A Dual Frequency VLF Timing System

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Abstract—**The use of high precision portable clocks and radio signals is discussed in relation to synchronization of remotely located clocks. The demonstrated inherent phase stability, approximately ten μs rms, of very-low-frequency (VLF) propagation and its low attenuation rate with distance, have led to various approaches to exploit these virtues in timing applications. The system considered here employs two carrier frequencies with timing information contained in their difference frequency to permit identification of a specific cycle of one of the carrier frequencies. Such a system makes stringent demands on phase stabilities of the transmitted signals and of the receiving system as well as that of the propagation medium itself.**

The present system, whose development has been supported jointly by NBS and NASA, makes use of NBS radio station WWVL at Fort Collins, Colo. Receivers are of the standard VLF phase tracking servo type. A special signal generator is used in conjunction with the local clock to simulate the transmitted signal in order to relate the local time scale to that at the transmitter.

One of the carrier frequencies is maintained at 20 kHz. With a second frequency (500 Hz removed from this frequency), carrier cycle identification was achieved on about 90 percent of the days for over a month on the path from Fort Collins, Colo., to Greenbelt, Md. Since January 4, 1966, the difference frequency has been 100 Hz, with somewhat more fluctuation in results. However, lower precision is required for the initial synchronization. The results of averaging to improve this performance will be discussed.

TWO BASIC methods are used for synchronizing remotely located high precision clocks. One is transportation of operating clocks between locations where clocks are to be synchronized; the other is propagation of radio signals which contain timing information.

Until recently, high-frequency (HF) propagation such as from radio station WWV, rather than the portable clock method, offered the higher practical precision of synchronization, with modest investment in receiving equipment. This system employs amplitude modulation of an HF carrier with time ticks once per second. The bandwidth used is limited by considerations of spectrum availability rather than any inherent equipment limitations. Propagation is by means of the *E* and *F* ionospheric layers, whose inherent instabilities limit precisions of synchronization to around one millisecond from day to day so that no additional advantage accrues from increasing the present bandwidth. On the other hand, improvements in the uniformity with which time scales can be kept, and the development of portable quartz crystal clocks and atomic oscillator clocks, now permit synchronization with precisions of the order of one microsecond using portable clocks [1], [2]. Since many of the demands in such areas as satellite and missile tracking, geodesy, seismology, and sophisticated communications systems are in the range which can only be satisfied by portable clocks, increasing use is now being made of these devices. The difficulty with this approach is that extreme reliability is required of clocks to maintain synchronization for long periods after they have been synchronized. Furthermore, even without clock failures it has been shown [3] that time kept by clocks driven by crystal oscillators may be expected to depart from that of a uniform standard in a random walk fashion, necessitating periodic resynchronization.

One obvious approach in solving this problem is to make use of the extreme stability of very-low-frequency (VLF) propagation compared to that of HF propagation. This stability is that of the ionospheric *D* layer by which VLF signals propagate and is such that day-to-day repeatabilities of better than 10 microseconds are possible. A further advantage is the reliability of this mode of propagation and its extremely low attenuation rate, permitting worldwide reception from a single station [4], [5]. The simplest way to make use of these advantages is to synchronize a remote clock by use of a portable clock, then maintain the remote clock's synchronization by manual adjustment of its oscillator frequency, obtaining the necessary corrections from VLF reception. For direct time synchronization, however, the VLF approach has serious limitations when carried out in the same way as the high-frequency case—that is, using pulse modulated timing information. This limitation is due to the combination of low carrier frequency and high transmitting antenna *Q* which severely restricts the useful bandwidth and prevents the broadcast of fast rise time pulses by VLF.

Essentially, then, the difference between HF methods and VLF methods arises from the comparatively narrow bandwidth capability but high propagation stability available at VLF which make new techniques necessary if the potential synchronization stability is to be reached. An early description of a VLF system having such a potential was given by Casselman and Tibbals at the 1958 IRE Conference on Military Electronics [6]. The original suggestion for this system, which is known as Omega, was attributed to J. A. Pierce of Harvard, who also first established that VLF transmissions possessed extremely good phase stability [7].

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The Omega system was proposed for and is currently under active study as a worldwide navigation system [8]. Here it is the position on the earth's surface which is to be determined, but the necessary measurement is a time comparison between CW signals received from pairs of transmitting stations. However, a single CW signal is insufficient, since time measurements on a CW signal would have ambiguities equal to the period of the signal (between about 30 μs to 100 μs for the VLF band) resulting in position ambiguities spaced by a few miles. In order to resolve these ambiguities, the navigator would require independent knowledge of his position to better than one-half the spacing of the ambiguities, which corresponds to previous synchronization uncertainty of less than one-half of a carrier period. Omega employs coherent, closely spaced, multiple frequency CW broadcasts to overcome this difficulty. In essence, the difference frequency between a pair of closely spaced carrier frequencies, having a longer period than either carrier frequency, is used to resolve ambiguities of the carrier phase. The difference frequency period fluctuations need only be less than one-half the carrier period to permit a particular carrier cycle to be unambiguously associated with a zero crossover of the difference frequency for this resolution. The Omega system carries out these resolutions in several steps, starting with a relatively small difference frequency period to resolve carrier ambiguities, and then resolving difference frequency ambiguities in turn with longer difference frequency periods. This process continues until position ambiguities are too widely spaced for the navigator to be uncertain as to which location he is nearest. This multi-step procedure is necessary in a navigation system where only a short averaging time is allowed for each determination of position, and where the measurement must be made at any time of day or night rather than at the time propagation conditions are best. The NBS-NASA timing system requirements are less stringent in that receiving locations are fixed and synchronization information is not needed so often as navigation position information. For example, at NASA tracking stations [9], employing atomic oscillators or ultra-stable crystal oscillators and the resultant highly uniform time scales, it is now thought sufficient to make a time synchronization determination once a day during the time of most stable propagation conditions.

NBS-NASA System Description

The NBS-NASA system, while using the same techniques as Omega, is simpler in that only one transmitter location is used, making transmission time sharing among stations unnecessary. Ultimately the system may employ more than one difference frequency, but tests have been made using only one at a time for a period of several months each. The frequencies used in the NBS-NASA tests are the 20.0 kHz standard frequency carrier, offset from the United States Frequency Standard (USFS) to conform to the coordinated Universal Time scale, with either 20.5 kHz or 19.9 kHz as the single auxiliary frequency for a given test period. These frequencies are all synthesized from a common source which is phase locked to the offset USFS. Carrier period ambiguities are thus 50 μs; difference frequency period ambiguities are either 2 ms or 10 ms. Frequency shift keying is employed to utilize the single transmitter and antenna at Fort Collins, Colo., allowing 10 seconds transmission time alternately for each frequency. In order to relate these transmissions to Universal Time (UTC) the phases of the two frequencies being broadcast are adjusted at the transmitter to go through zero simultaneously in a positive direction coincident with a seconds tick of UTC. These phase relations are maintained, upon reception, with the time of simultaneous phase agreement delayed by an amount depending on the phase velocity of the signals and on the distance of receiver from transmitter. Since there is a slight dependence of phase velocity on frequency, an additional small phase shift of the difference frequency will occur. In order to extract timing information from the signals, one or, with a little more convenience, two conventional phase tracking VLF receivers may be used.

It is necessary to relate the phases of the received signals to the time scale operating at the receiving site in order to perform a time synchronization. This is done by making use of an auxiliary device, referred to as a calibrator, to generate from the local time scale the same two frequencies as transmitted, with the same simultaneous phase relationship conditions. The calibrator actually consists of a sawtooth waveform generator, which is synchronized by a 100 pulse-per-second rate taken from the local time scale divider chain. This waveform has the desired properties that all harmonics of the fundamental frequency exist, (in particular, 19.9 kHz, 20.0 kHz, and 20.5 kHz) and all go through zero in a positive direction coincident with each pulse of the 100 pps clock output which is in turn synchronized with the 1 pps clock output. An interchange of signals from the receiving antenna with the calibrator output permits a measurement to be made of the pair of phase differences. From these two measured quantities, the relation between transmitter time scale and receiver time scale may be calculated, provided only that the carrier propagation phase delays are known. In order to study propagation fluctuations on the path from Fort Collins, Colo., to the Goddard Space Flight Center at Greenbelt, Md., however, the clocks at the two locations have been kept synchronized by the use of portable clocks and propagation delay[1] treated as the unknown quantity. If Δt_1 denotes the measured time difference between the phase

[1] For purposes of simplicity, the treatment of this paper assumes that propagation is dispersionless so that no distinction between phase velocity and group velocity is made. The term propagation delay will thus be used throughout in referring to either phase or group delay; effects of dispersion will be treated later as a correction.

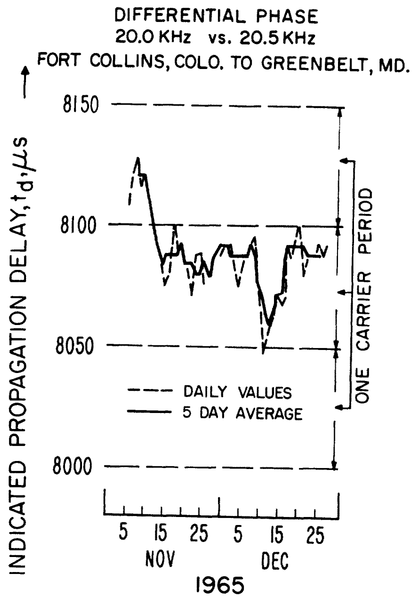

Fig. 1.

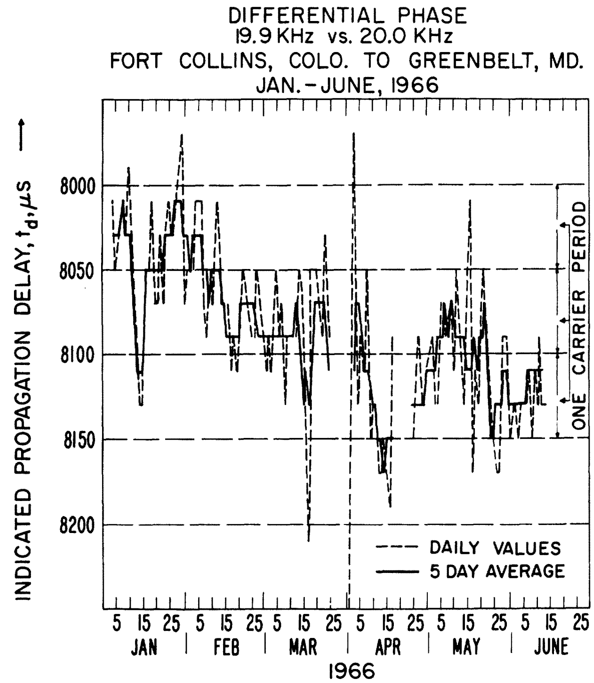

Fig. 2.

of the received signal and locally generated signal of the lower frequency f_1, and Δt_2 denotes measured time difference similarly for the higher frequency f_2, then it is shown in the Appendix that when the transmitter and receiver clocks are synchronized, the propagation delay t_d, as determined from the difference frequency, is

$$t_d = (\Delta t_2 - \Delta t_1)\left[\frac{f_1}{f_2 - f_1}\right] + \Delta t_2.$$

Using this formula with measurements made around noon each day at Greenbelt, Md., propagation delay has been calculated for data taken from late October 1965 to June 1, 1966. From October to January 4, the two frequencies broadcast were 20.0 kHz and 20.5 kHz. From January 4 to June the frequencies were 20.0 kHz and 19.9 kHz. Results are shown in Figs. 1 and 2, which display a plot of indicated propagation delay as derived from the difference frequency measurements. The data were obtained from the μs-counter dial of a pair of commercial VLF receivers which have a resolution of 0.1 μs. This accounts for the steps in indicated propagation delay, and also points out the severe requirement on relative phase stability of the two carrier frequencies. Even though each transmitted frequency is nominally phase-locked to a common reference, there has been some long term differential phase fluctuation. This made daily local measurement of differential phase, as transmitted, necessary. The data in Figs. 1 and 2 have been corrected for this fluctuation. On May 24 an improved AGC circuit was installed in the transmitter phase control servos, resulting in considerable improvement in the transmitted phase stability as shown in Fig. 2. In order to gain a knowledge of longer term propagation fluctuation, the data has been smoothed by making five-day averages. These results are also shown in Figs. 1 and 2. The value for $(\Delta t_2 - \Delta t_1)$ for the months of February through early June 1966 approximates -9.5 μs, which corresponds to a propagation delay of approximately 8100 μs. Portable clock measurements indicate a value for propagation delay of approximately 8050 μs. This difference of 50 μs between VLF and portable clock measurements is equivalent to a 0.25 μs bias in $(\Delta t_2 - \Delta t_1)$ which may be due to the variation of phase velocity with frequency.

According to mode theory calculations of Wait and Spies [10], propagation phase velocity decreases with increasing frequency in the neighborhood of 20.0 kHz, resulting in about 0.2 μs delay of the 20.0 kHz signal over that of 19.9 kHz and about 0.7 μs delay of the 20.5 kHz signal over that of 20.0 kHz signal. These values of 0.2 μs and 0.7 μs, when applied to the measured data shown in Figs. 1 and 2, do indicate a value of approximately 8050 μs for propagation time, thus bringing the VLF and portable clock data into close agreement. This correction is of theoretical interest, and arises from the dispersive properties of the earth ionosphere waveguide which cause the phase and group velocities to be different. It is necessary in making predictions of propagation delay to receiving points for which portable clock measurement is not readily accessible. At present, the measured values of propagation delays must also be extrapolated with care for another reason. Measurements made using portable receiving equipment indicate that as much as one microsecond bias error in the phase of the difference frequency may occur as a result of close proximity of the receiving antenna to long metallic structures, such as power lines, pipes, or buildings. This error in phase could cause an indicated propagation delay error of as much as 200 μs and makes evident that, where precise synchronization is required, propa-

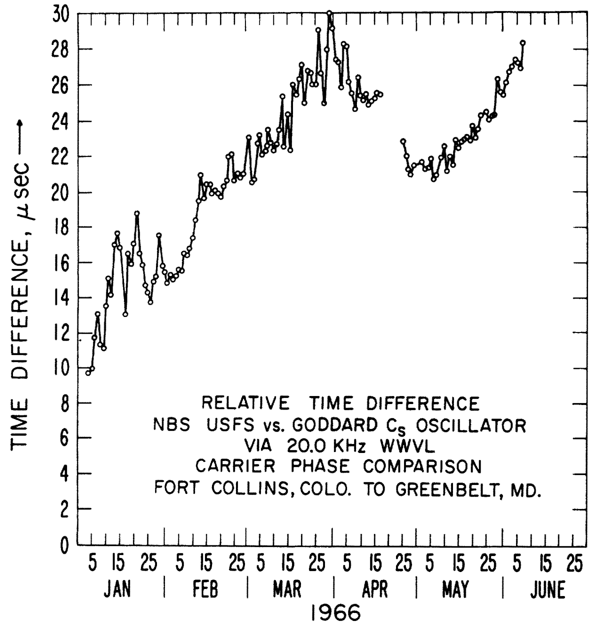

Fig. 3. Residual daily VLF time fluctuations assuming correct carrier cycle identification each day.

gation delay should be determined with a portable clock.

As can be seen in Fig. 2, phase stability of the received 100 Hz difference frequency was not sufficient to permit identification of the same carrier cycle every day measurements were made. For this to have occurred, the measurements would have to all be between two of the indicated dashed horizontal lines. If the system were being used for timing, an error of 50 μs would have occurred in indicated time each day the measurements shifted from one band into an adjacent band. Taking five-day averages improves the synchronization at the expense of a delay in making a determination. The degree of objection to such delay depends on the uniformity which can be ascribed to the local time scale. Catastrophic failures would, of course, make a complete resynchronization necessary.

In order to evaluate the maximum stability of synchronization of the system over the path under discussion, the day-to-day time variation of the received carrier is plotted in Fig. 3. This would be the system synchronization capability if the same carrier cycle had been identified each day. The capability of VLF in comparing frequencies to better than 1 part in 10^{12} is readily apparent from this figure. Between January and March the mean slope is about 2 parts in 10^{12}.

In order to permit positive carrier cycle identification continuously during day and night, it is apparent that several difference frequencies must be transmitted simultaneously. More effort is needed to determine the optimum frequencies; but it appears that if both 500 and 100 cycle frequencies had been used simultaneously, cycle identification could have been accomplished nearly every day in two steps, with an initial synchronization certainty of ± 5 milliseconds from WWV transmissions. A completely self-sufficient VLF system is clearly preferable; possibly this system would provide the convenient frequencies of 1 Hz, 100 Hz, and 1000 Hz in a multiple transmission from an NBS VLF station, and we believe would permit time synchronization of better than 10 μs throughout most of the world.

Appendix

In deriving an expression for the propagation phase delay of the difference frequency, it is convenient to refer to Fig. 4, which displays phase relationships of the two carrier frequencies at significant times at transmitter and receiver. The dotted line represents the traveling wave of the signal from the transmitter as it progresses in distance, on the vertical axis, and in time, on the horizontal axis. The slope of the dotted line then indicates phase velocity. The time origin is $t=t_0$, the time of occurrence of a seconds tick. This time is synchronized at transmitter and receiver locations using a portable clock.

The phases of f_1 and f_2, the lower and upper frequency carriers, are identically zero at $t=t_0$ from both the transmitter, represented by Φ_{1tx} and Φ_{2tx}, and from the calibrator at the receiver, represented by $\Phi_{1\text{cal}}$ and $\Phi_{2\text{cal}}$. Assume that the phase relationships of two signals emitted from the transmitter at time $t=0$ may be observed during propagation to the receiver. If propagation phase velocity is the same for each frequency, their phases will remain at zero and they will arrive at the receiver with that relationship as represented by Φ_{1rx} and Φ_{2rx}. Meanwhile, the phase of each signal emitted from the calibrator will have advanced. The observed phase difference between received signal and calibrator signal of f_1 will be

$$\Phi_{1rx} - \Phi_{1\text{cal}} = \phi_1 \tag{1}$$

and of f_2 will be

$$\Phi_{2rx} - \Phi_{2\text{cal}} = \phi_2. \tag{2}$$

We denote the total phase advance of calibrator phase of f_1 during propagation of the signal from transmitter to receiver by θ_1 and, similarly, the total phase advance of f_2 by θ_2. Now θ_2 will advance more rapidly as a function of distance than θ_1, since $f_2>f_1$. Let the distance d of the receiver from the transmitter be sufficiently small so that during propagation time t_d from transmitter to receiver θ_2 has advanced over θ_1 by less than one complete cycle. The consequences of removing this restriction will be considered later. Therefore, let the number of complete cycles advance by either θ_1 or θ_2 from the calibrator during t_d be denoted by N. Then

$$\theta_1 = 2\pi N + \phi_1 \tag{3}$$

and

$$\theta_2 = 2\pi N + \phi_2, \tag{4}$$

where $\phi_2>\phi_1$. From (3) and (4),

$$\theta_2 - \theta_1 = \phi_2 - \phi_1. \tag{5}$$

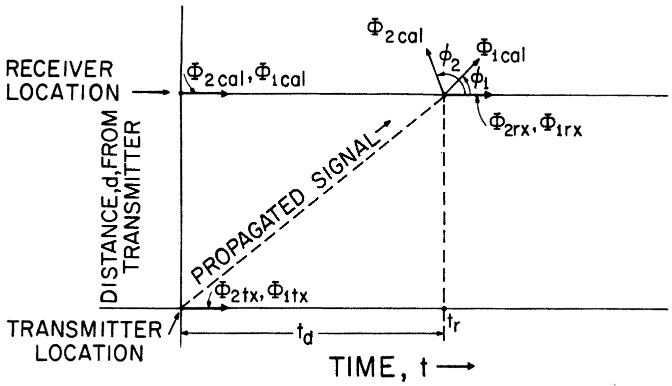

Fig. 4. Phase relationships of calibrator signal and propagated signal.

The propagation time t_d is equal to calibrator phase advance in cycles at either frequency multiplied by the period τ_1 or τ_2 of that frequency; hence,

$$t_d = \theta_1 \frac{\tau_1}{2\pi} \tag{6}$$

$$= N\tau_1 + \frac{\phi_1 \tau_1}{2\pi} \tag{6a}$$

from (3), or

$$t_d = \theta_2 \frac{\tau_2}{2\pi} \tag{7}$$

$$= N\tau_2 + \frac{\phi_2 \tau_2}{2\pi} \tag{7a}$$

from (4). Then

$$\theta_2 = \theta_1 \frac{\tau_1}{\tau_2} \tag{8}$$

from (6) and (7). Substituting for θ_2 in (5) and solving for θ_1,

$$\theta_1 = \frac{\phi_2 - \phi_1}{\frac{\tau_1}{\tau_2} - 1}. \tag{9}$$

Substituting θ_1 into (6),

$$t_d = \frac{(\phi_2 - \phi_2)}{\frac{\tau_1}{\tau_2} - 1} \frac{\tau_1}{2\pi} \tag{10}$$

$$= \frac{\phi_2 - \phi_1}{f_2 - f_1} \cdot \frac{1}{2\pi}$$

which is seen to be group delay [11]. The quantities ϕ_2 and ϕ_1 are generally not measured directly by VLF receivers. Instead, the corresponding time changes Δt_1 and Δt_2, where

$$\phi_1 = 2\pi \frac{\Delta t_1}{\tau_1} \tag{11}$$

and

$$\phi_2 = 2\pi \frac{\Delta t_2}{\tau_2}, \tag{11a}$$

are often obtained in the following way.

VLF receivers are commonly equipped with counters which totalize in steps of 0.1 μs changes in time between the phase of the received carrier and the local oscillator reference. This is a useful form of presentation when the receivers are used to measure time accumulation between local clock and transmitter clock, since this quantity does not depend on carrier frequency while phase accumulation does.

Using a receiver so equipped in order to make a time measurement, the time readings corresponding to received phases Φ_{1rx} and Φ_{2rx} are noted. Interchanging receiver VLF input from antenna to calibrator, the time readings corresponding to calibrator phases Φ_{1Cal} and Φ_{2Cal} are noted. Only the differences Δt_1 and Δt_2 between these sets of readings are meaningful, since the measurements themselves depend on unknown receiver phase shifts which are assumed constant throughout the measurement and therefore cancel out.

Substituting (11) and (11a) into (10) and rearranging,

$$t_d = (\Delta t_2 - \Delta t_1) \left[\frac{\tau_1}{\tau_1 - \tau_2} \right] + \Delta t_1. \tag{12}$$

Using the definitions

$$\tau_1 = \frac{1}{f_1} \tag{13}$$

and

$$\tau_2 = \frac{1}{f_2}, \tag{13a}$$

(12) becomes

$$t_d = (\Delta t_2 - \Delta t_1) \left[\frac{f_2}{f_2 - f_1} \right] + \Delta t_1, \tag{14}$$

or

$$t_d = (\Delta t_2 - \Delta t_1) \left[\frac{f_1}{f_2 - f_1} \right] + \Delta t_2. \tag{14a}$$

This is the time corresponding to propagation delay as determined from the difference frequency. Once determined, it is used to resolve carrier cycle ambiguities in the following way.

First, round the first term in (14a) to the nearest integral multiple of the carrier period τ_2, 50 μs. That is the coarse time determination and follows recognition that, from (6a), (11a), and (14a),

$$N\tau_2 = (\Delta t_2 - \Delta t_1) \left[\frac{f_1}{f_2 - f_1} \right]. \tag{15}$$

If the errors in determination of Δt_2 and Δt_1 produce an error of less than $\frac{1}{2}\tau_2$, this roundoff procedure will produce the correct value for N_{T2} and avoid error due to the magnification term $[f_1/(f_2-f_1)]$. Then the time Δt_2, the second term of (14a), is added to this integral number of periods to give the final answer or the fine time determination.

The case where the phase of f_2 advances by more than one cycle over that of f_1 will be considered. This evidently occurs when distance d is so large that propagation delay t_d is greater than the period T_{diff} of the difference frequency. This period corresponds to a distance of one wavelength λ_{diff} of the difference frequency, where approximately

$$\lambda_{\text{diff}} = cT_{\text{diff}}$$

with c = velocity of light. In this case, when calculated by (14a), t_d is too small by one or more periods of the difference frequency. This situation causes no problem for difference periods under discussion here. For instance, the two ms periods correspond to one complete wavelength of difference frequency, being somewhat less than 400 miles. If d is uncertain by less than half this amount, then the correct value of t_d can be readily determined by observing which value of n, for $n=0, 1, 2, 3, \ldots$ results in (t_d+nT_{diff}) being nearest the value of propagation delay calculated from d and the velocity of light. This situation is illustrated in Fig. 5. where the distance and time scales have been lengthened to include two periods of the difference frequency. Ambiguities of distance occur at $n=1$ and $n=2$ where all phases return to their values at $t=t_0$. When t_d is $>\frac{1}{2}T_{\text{diff}}$, values of t_d calculated from (14a) may be negative. This merely means that they are to be subtracted from $(n+1)T_{\text{diff}}$, rather than added to nT_{diff}, and illustrates again that distance must be known to better than half a wavelength of the difference frequency.

A sample set of data taken at Goddard Space Flight Center is given in Table I to illustrate the above procedure.

From these data $\Delta t_1 = 22.3$ μs; $\Delta t_2 = 12.8$ μs; $\Delta t_2 - \Delta t_1 = -9.5$ μs; and $f_1/(f_2-f_1) = 199$; so that, from (14a) first term, $t_d \cong -1890.5$ μs. With a difference frequency of 100 Hz, $T_{\text{diff}} = 10\,000$ μs. The distance from Fort Collins, Colo., to Greenbelt, Md., is about 2400 km so that nT_{diff} is closest to a calculated propagation delay of about 8000 μs when $n=1$. Subtracting the measured value of 1890.5 μs from 10 000 μs results in a measured value for propagation delay of 8109.5 μs which rounds to 8100 μs as the time corresponding to the nearest integral number of 20.0 kHz carrier periods. To this is added time Δt_2, 12.8 μs so that measured propagation delay, finally, is 8112.8 μs for this example.

It remains to point out how this system could be used for time synchronization if propagation delay t_d were known. In that case

$$\Delta T = (\Delta t_2 - \Delta t_1)\left[\frac{f_1}{f_2 - f_1}\right] + \Delta t_2 - t_d, \qquad (16)$$

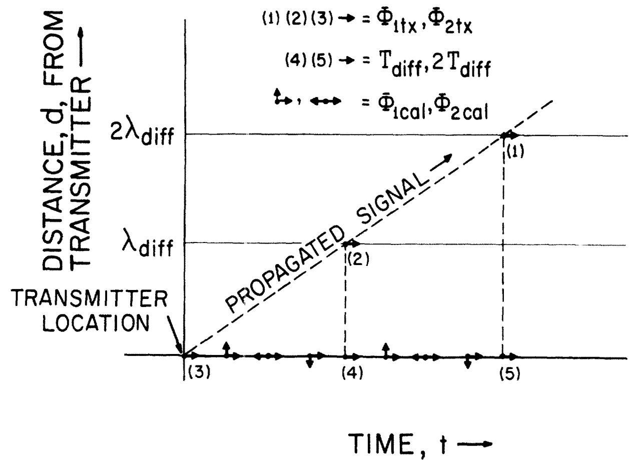

Fig. 5. Phase repetition relationships of calibrator signal and propagated signal showing ambiguities.

TABLE I

	Receiver Counter Reading	
Frequency	19.9 kHz	20 kHz
Propagated Signal	1306.7 μs	1302.4 μs
Calibrator Signal	1284.3 μs	1289.6 μs

where ΔT is time delay of the receiver clock compared to the transmitter clock. Ambiguities would now appear as before but would be in time, rather than distance, as illustrated by identical phases at times T_{diff} when $n=1$ and $2T_{\text{diff}}$ when $n=2$. In order to make a correct synchronization, the receiver clock would need to be synchronized independently to better than $\frac{1}{2}T_{\text{diff}}$. For the case of $T_{\text{diff}}=10$ ms this could be achieved with WWV when it can be received. For $T_{\text{diff}}=2$ ms use of WWV is questionable.

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