_A = \underline{S} - \underline{A} \text{ and } \underline{R}_B = \underline{S} - \underline{B}.$$

Let $\hat{e}_A$ and $\hat{e}_B$ be the unit vectors in the directions of $\underline{R}_A$ and $\underline{R}_B$ respectively. Then the ranges are:

$$r_A = \hat{e}_A \cdot (\underline{S} - \underline{A}) \text{ and } r_B = \hat{e}_B \cdot (\underline{S} - \underline{B}).$$

If $\underline{S}$ is the satellite position according to its ephemeris, but the true position is $\underline{S} + \underline{\Delta S}$ then the new unit vectors, $\hat{e}'_A$ and $\hat{e}'_B$, are the same as the old to first order:

$$\hat{e}_A \cdot \hat{e}'_A = 1 - \frac{\alpha^2}{2} + \cdots = \cos(\alpha),$$

where α is the angle between $\hat{e}_A$ and $\hat{e}'_A$. So, to first order, the new ranges are:

$$r'_A = \hat{e}_A \cdot (\underline{S} + \underline{\Delta S} - \underline{A}) \quad r'_B = \hat{e}_B \cdot (\underline{S} + \underline{\Delta S} - \underline{B}).$$

Thus, the range errors are approximately:

$$\Delta r_A = r'_A - r_A = \hat{e}_A \cdot \Delta\underline{S} \text{ and}$$
$$\Delta r_B = r'_B - r_B = \hat{e}_B \cdot \Delta\underline{S}$$

so:

$$\Delta\tau_{AB} = (\Delta r_B - \Delta r_A)/c = \frac{1}{c}(\hat{e}_B - \hat{e}_A) \cdot \underline{\Delta S}.$$

We see that the time transfer error increases as the vectors pointing to the satellite from the ground stations become less parallel up to the maximum of $\sqrt{2}$ times the ephemeris error when they are perpendicular, down to zero when they are parallel. Because of the dot product, some interesting and very helpful situations may arise. For example, if the path of the satellite were at right angles to the line between stations A and B and were half-way in between the two stations, the effect of the ephemeris errors due to radial and on-track go to zero! Since the GPS satellites are so far out, 4.2 earth radii approximately, the direction vectors pointing to the satellite tend to be close to parallel, thus cancelling most of the ephemeris error in all cases where common-view is available.

Errors Resulting from Ionosphere

The ionospheric time delay is given by $\Delta t = (40.3/cf^2) \cdot \text{TEC}$ (seconds) where TEC is the total number of electrons, called the Total Electron Content, along the path from the transmitter to the receiver, c is the velocity of light in meters per second, and f is the carrier frequency in Hz. TEC is usually expressed as the number of electrons in a unit cross-section column of 1 square meter area along the path and ranges from 10^{16} electrons per meter squared to 10^{19} electrons per meter squared. At the 1.575 GHz C/A carrier frequency for the GPS satellite system and for a TEC of 10^{18} electrons per meter squared, one computes the delay of 54 ns which is possible for low latitude parts of the world. For these low latitudes and solar exposed regions of the world, time delays exceeding 100 ns are possible especially during periods of solar maximum. Clearly, the TEC parameter is of great importance in the GPS system. Shown in Fig. 19 is a reproduction of a figure taken from a paper by J. A. Klobuchar (6), this figure clearly shows during a solar maximum year, 1968, that the range of delays vary from about 5 to 40 ns, being maximum near the equator and near the noon path. This figure refers to the extra delay that would be encountered if the satellite was directly overhead. At lower elevation angles, the slant path through the ionosphere lengthens, which increases the ionospheric component of delay. If τ_0 is the total path delay when the satellite is overhead, then the path delay, τ, at other elevation angles, E, is given to a good approximation by

$$\tau = \tau_0 \, [\mathrm{CSC}[E^2 + 20.3]]^{\frac{1}{2}} \quad \text{(ref. (7))}.$$

Fig. 20 is also from Klobuchar's paper and shows the actual vertical electron content at Hamilton, MA looking towards the ATS-3 satellite for every day of the year, and here again one sees the variations from the order of 5 ns to 40 ns.

In studying these graphs, one observes two very important things: 1) the total delay at nighttime and/or high latitude is much smaller than at daytime, and 2) the correlation in absolute delay time covers much larger distances when one moves away from the equator and the vicinity of noon; the conclusion being that a significant amount of common-mode cancellation will occur through the ionosphere at short or long baselines between A and B if observations are made at either high latitudes and/or at nighttime. These cancellation effects, as can be seen from Fig. 20 over several thousand km, will cause errors of less than 5 ns. For short baselines less than 1000 km, this common-mode cancellation will cause errors of the order of or less than about 2 ns.

Clearly, this gives a definite direction as to how one should proceed using the common-view GPS time and frequency transfer technique proposed in this paper. Even though the total ionospheric delay may be very large at certain times and places, there are ways to pick and choose, which would allow one to get large amounts of common-mode cancellation and which would allow one to achieve with some care, time and frequency transfer accuracies approaching a nanosecond.

Beyond the common-mode cancellation, if one had access to the measurements of the total electron content, then clearly one could use the model to actually calculate the delay over the two paths of interest, or if the monitor stations for the TEC were nearby, given reasonable correlations from one monitor station to another, one could interpolate the TEC so that on an ongoing basis, the differential delay variations could be calculated again to the order of a nanosecond. Also, if one used both the L_1 and L_2 frequencies from the GPS satellite, the TEC could be calculated.

Errors Resulting From Troposphere

In transferring time between ground stations via common-view satellite, one records the time of arrival of the signal and computes the time of transmission by subtracting the propagation time. The propagation time is found by dividing the range to the satellite by the velocity of light. However, moisture and oxygen in the troposphere have an effect on the velocity of propagation of the signal, thus affecting the computed time of transmission and therefore, the time transfer. This effect is dependent on the geometry, the latitude, the pressure, and the temperature, and may vary in magnitude from 3 ns to 300 ns (7). However, by employing reasonable models and using high elevation angles, the uncertainties in the differential delay between two sites should be well below 10 ns. Later on, if needed, the magnitude of the troposphere delay can be calculated with uncertainties which will approach a nanosecond.

Relativistic Corrections

In navigational and time transfer systems where great accuracy is required, relativistic effects become important. The relativistic correction is comprised of three components: a correction for gravitational potential; another for clock motion; and finally, for earth rotation. In many instances, all three components must be considered. We refer the reader to ref. (8), which provides numerous examples in a variety of applications. However, in this paper, which focuses on the common-view time transfer technique and on passive reception of GPS signals for position location, only the last component is important --the correction for earth rotation. Figure (21) illustrates why this correction is needed.

Let $\underline{r}_A'$ be the position of the satellite at the instant of transmission and $\underline{r}_A$ be the earth station position at the same instant. The difference between τ and the time, τ', it would take for the signal to propagate if the ground station had not moved due to earth rotation is given approximately by

$$\tau - \tau' = -\omega \, \underline{r}'_A \cdot \hat{k} \times \underline{r}_A / c^2$$

where τ and τ' are as shown in figure 21, and where ω is the rotational angular velocity of the earth, c the speed of light, and $\hat{k}$ is a unit vector in the direction of the earth's rotation axis. If there were no earth rotation ($\omega = 0$), $\tau = \tau'$ as one would expect. As the equation shows, the magnitude of $\tau - \tau'$ depends upon the relative positions of the earth station, the satellite, and the direction of the axis of earth rotation. For GPS satellites with a nominal 12 hour circular orbit, if one assumes $\tau = \tau'$, then errors of nearly 100 ns for the "worst case" geometry could result.

This calculation can be reduced to a few simple operations. The satellite position vector, $\underline{r}'_A$, can be computed in earth-fixed coordinates from the GPS C/A data. The earth station position vector, $\underline{r}_A$, should be known. Then the distance between them at the instant of transmission is:

$$R = \| \underline{r}'_A - \underline{r}_A \| \text{ , so } \tau' = R/c.$$

The correction term can be simplified if we consider the vector identity:

$$\underline{r}'_A \cdot \hat{k} \times \underline{r}_A = \hat{k} \cdot (\underline{r}_A \times \underline{r}'_A) \quad .$$

The term on the right is simply the z-component of the $\underline{r}_A \times \underline{r}'_A$ cross product which equals:

$$r_{Ax} \, r'_{Ay} - r_{Ay} \, r'_{Ax} \quad .$$

Thus, we find:

$$\tau = \frac{\| \underline{r}'_A - \underline{r}_A \|}{c} - \frac{\omega}{c^2} \left(r_{Ax} \; r'_{Ay} - r_{Ay} \; r'_{Ax} \right) \quad .$$

For this, the values $\frac{1}{c}$, $\underline{r}_A$, and $\frac{\omega}{c^2}$ are already known. All we need compute is $\underline{r}'_A$ and perform the few indicated operations.

A way to calculate and to conceptualize this correction due to earth rotation (sometimes called the Sagnac effect) is the following. Consider a vector with its tail at the center of the earth and its head at the location of the GPS satellite at the time of transmission of a signal. Let the head of the vector follow the signal to its reception point--say at some ground station receiver. The vector in so doing will sweep out an area, A, which has a projection on the equatorial plane, A'. The correction is given by:

$$\tau = \tau' + 1.6 \times 10^{-6} \text{ ns/(km)}^2 \cdot A' \quad ,$$

where A' is measured in kilometers squared and is considered positive (negative) when the area swept is eastward (westward).

Error Consideration in Receiver Design

Since the primary goal of the NBS receiver design is accuracy in time and frequency transfer, the approach taken tends to be somewhat different than perhaps may be considered in a navigation receiver. The fundamental concern is that whatever time delay exists within the receiver be extremely stable (of the order of a nanosecond). This, of course, can be most easily accomplished if the total additional delay (beyond cables) is minimized through the receiver. We also are working toward minimum parts cost, while still providing full automation capability. In addition, we are designing into the current units being built by NBS, self contained microprocessor control and a (1 ns) time interval counter.

The total receiver will have high accuracy, is designed to be very stable, and will be totally automated and self-contained. This allows one to take maximum advantage of appropriate seeing time of the satellites, minimize ionospheric delay and delay variations, and maximize the common-mode cancellations between two sites. We estimate a total receiver delay, excluding cables, to be less than 30 ns and the receiver instability to be less than 2 ns. This stability assumes a 16 db antenna, which tracks the satellite, and a 1 Hz receiver loop bandwidth. Receivers can be straightforwardly calibrated in a side-by-side mode as to the differential delay, and since one uses the concept of common-mode between two sites, only the differential delay is important for accurate time and frequency transfer between sites A and B.

CURRENT AND FUTURE SYSTEM ACCURACY POTENTIAL AND SYSTEM COST

When all of the estimated errors from any of the potential error sources are combined, one obtains an absolute accuracy of time transfer of better than 10 ns, and a time stability of the order of a nanosecond. This means that on a 24 hour basis, one could measure absolute frequency differences between remote sites to a few parts in 10^{14}. We anticipate a front end parts and assembly cost (not including development costs) of well under $10,000. This includes the computer and automatic control system as well as a 1 ns time interval counter; but, of course, does not include the necessary testing, documentation, and costs incurred by a vendor if they are to develop and put into production such a system. The concept being developed has the significant advantage that the main costs will be front end costs as the system should not be labor-intensive after being set in operation. It also has the significant advantage over two-way satellite systems of operating in the receive only mode, which should allow a much larger user audience for this kind of receiver as well as avoiding all of the problems of clearances for operating a transmitter, which are necessary for a two-way satellite system. There have been some indications that, because of the excellent signal-to-noise on the C/A code, the signal strength would be degraded, so that adversary users would be denied the full accuracy of the system. From a time and frequency point of view, this may or may not be a serious problem depending on how and when all this is done. For example, if there was a degradation in signal-to-noise, one could simply do averaging since there is plenty of time to average over a pass, and in this case, one would still get comparable accuracy results.

The future accuracy potential is quite exciting because there is significant anticipated improvement in the accuracy of the ephemerides for the satellites, and that error contribution should be reduced considerably. The ionospheric delay can, in fact, be calibrated at or below the nanosecond level, and the tropospheric delay can also be modeled to a few nanoseconds. As we gain more experience with receiver design and total delay and delay stability, it is believed that its accuracy can also be improved to the nanosecond level or below. Ultimately, over the next several years this common-view approach could be developed with accuracies of the order of a nanosecond.

POSITION LOCATION

In the timing mode, it is assumed that the satellite and the user's geographic locations are known, so that range delay can be calculated. In the navigation mode, the user's position is determined by making observations of signals from several satellites.

If the user's clock is synchronized to GPS system time (or the equivalent if the offset is known), the range between user and a particular satellite can be determined directly by simply measuring the time it takes a signal to travel from the satellite, whose position is known, to the user. One such measurement puts the user on the surface of a sphere concentric with the satellite and with radius equal to the range to the satellite. Three such measurements to three different satellites determine the user's position as being at the point of common intersection of the three spheres.

If the user does not know GPS system time, then his position, as well as the relation of his clock to system time, can be determined from observations of signals from four satellites. In mathematical terms, there are four unknowns (the user's location in space (x, y, and z) and the unknown time offset) and four equations which relate the user's position, the satellite positions, and the clock offset, so that x, y, z, and t can be determined uniquely.

Ideally, one would like to make all measurements simultaneously. However, this, in effect, requires as many receiving and processing channels as there are satellites to be measured. The alternative is make measurements serially in time with just one receiver. However, this requires that the several measurements be "coherent" in time. As a simple illustration, consider a user at locations x, y, and z and two satellites , S_1 and S_2, at x_{S1}, y_{S1}, z_{S1} and x_{S2}, y_{S2}, z_{S2} (see figure 21). Suppose the user first measures a signal from S_1 at time t_1. If the user clock and GPS system time are synchronous, then, as we stated previously, the range to the satellite can be determined directly by simply measuring the time it takes the satellite signal to reach the user. However, in the general case, there will be some unknown offset, t, between system time and user time. Thus, what is measured is not the range, but what is called the "pseudo-range," which contains a term expressing the error due to clock offset. For the coordinates defined above, the pseudo-range, r_1, to S_1 is:

$$r_1 = [(x_{S1} - x)^2 + (y_{S1} - y)^2 + (z_{S1} - z)^2]^{\frac{1}{2}} + ct \tag{1}$$

where c is signal speed.

Suppose now the pseudo-range to S_2 is measured at time t_2, later than t_1. Since no clocks are perfect, the clock offset will change by an amount, Δt, over the interval $t_2 - t_1$, so that the pseudo-range r_2 to S_2 is

$$r_2 = [(x_{S2} - x)^2 + (y_{S2} - y)^2 + (z_{S2} - z)^2]^{\frac{1}{2}} + c(t + \Delta t).$$

More generally, the pseudo-range, r_n, to the nth satellite is

$$r_n = [(y_{Sn} - x)^2 + (y_{Sn} - y)^2 + (z_{Sn} - z)^2]^{\frac{1}{2}} c(t + \Delta t_1 + \Delta t_2) \cdots$$

where Δt_{n-1}, $\ell = 1,2,3, \cdots n-1$ represent the clock offset change between successive measurements. If the Δt_i are unknowns, then there are always more unknowns than there are measurments, and the user's location cannot be determined. However, a good deal is known about the clock performance, and it is usually possible to express the amount of clock offset change between two measurements.

There is much in the literature on clock performance, so we will not pursue it here, except to say that most models include terms for systematic and random effects. Thus, formally there may be some initial fractional frequency offset, which changes systematically in time along with the random components.

With this model then, we can express time error accumulation, T(t), in the form

$$T(t) = R_o \cdot t + \tfrac{1}{2}Dt^2 + \varepsilon(t) \tag{2}$$

where $\varepsilon(t)$ is the random component, D is a coefficient which defines the systematic frequency drift, and R_o is the initial frequency offset (9). An expression of this type, coupled with the pseudo-range measurements, provides enough information to solve for the user's position and initial clock offset.

As a practical matter, however, clock errors accumulated between measurements may be small enough to ignore, when compared to other error sources. The receiver described in this paper requires at most 10 minutes to "acquire" a GPS signal and measure the pseudo-range to 4 GPS satellites. If we assume that the frequency offset, R_o, and systematic frequency drift, D, are known, then the only term of concern is $\varepsilon(t)$; the random component. With typical commercial frequency standards available today (both rubidium and crystal), the amount of accumulated error for a period of 10 minutes is less than a nanosecond, considerably under the uncorrected ionospheric and tropospheric errors. If careful corrections are made for these effects, then clock error accumulation will have a magnitude which is comparable to the corrected atmospheric delay effects as discussed earlier.

As we pointed out, the system was primarily designed for common-mode time transfer between fixed locations. Thus, the emphasis was on receiver stability and not on absolute receiver delay. However, with measurements to four satellites where only a position fix is required, it is not necessary to know absolute receiver delay. Receiver delay represents a common bias for all measurements, so that the term (ct) in equation 1 could be rewritten in the form, $c(t + t_R)$, where, as before, t is clock offset and t_R is receiver delay. Now, when the equations relating the four measurements are simultaneously solved, the quantities obtained are x, y, z, and $c(t + t_R)$. Of course, if one wants to know clock offset t, it is necessary to know t_R. With the present system, it is estimated that t_R is of the the order of 30 ns and can be measured with a precision of ±5 ns.

In many position location problems, it may not be practical to use a tracking antenna of the kind we envisioned for the time transfer application. First, the user may not know his position well enough to point the antenna to the satellite, and second, such an antenna may be unwieldy for mobile applications. If we assume an omnidirectional, 0 db gain antenna, then the signal stability will be degraded from 2 ns to about 30 ns, which is of the same order of delay error introduced by the tropospheric and ionospheric delay.

When the satellites are used in the time transfer mode, range error translates directly into time transfer error, while in the navigational mode, the geometric arrangement of the satellites and the user must be taken into account. The impact of geometry on position fix accuracy has been studied extensively (see Ref. 10 for a good summary) and is usually referred to as "geometric dilution of precision" (GDOP). The position fix error can be expressed as the product of GDOP and the satellite range error (assuming it is the same to all satellites). Small values of GDOP indicate a good geometrical arrangement of the satellites, while large values indicate poor geometrical arrangements. Most users, after all 18 satellites have been launched, should be able to select four satellites such that the GDOP never exceeds 4. Thus, if the rms time error is 3 ns (about 1 meter) and GDOP is 4, then the 1 σ radial error in three-dimensional space is 1 meter x 4 = 4 meters.

The degree to which the user will want to correct for the various sources of error will depend upon the position fix accuracy required. Let's consider what might be termed a "worst case" example. Normally, if satellites below 10 degrees elevation are not observed, ionospheric delay will not exceed 90 ns and tropospheric delay will be under 80 ns. Assuming 0 db antenna gain and a 1 Hz loop bandwidth, the signal instability is about 30 ns, as we stated earlier. We must also consider the error due to the Sagnac effect. As we saw in the previous discussion, it depends upon the geometry of the user and the array of satellites he is observing. Further, the correction may be negative or positive for a particular satellite. For the sake of our illustration, we will assume that the error due to the Sagnac effect for all the observations produces a composite error of 100 ns. The total error due to all of these sources is 300 ns or about 100 meters. If we assume a "worst case" GDOP of 4, then the position fix error in three-dimensional space is approximately 400 meters. Considering the nominal cost of the equipment, this is quite a satisfactory result and is adequate for many applications. Taking the Sagnac effect into account and with even rather crude modeling of the ionospheric and tropospheric delays, the position fix can probably be improved by at least a factor of 3. Of course, at a fixed location, it is possible to average many measurements over a long period of time, so that modeling errors become the limiting factor--not signal instability.

In the discussion so far, we have assumed that the user is fixed in space--which may or may not be the case. If the user movement is small over the observation time compared with the position fix accuracy required, then the movement can be ignored, just as we ignored accumulating clock error over sufficiently short observation periods. However, if the motion cannot be ignored within the required position fix accuracy, then extra information is required to make the observations spatially coherent, just as we needed a model for clock error (Eq. 2) to provide time coherence when the observations are not sufficiently close in time. If the user's motion is constrained to the surface of the earth (boats, etc.), then local heading and speed provide the necessary information. Of course, if the user is moving in three-dimensional space, an extra "dimension" of information is needed. As a final point and as we stated earlier, there has been some discussion that the C/A signal-to-noise might be degraded. As in the time transfer case, this presents no problem for determining the location of fixed positions. However, for moving vehicles, this could be a serious problem, since there would not be sufficient time to average the signal to obtain the desired position fix accuracy.

CONCLUSIONS

In conclusion, we have shown that one-way satellite transmission from a GPS satellite in common-view at two sites allows one to do accurate time transfer to 10 ns or better. This accuracy is achieved because of common-mode cancellations of several contributing errors in the system. The system furthermore has the potential to achieve accurate time transfer of the order of a nanosecond. The estimated stability of the receiver delays and all contributing error delays should yield stabilities of the order of 1 ns, which means that on a 24 hour basis, frequency transfer can occur with an accuracy of the order of a part in 10^{14}.

We have also shown how the receiver for time transfer could be used for position fixing. Even with "worse case" geometry and no corrections for ionospheric, tropospheric, and receiver delay, it appears as though the three-dimensional error would not exceed 400 m. With crude modeling, it should be possible to reduce the error to about 100 m. With averaging at a fixed location, even better accuracy can be obtained. The present receiver is software controlled and there is sufficient unused memory to incorporate the necessary mathematical steps to solve for the user's location (and clock offset from GPS system time, if desired). Two prototypes are being built at the National Bureau of Standards to test these ideas. If these prototypes work as planned, the above approach would provide a very cost-effective navigation approach available also to the civilian sector.

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5. Private Communication, Albert Bierman, Aerospace Corp.
6. Klobuchar, J. A. (1978), "Ionospheric effects on Satellite Navigation and Air Traffic Control Systems," NATO AGARD proceedings, Lecture Series No. 93, Recent Advances in Radio and Optical Propagation for Modem Communication, Navigation, and Detection Systems.
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8. Ashby, Neil and Allan, David W., Practical Implications of Relativity for Global Coordinate Time Scale," Radio Science 14:2, 649-669 July-August 1979.
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10. Milliken, R. J. and Zoller, C. J., "Principle of Operation of NAVSTAR and System Characteristics," Navigation 25:2, 95-106 Summer 1978.

FIGURE CAPTIONS

Figure 1. Four methods of time transfer and their approximate accuracies using GPS:

Upper left, using data from the satellite to find GPS time and comparing a local clock with the GPS time scale.

Upper right, using one satellite to decode GPS time at two different locations and times to compare both clocks with the GPS time scale and hence with each other.

Lower left, measuring the time of arrival of a common signal from a satellite at two locations to compare the computed time of transmission according to the two clocks and thus compare the clocks.

Lower right, recording signals from four satellites at two stations to determine locations and time differences.

Figure 2. Time transfer via a satellite in common view of two ground stations indicating that fairly large errors (100 m = 333 ns radial error or 10 m = 33 ns in-track or cross-track error) in satellite ephemeris can cancel to a few ns time transfer error.

Figures 3-18. Contour graphs of the error in common-view time transfer for various choices of ground stations, satellite direction, and ephemeris error. The odd-numbered figures use current ephemeris error estimates: 10 m in-track, 7 m cross-track, and 2 m radial corresponding to 41.23 ns rms (square root of the sum of the squares divided by the speed of light). The even-numbered figures use error values projected for 1985: 7 m in-track, 3 m cross-track, and 0.6 m radial corresponding to 25.46 ns rms. The satellite direction is always northerly in the "a" figures and southerly in the "b" figures. The ground station locations are marked with an "x". The contours in a given figure are spaced for equal error values with error increasing as one goes from dotted to dashed to solid to dotted lines. Figures 3a, 3b, 4a, and 4b are examples of all four combinations; the odd numbered "a" figures and the even numbered "b" are deleted thereafter because their contour may be inferred from studying Figures 3a, 3b, 4a, and 4b along with the station combination of interest.

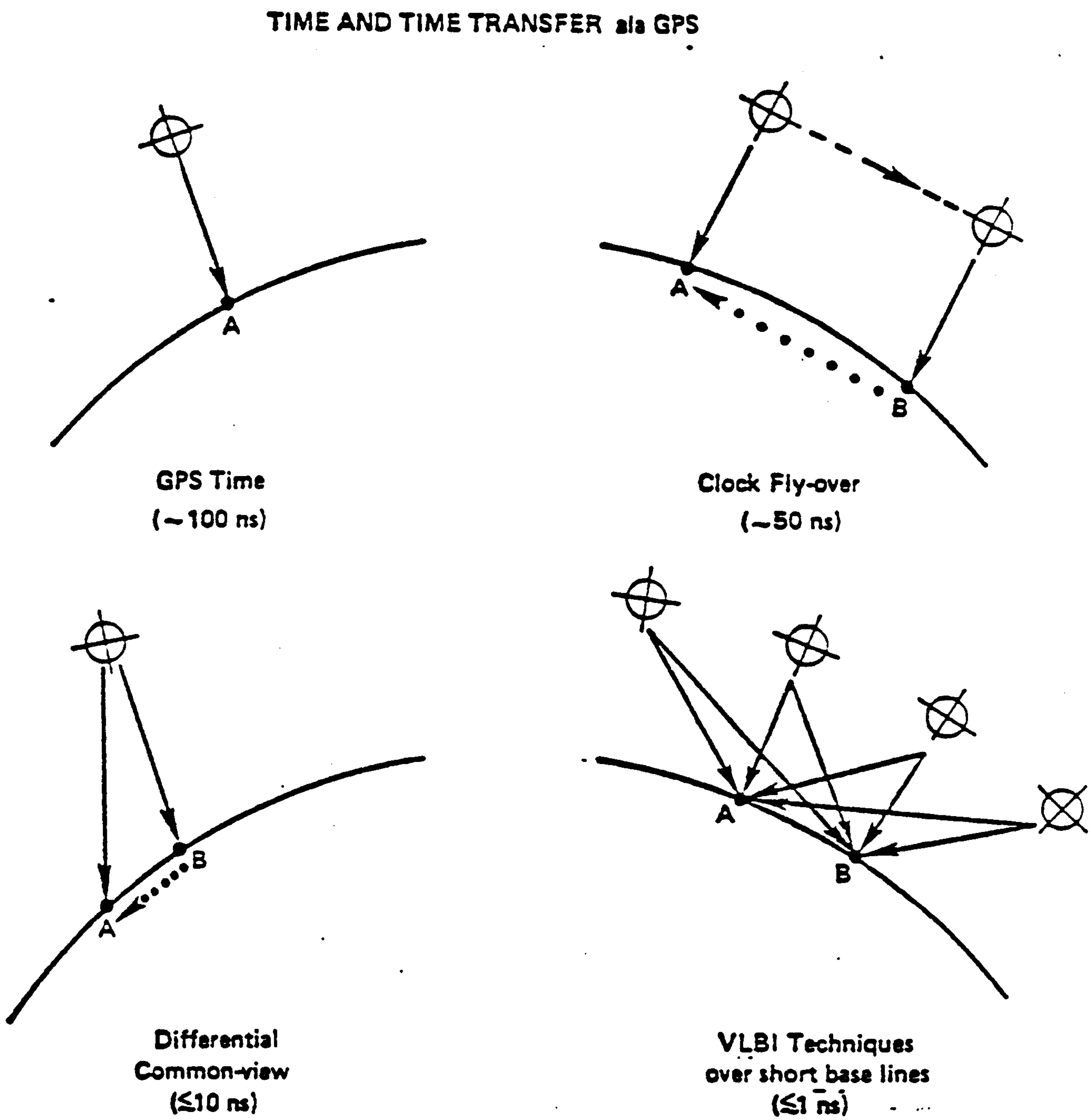

TIME AND TIME TRANSFER ala GPS
A
GPS Time
(~100 ns)
A
B
Clock Fly-over
(~50 ns)
B
A
Differential
Common-view
(≤10 ns)
A
B
VLBI Techniques
over short base lines
(≤1 ns)

Figure 1.

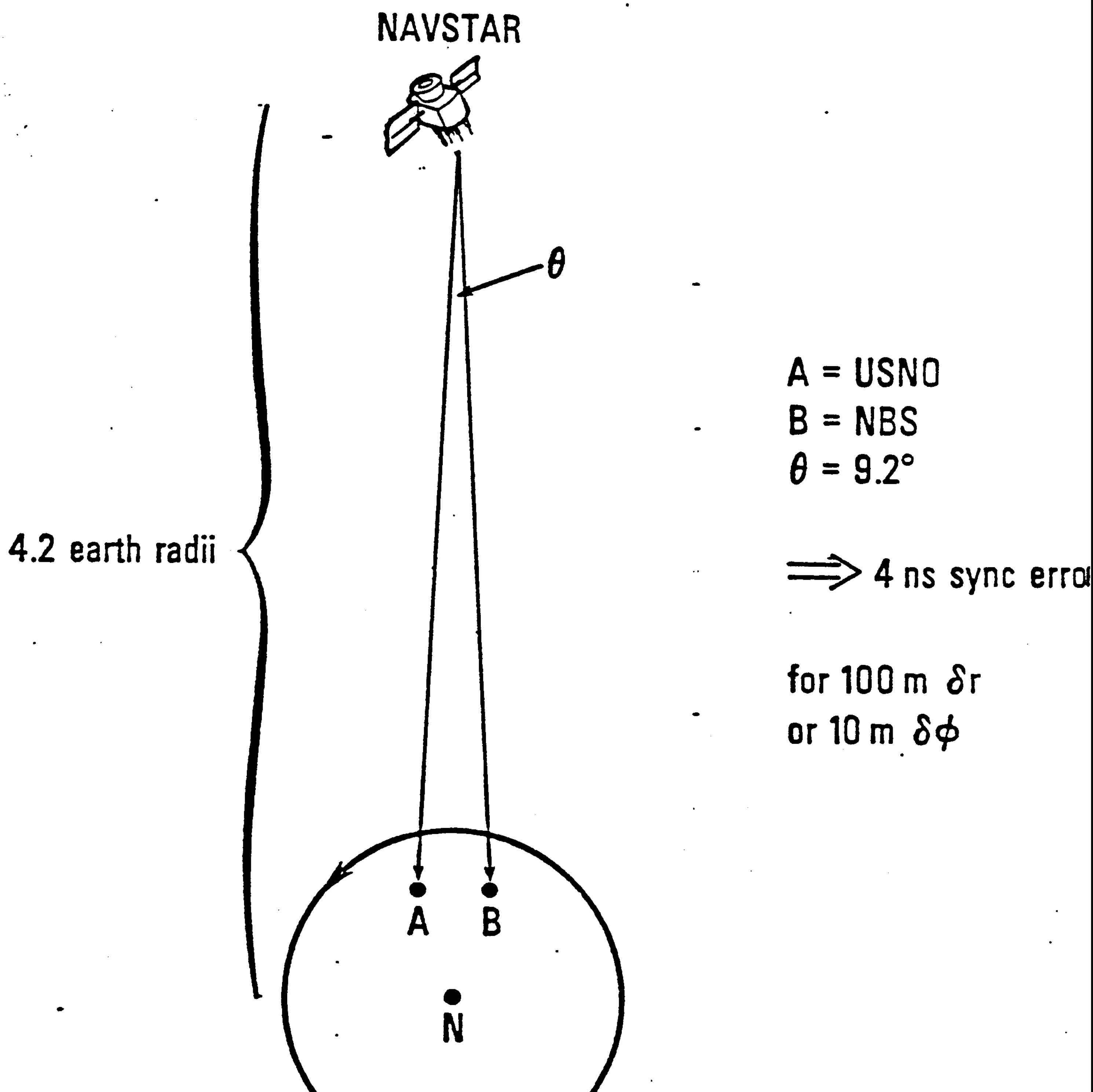

NAVSTAR
θ
A = USNO
B = NBS
θ = 9.2°
⟹ 4 ns sync erro
for 100 m δr
or 10 m δφ
4.2 earth radii
A
B
N

Figure 2.

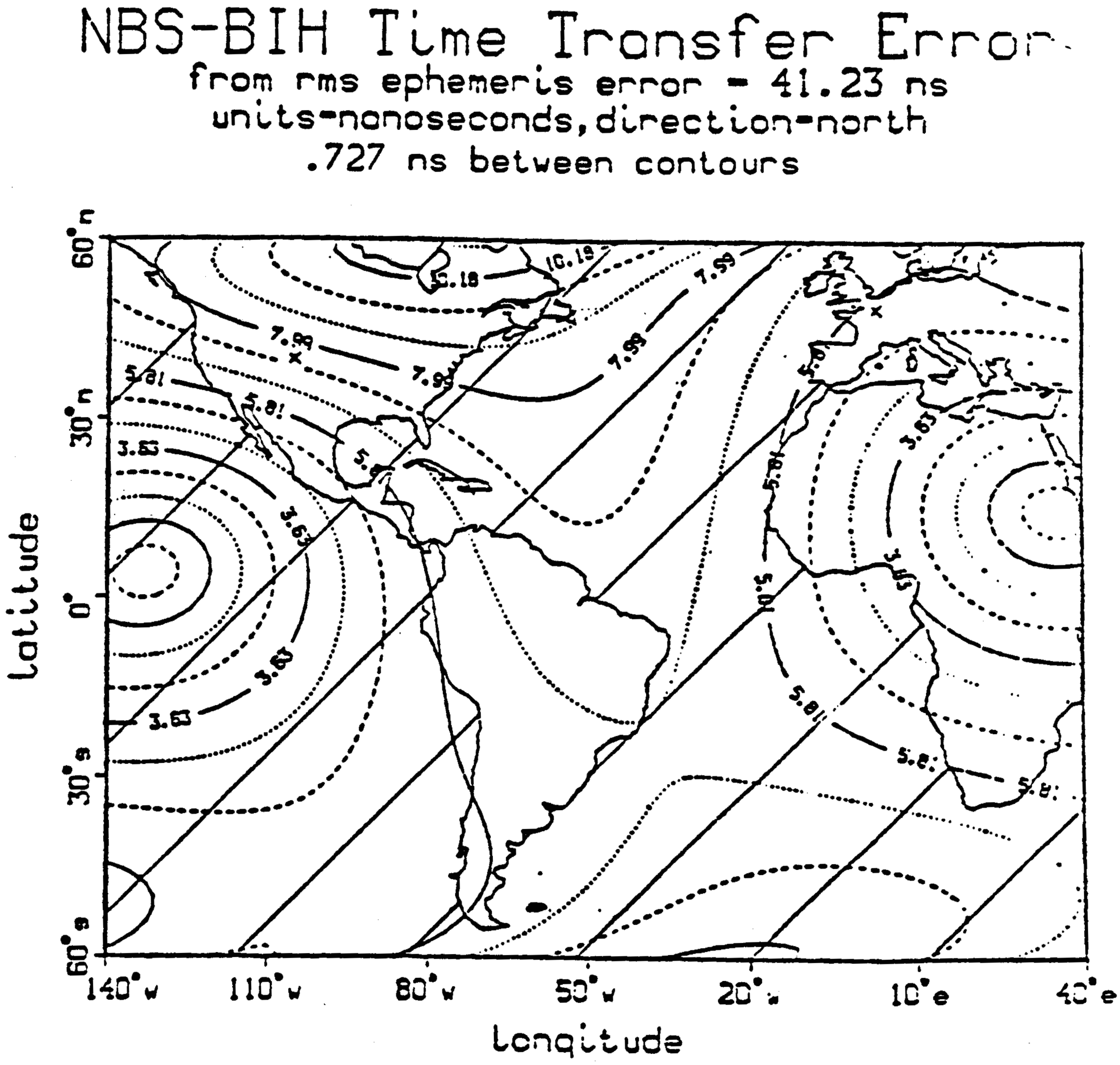

NBS-BIH Time Transfer Error
from rms ephemeris error = 41.23 ns
units=nanoseconds, direction=north
.727 ns between contours
Latitude
60° n
30° n
0°
30° s
60° s
140° w
110° w
80° w
50° w
20° w
10° e
40° e
Longitude
10.19
7.99
5.81
3.63

Figure 3a.

NBS-BIH Time Transfer Error
from rms ephemeris error = 41.23 ns
units=nanoseconds, direction=south

.685 ns between contours

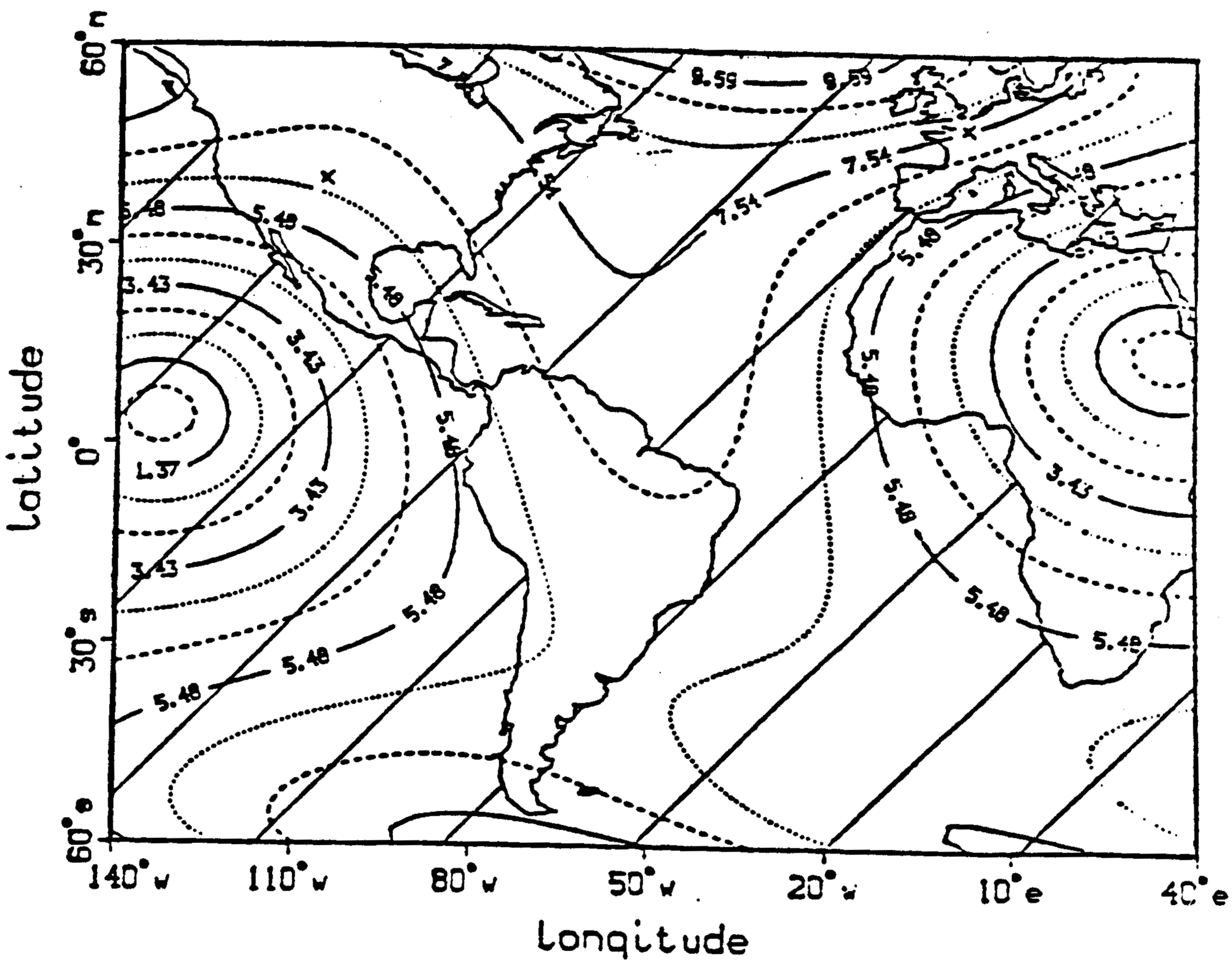


Figure 3b.

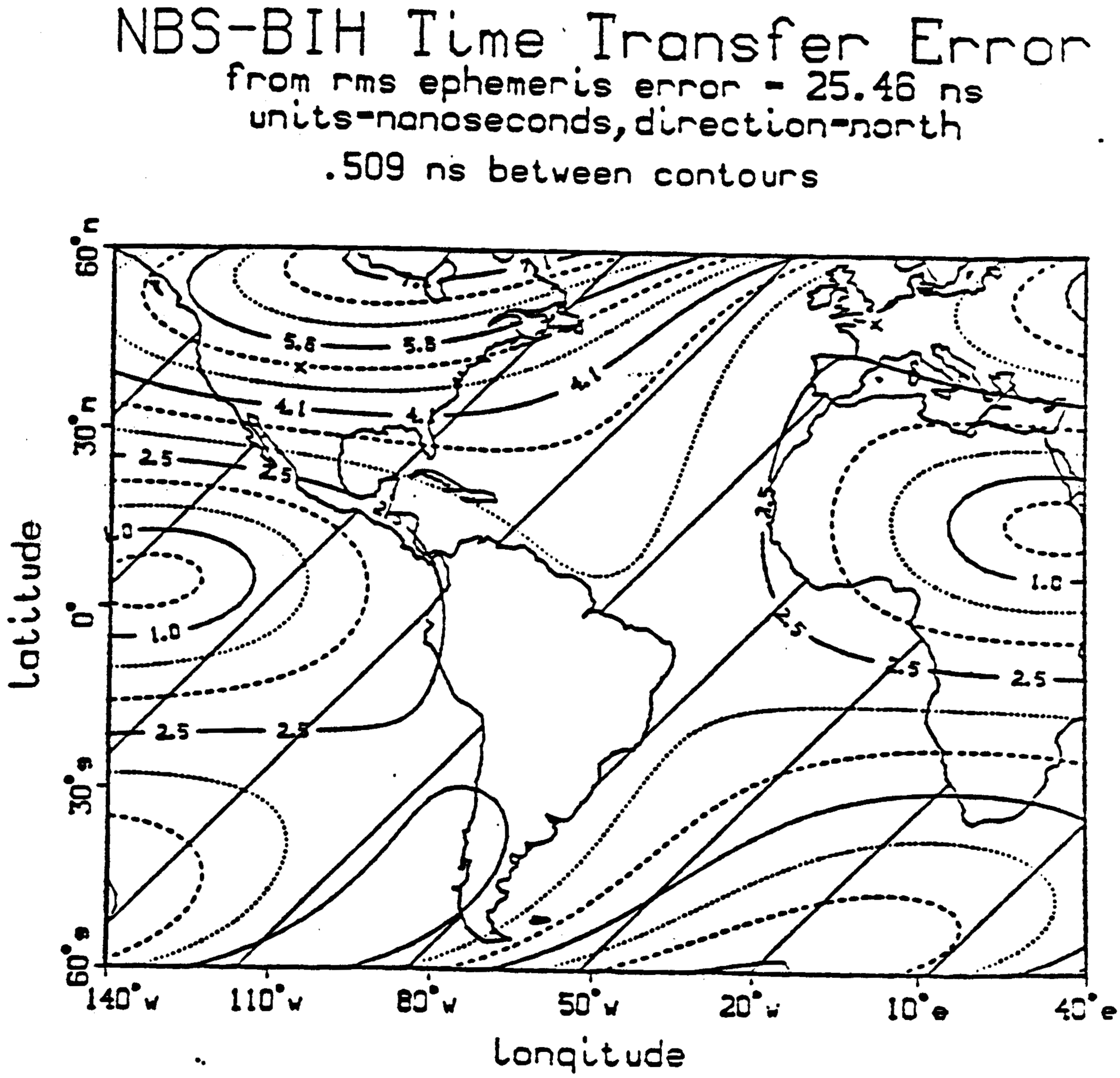

NBS-BIH Time Transfer Error
from rms ephemeris error = 25.46 ns
units=nanoseconds, direction=north
.509 ns between contours
5.8
4.1
2.5
1.0
60°n
30°n
0°
30°s
60°s
Latitude
140°w
110°w
80°w
50°w
20°w
10°e
40°e
Longitude

Figure 4a.

NBS-BIH Time Transfer Error
from rms ephemeris error = 25.46 ns
units=nanoseconds, direction=south
.473 ns between contours

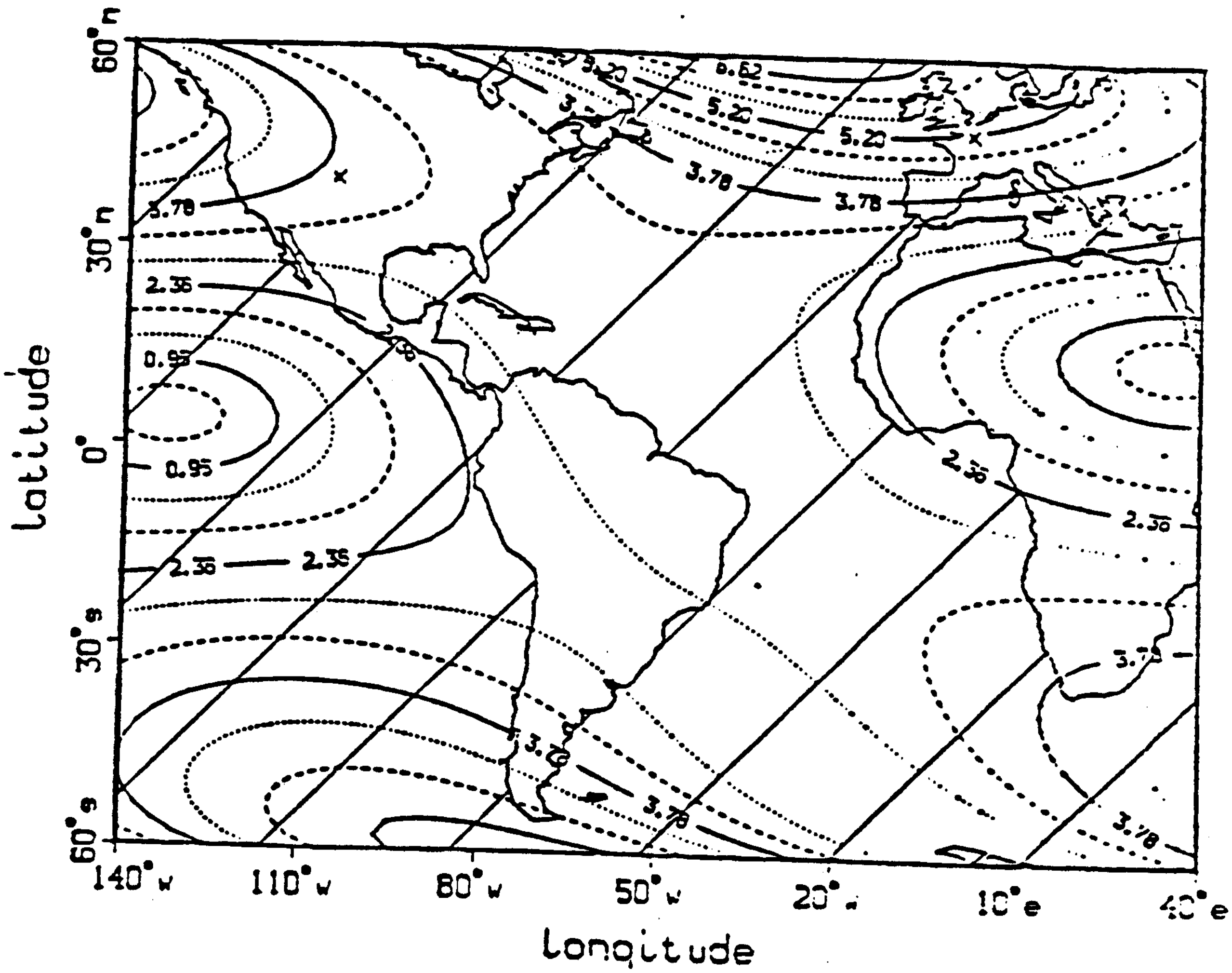


Figure 4b.

NBS-NRC Time Transfer Error

from rms ephemeris error = 41.23 ns

units=nanoseconds, direction=south

.200 ns between contours

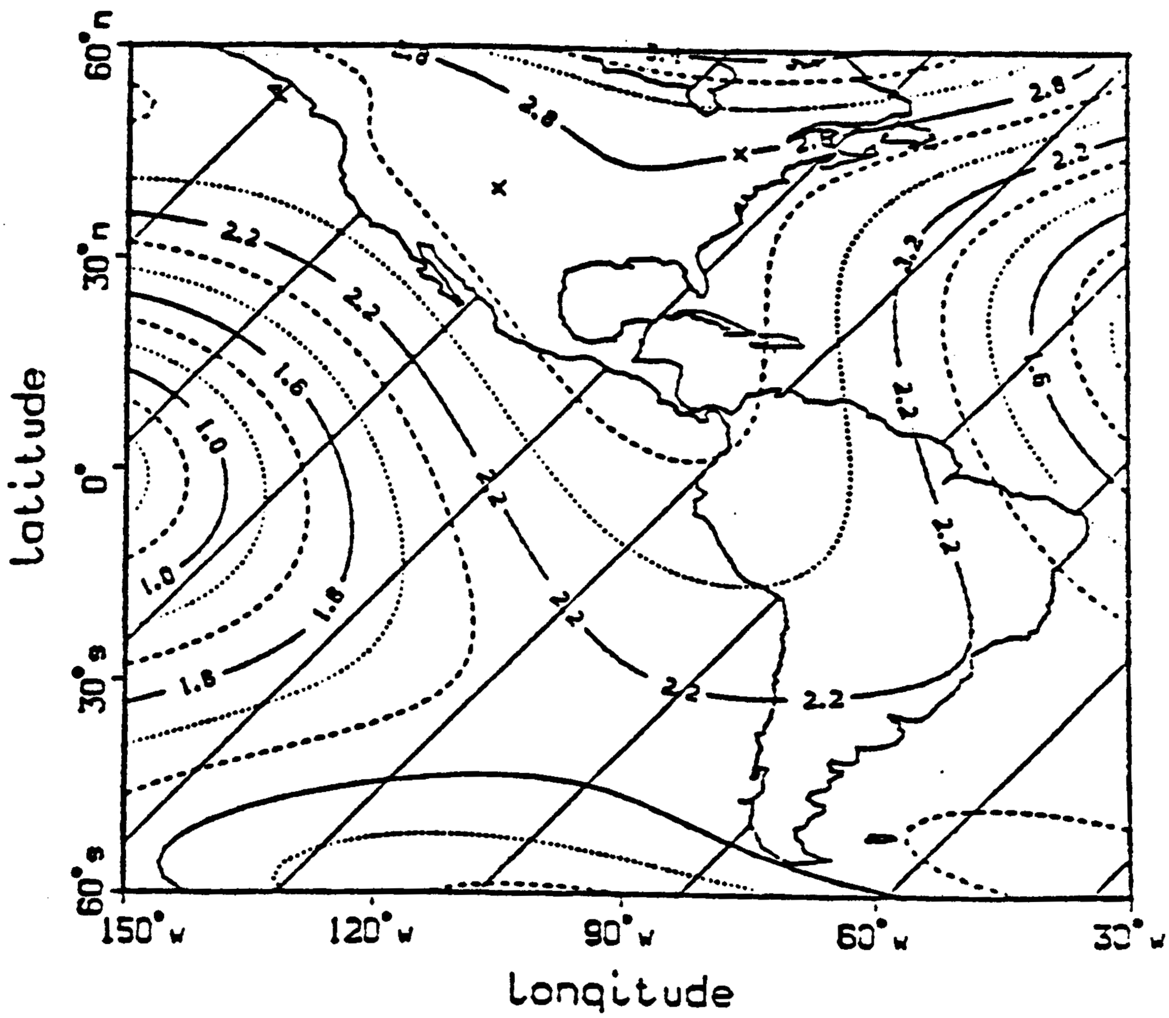
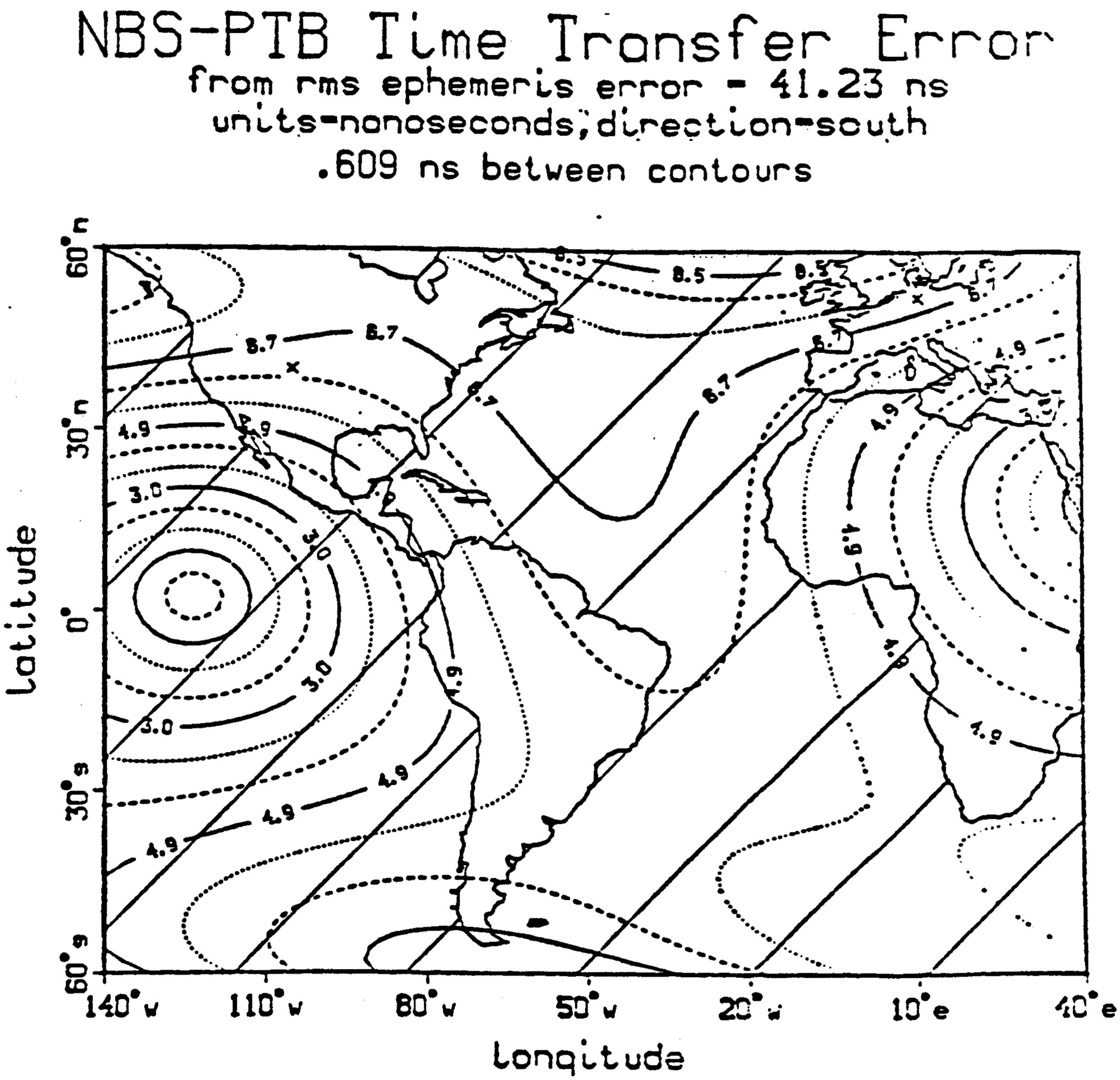


Figure 5b.


NBS-PTB Time Transfer Error
from rms ephemeris error = 41.23 ns
units=nanoseconds; direction=south
.609 ns between contours
Latitude
60° n
30° n
0°
30° s
60° s
140° w
110° w
80° w
50° w
20° w
10° e
40° e
Longitude
8.5
8.7
4.9
3.0
Figure 7b.

NBS-NRC Time Transfer Error

from rms ephemeris error = 25.46 ns
units=nanoseconds, direction=north

.167 ns between contours

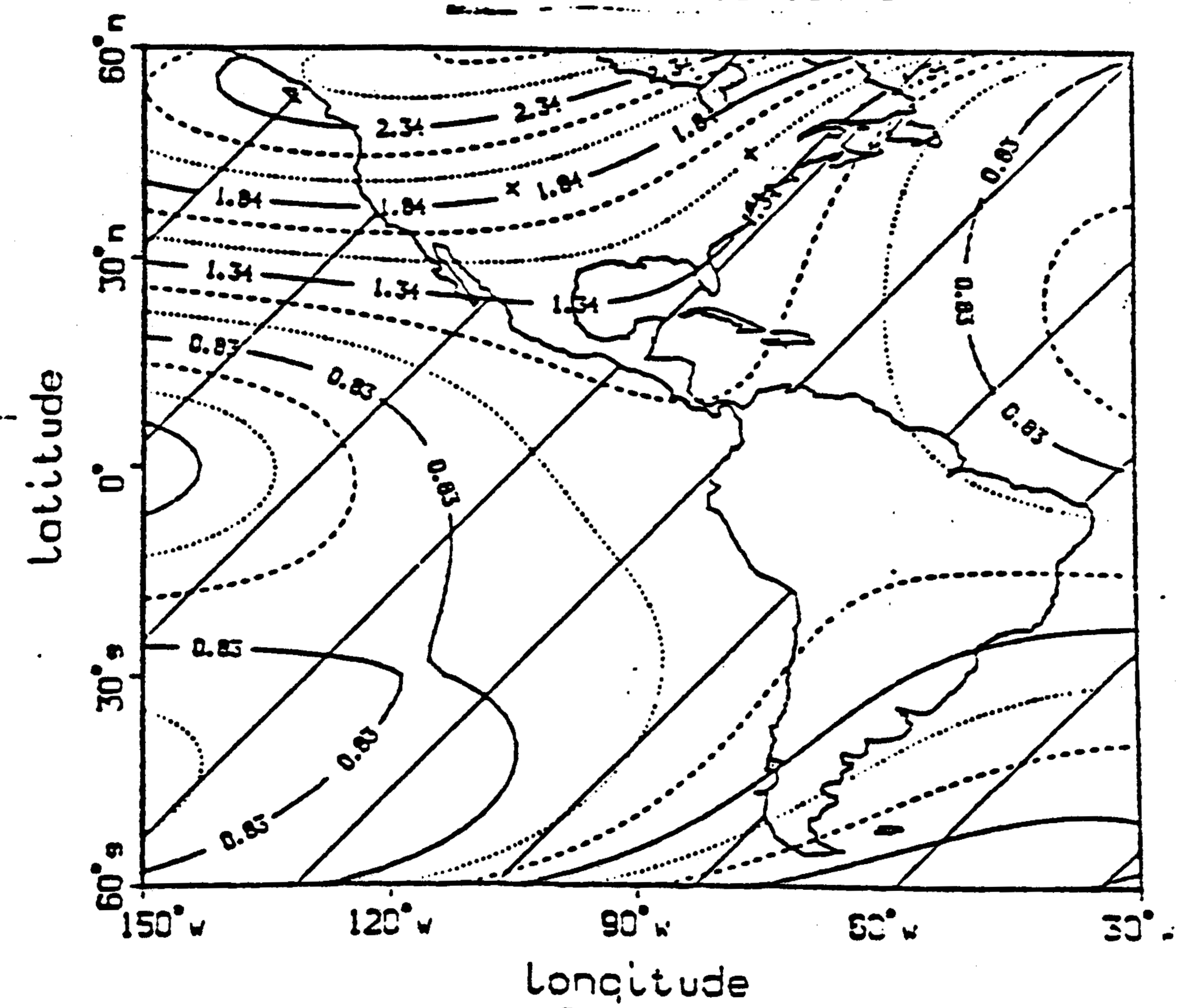


Figure 6a.

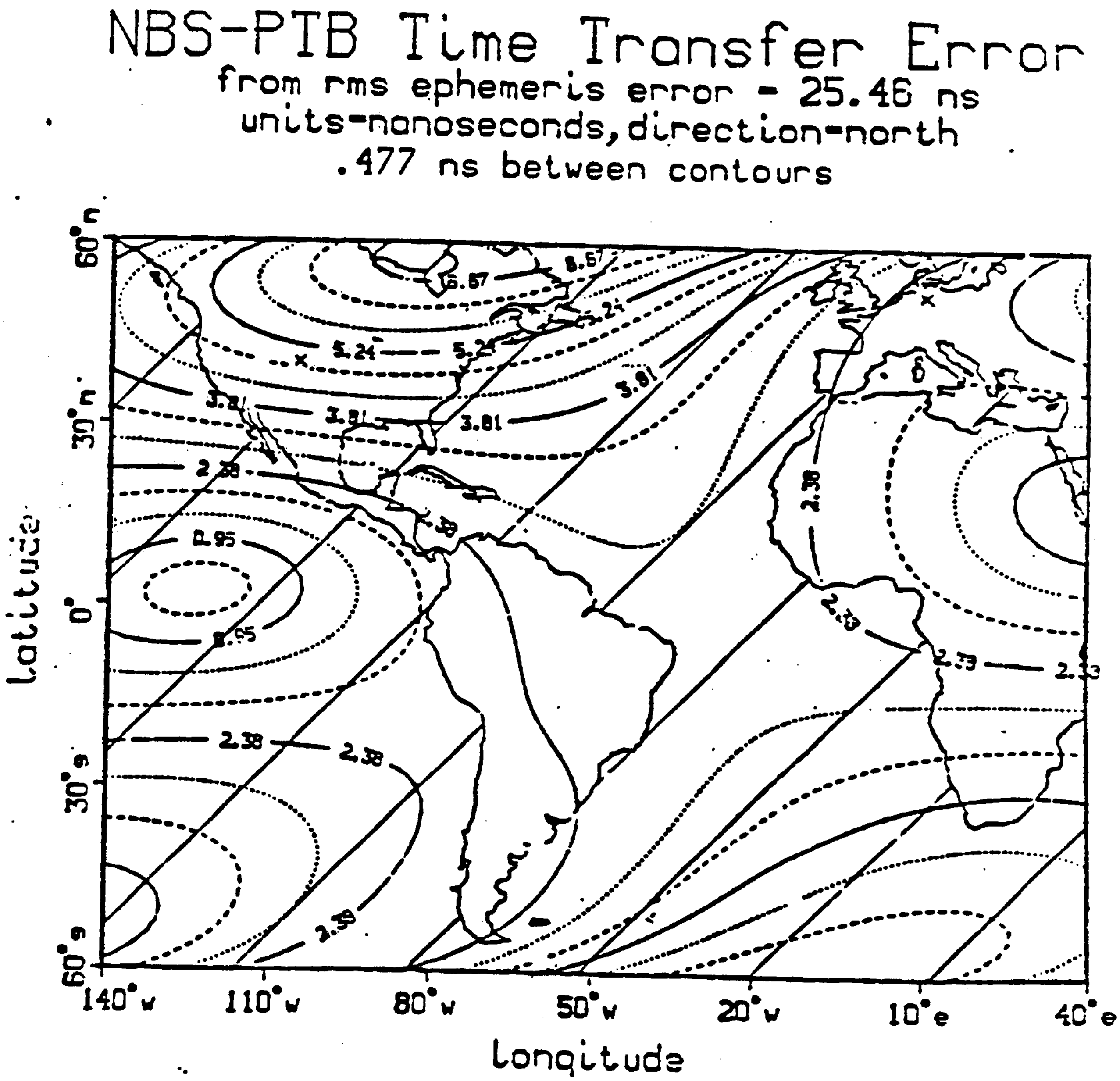

NBS-PTB Time Transfer Error
from rms ephemeris error = 25.46 ns
units=nanoseconds, direction=north
.477 ns between contours
Latitude
60° n
30° n
0°
30° s
60° s
140° w
110° w
80° w
50° w
20° w
10° e
40° e
Longitude
6.67
5.24
3.81
2.38
0.95
2.33

Figure 8a.

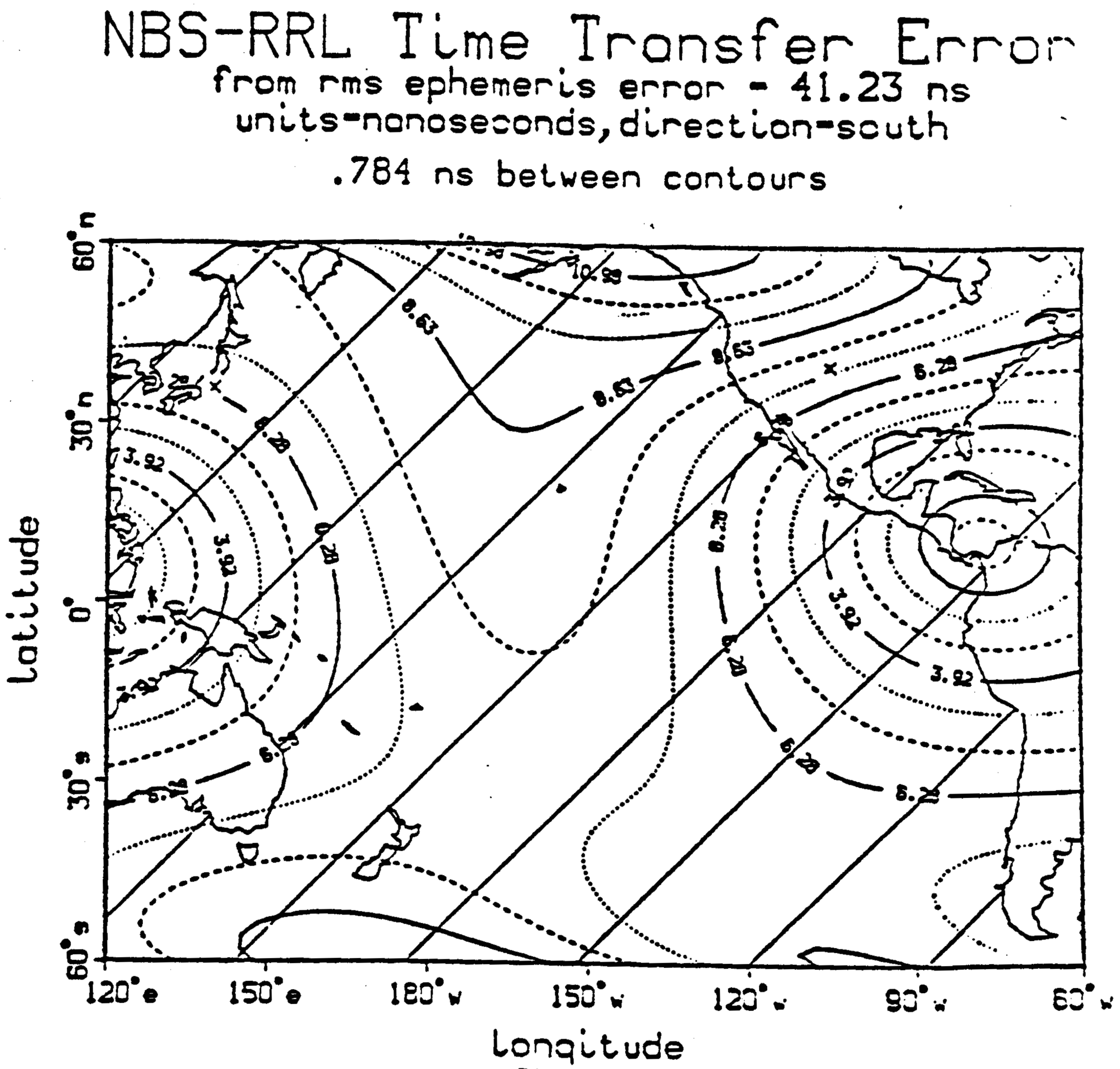

NBS-RRL Time Transfer Error
from rms ephemeris error = 41.23 ns
units=nanoseconds,direction=south
.784 ns between contours
60°n
30°n
0°
30°s
60°s
Latitude
120°e
150°e
180°w
150°w
120°w
90°w
60°w
Longitude
10.98
8.63
6.28
3.92

Figure 9b.

NBS-RRL Time Transfer Error

from rms ephemeris error = 25.46 ns
units=nanoseconds, direction=north

.561 ns between contours

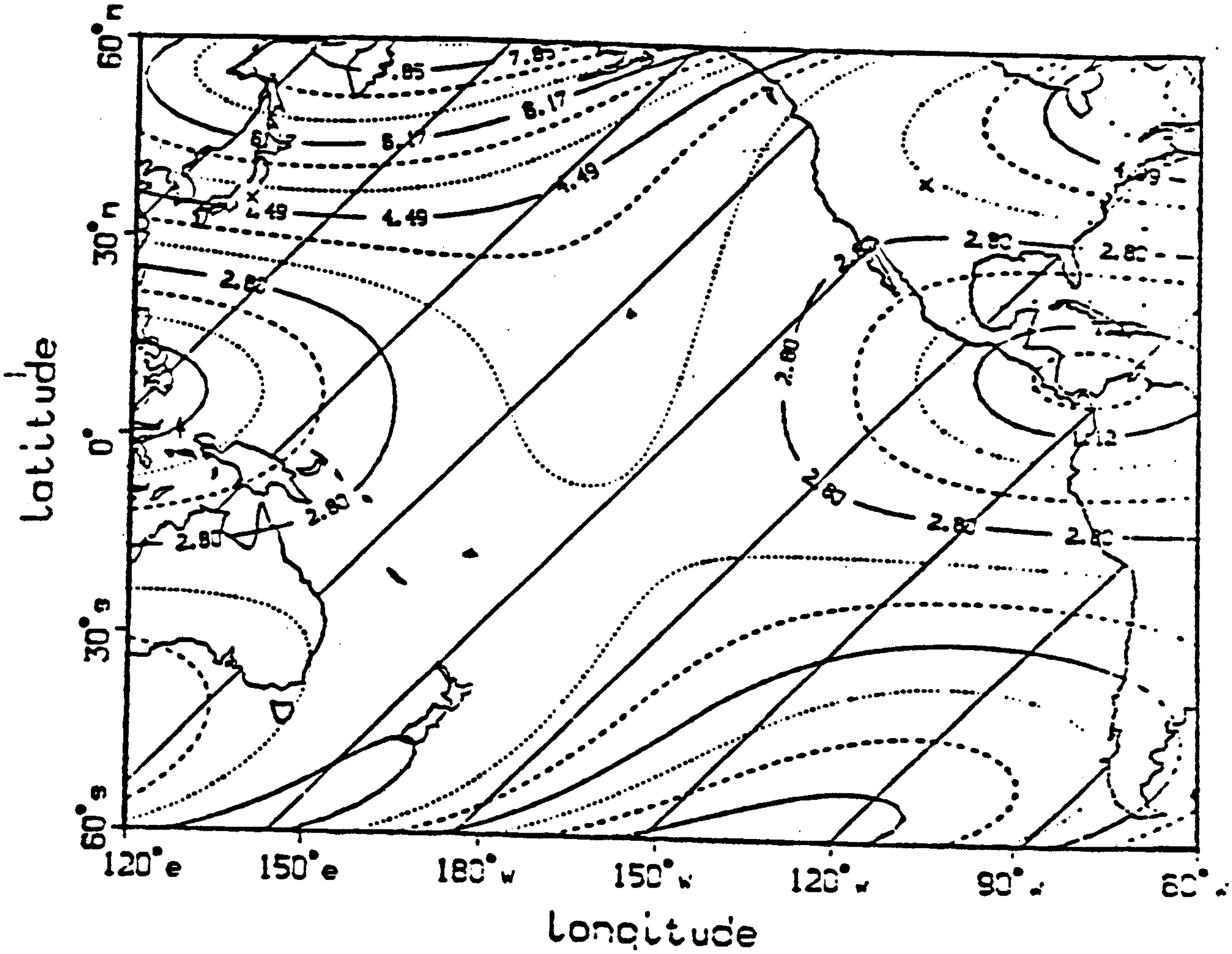


Figure 10a.

NBS-USNO Time Transfer Error

from rms ephemeris error = 41.23 ns
units=nanoseconds, direction=south

.202 ns between contours

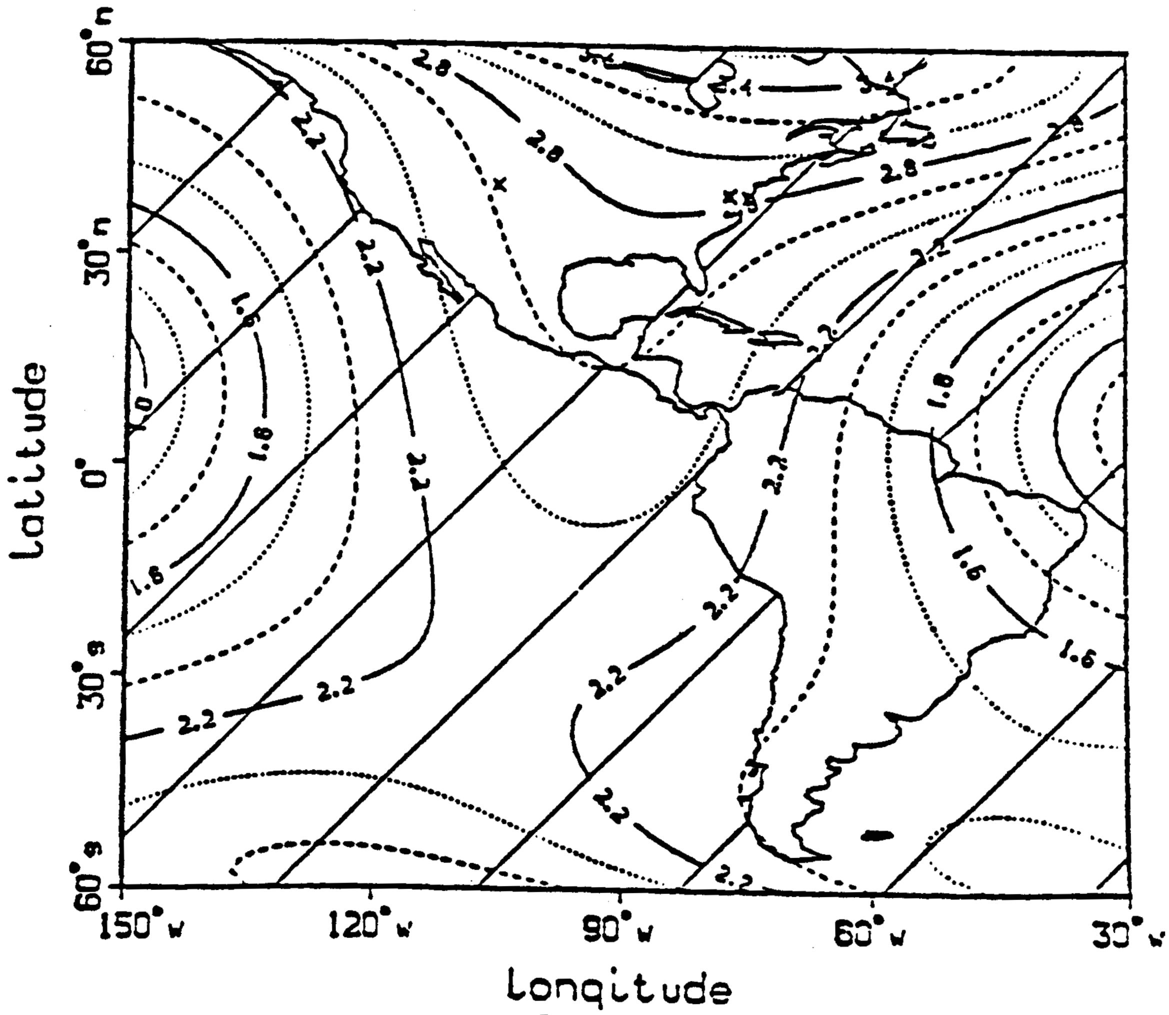


Figure 11b.

NBS-USNO Time Transfer Error

from rms ephemeris error - 25.46 ns
units-nanoseconds, direction-north
.147 ns between contours

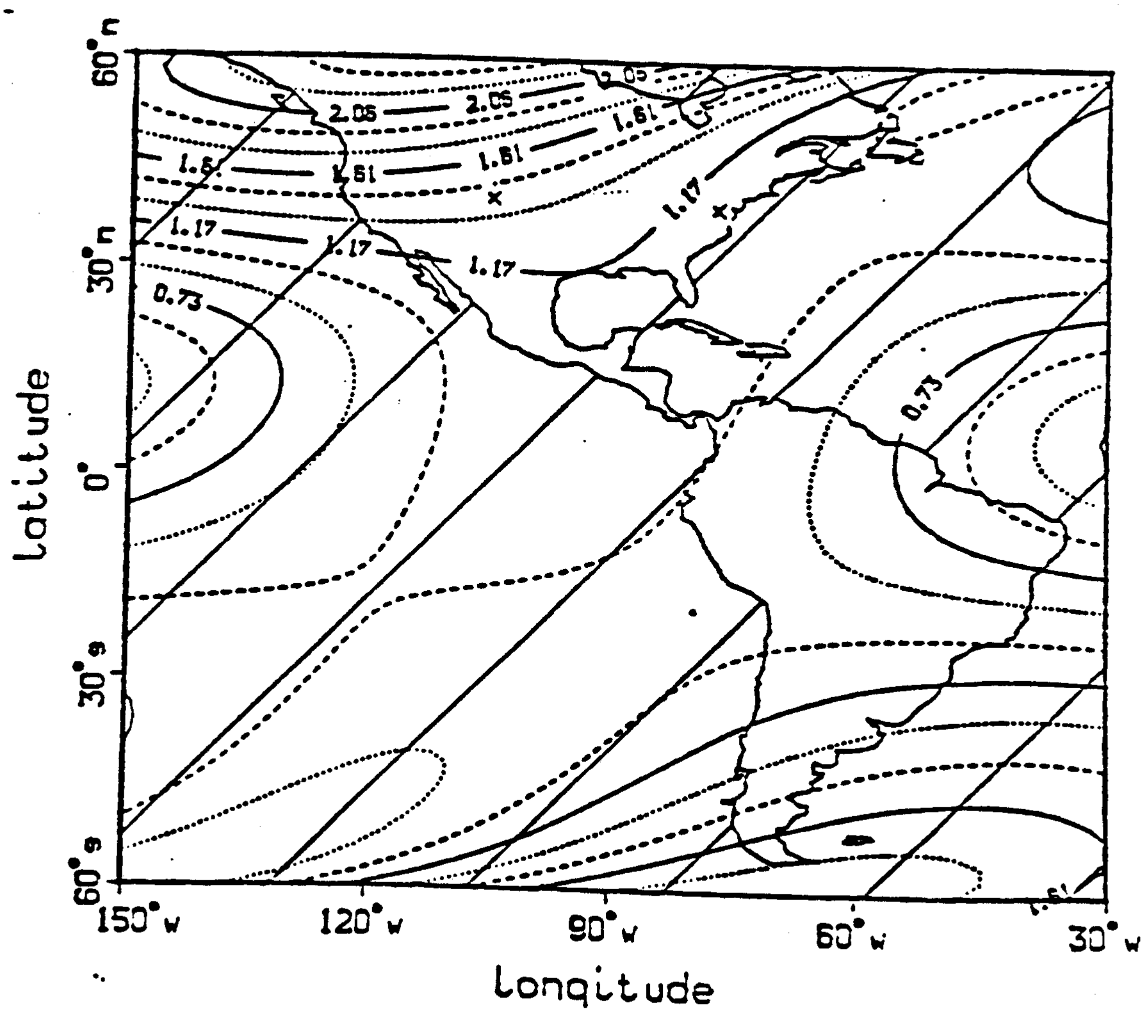


Figure 12a.

NBS-Vandenberg Time Transfer Error

from rms ephemeris error = 41.23 ns
units=nanoseconds, direction=south

.102 ns between contours

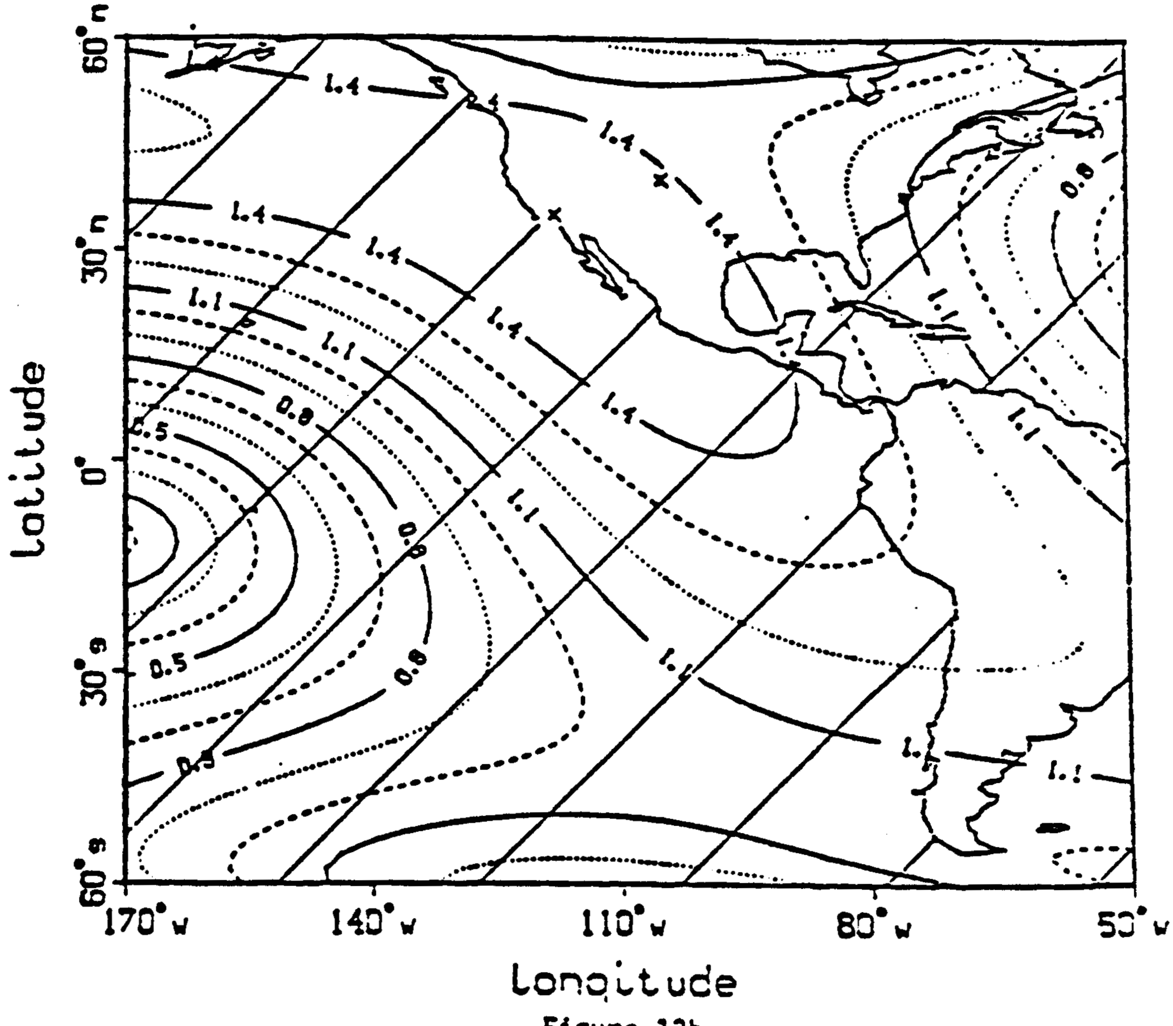


Figure 13b.

NBS-Vandenberg Time Transfer Error

from rms ephemeris error = 25.46 ns
units=nanoseconds, direction=north

.091 ns between contours

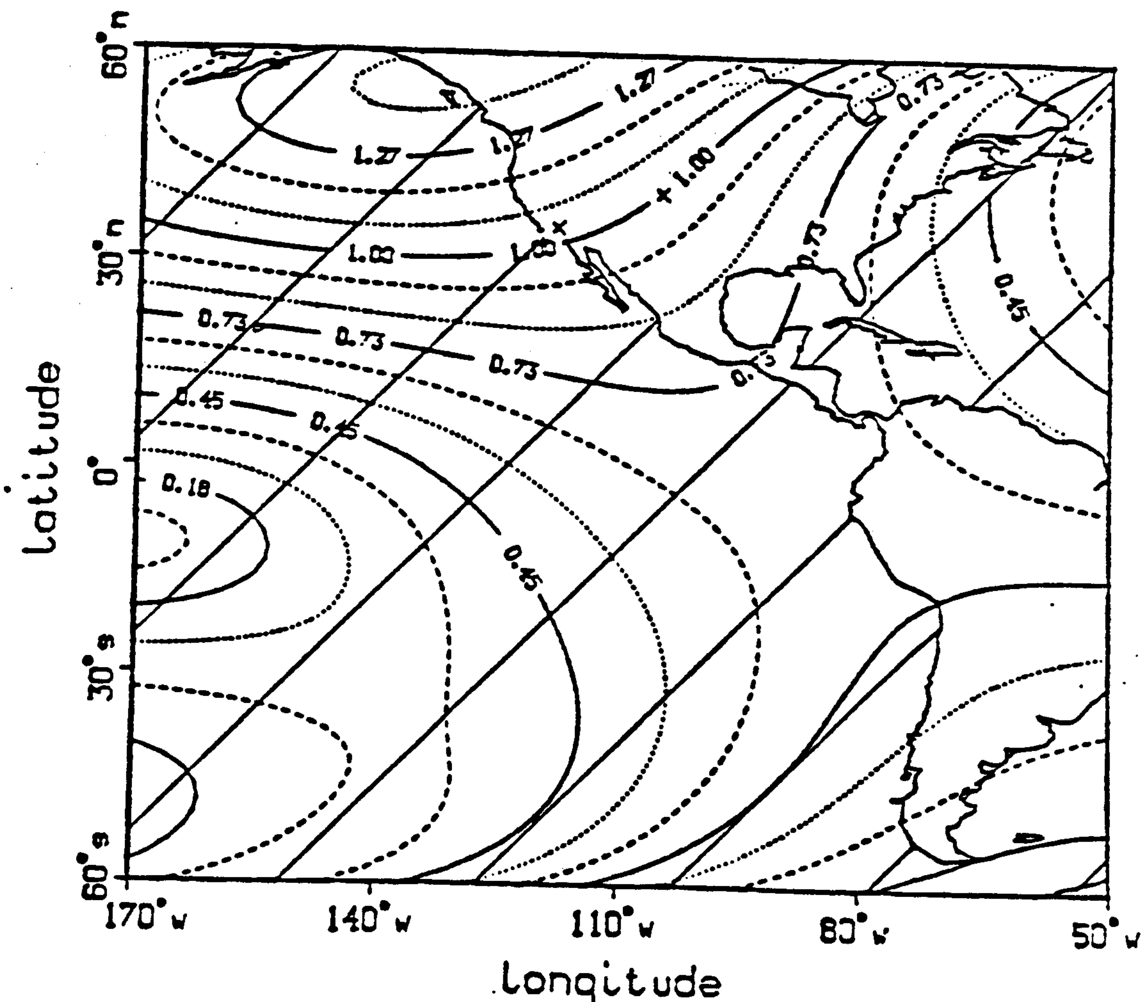


Figure 14a.

NRC-PTB Time Transfer Error

from rms ephemeris error = 41.23 ns
units=nanoseconds, direction=south

.421 ns between contours

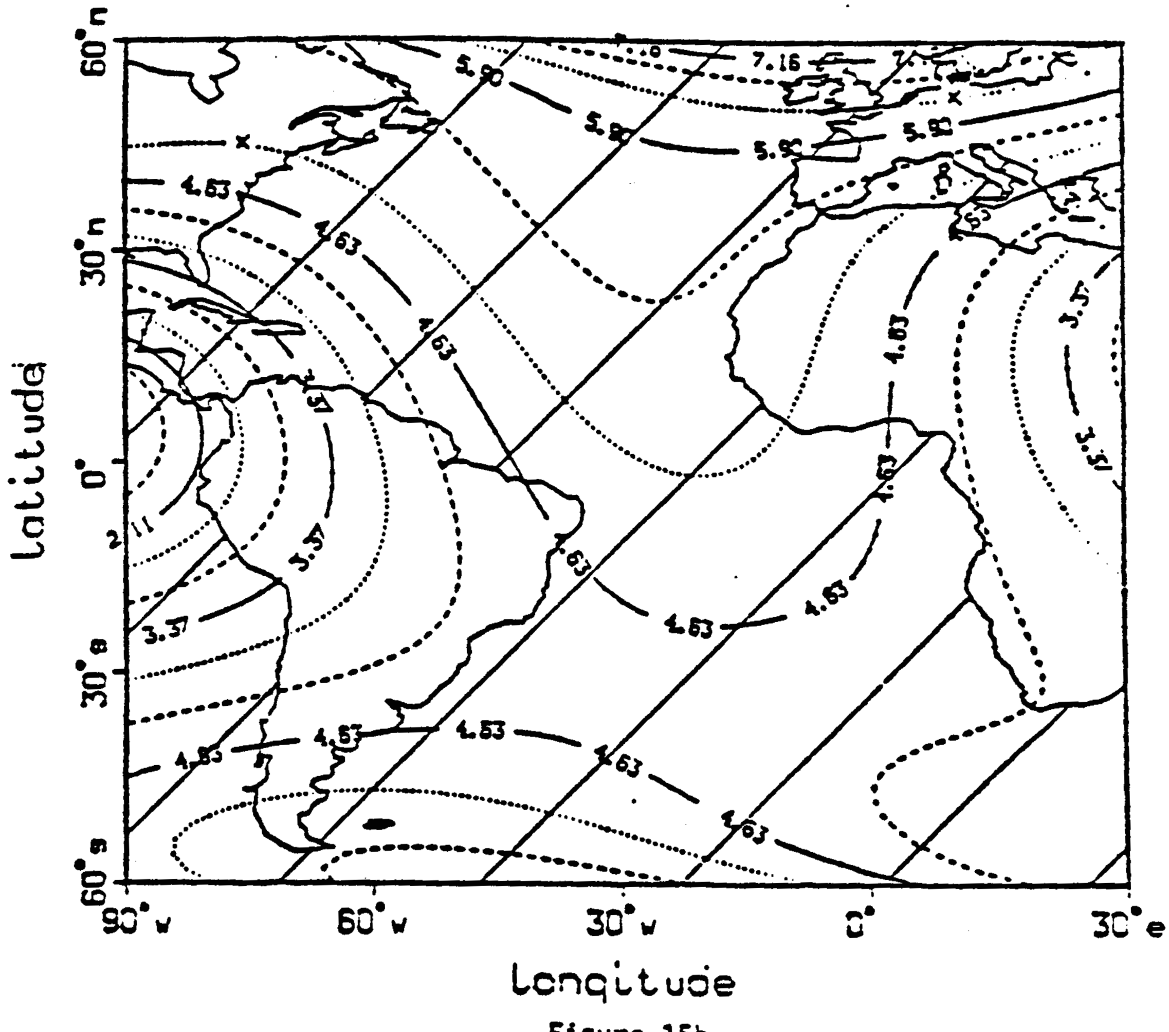


Figure 15b.

NRC-PTB Time Transfer Error

from rms ephemeris error = 25.46 ns
units=nanoseconds, direction=north

.347 ns between contours

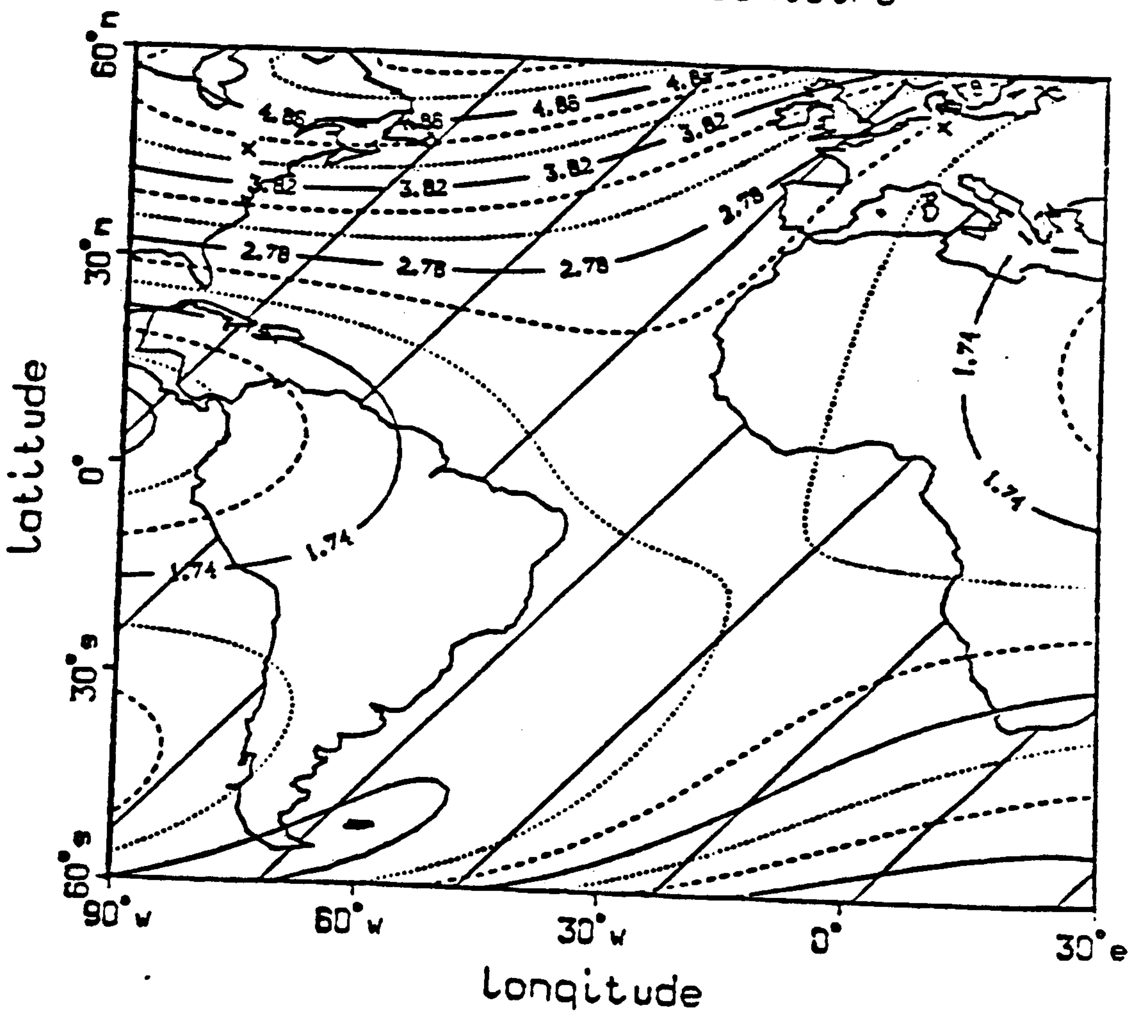


Figure 16a.

USNO-BIH Time Transfer Error

from rms ephemeris error = 41.23 ns
units=nanoseconds, direction=south
.468 ns between contours

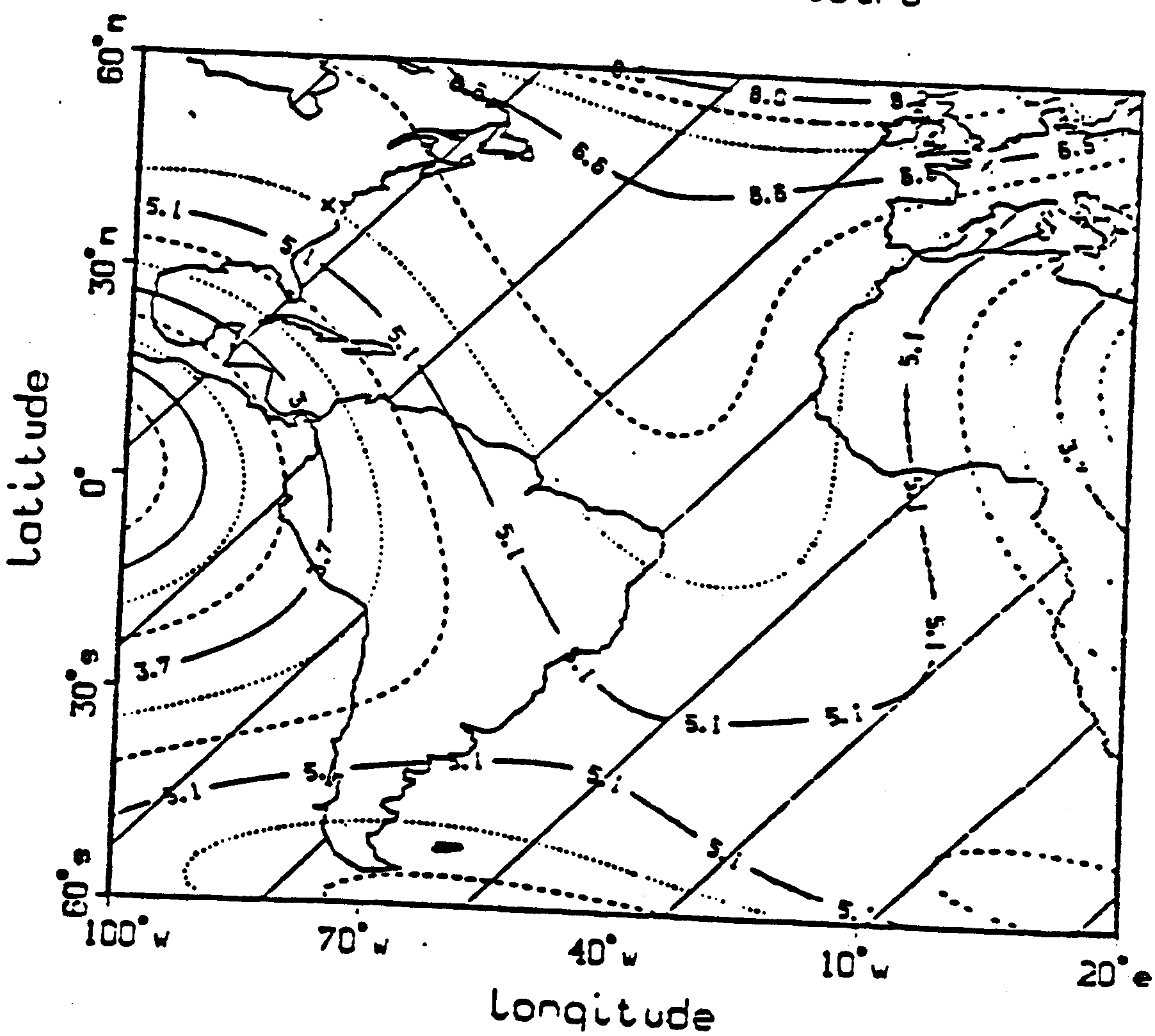


Figure 17b.

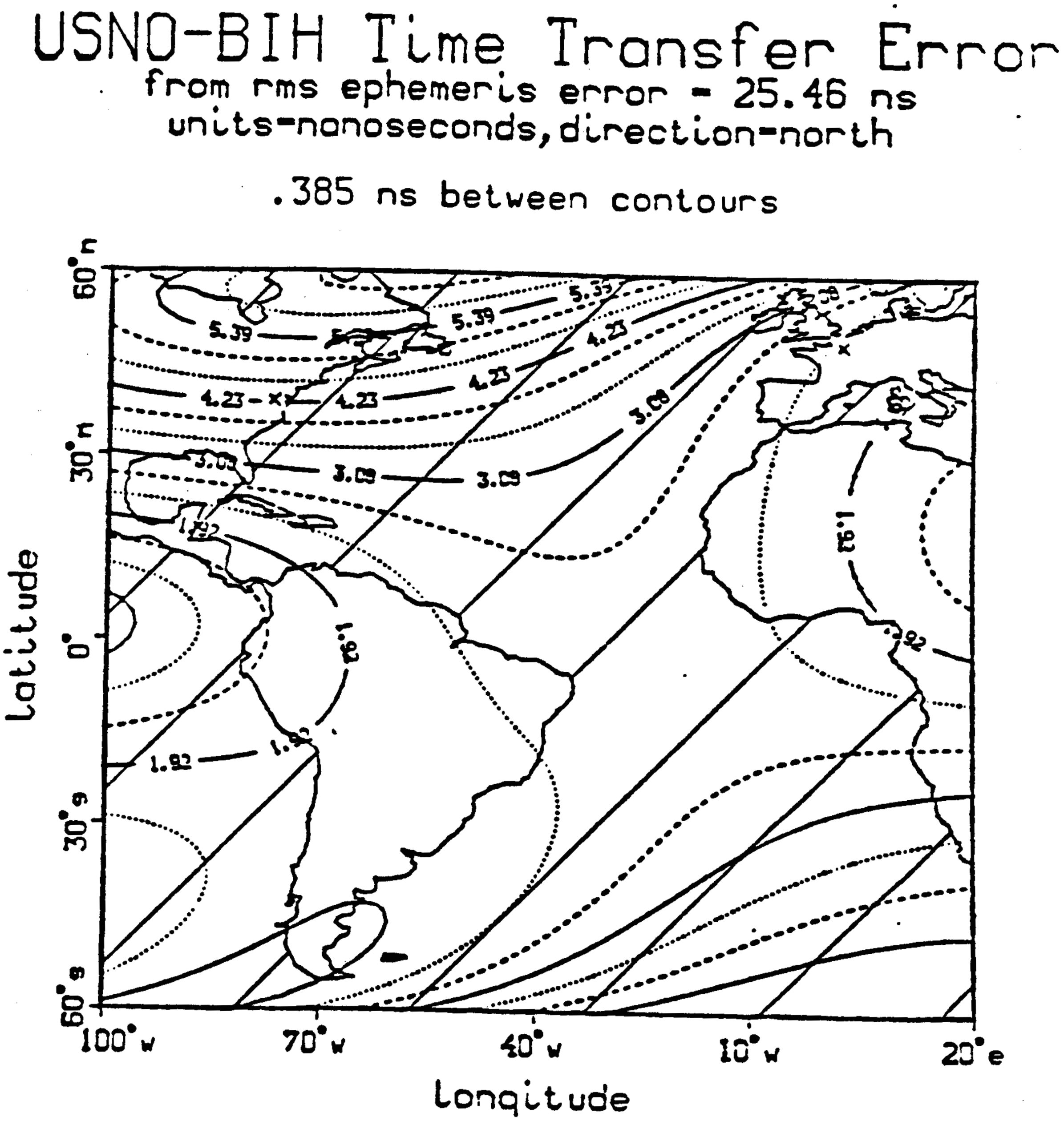
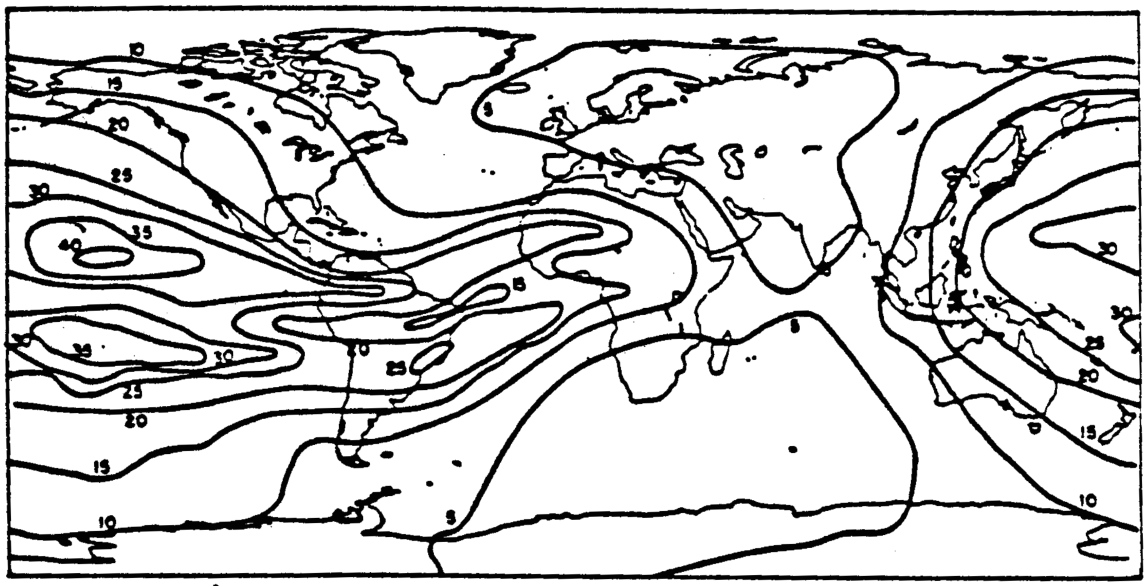

USNO-BIH Time Transfer Error
from rms ephemeris error = 25.46 ns
units=nanoseconds, direction=north
.385 ns between contours
5.39
4.23
3.08
1.92
60°n
30°n
0°
30°s
60°s
Latitude
100°w
70°w
40°w
10°w
20°e
Longitude

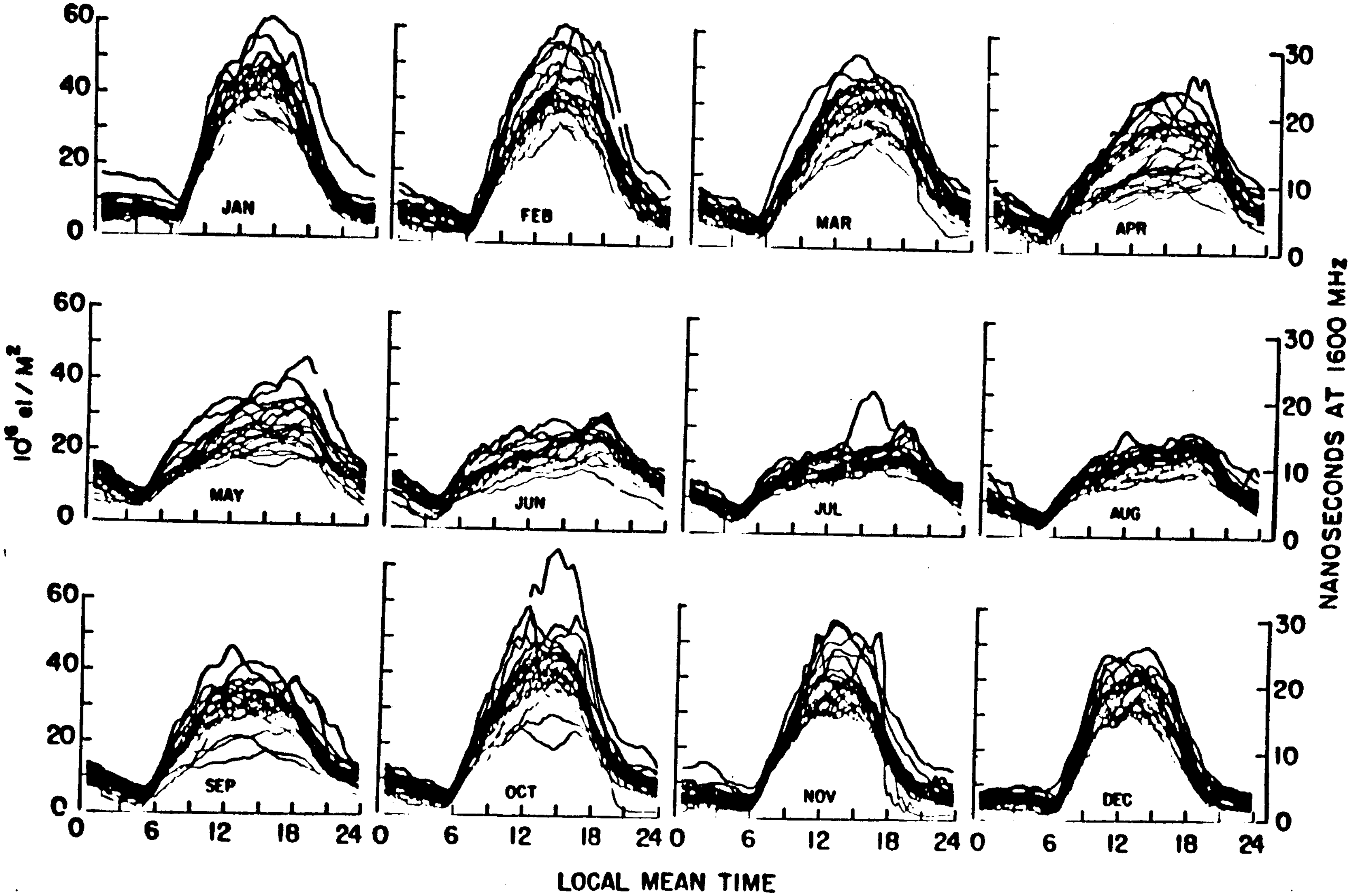
Figure 18a.

IONOSPHERIC TIME DELAY (BENT MODEL)
(NANOSECONDS AT 1.6 GHz) 00 h UT, MARCH 1968



Worldwide average vertical ionospheric time delay at 1.6 GHz for 00 hours universal time March, 1968, an average solar maximum year.

Figure 19.



TOTAL EQUIVALENT VERTICAL ELECTRON CONTENT FROM HAMILTON, MASS. (looking towards ATS-3), 1968

Monthly overplots of equivalent vertical TEC from Hamilton, Massachusetts for the year 1968, a solar maximum year.

Figure 20.

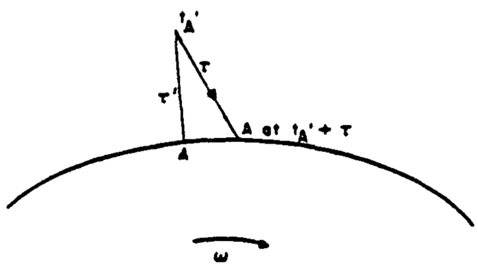


Fig. 21 Corrections due to earth rotation for one-way satellite transmissions, such as in the GPS system [*Schuchman and Spiker*, 1977; *Buisson et al.*, 1977].