Obscurities of Oscillator Noise*

In a recent paper[1] (which will be referred to as paper 1) the origin of crystal oscillator phase noise has been discussed, and an analysis of the measurements reported therein was used to support the hypothesis that crystal oscillator fluctuations arise as a result of thermal noise in the circuitry associated with the oscillator.

Because of the need for knowledge of oscillator noise characteristics and because the results given in paper 1 conflict with those given in a recent paper by the present authors,[2] it was decided that a separate analysis of the data presented in paper 1 should be made in order to ascertain whether any alternative conclusions might be reasonable.

The data analyzed are those presented in Fig. 12 of paper 1, in which good agreement with the calculated curve of Fig. 11, paper 1, has been taken as experimental verification of the hypothesis that the source of oscillator noise is predominantely thermal. Particular importance is to be attached to the higher-frequency portion of the curve to which a single straight line segment was fitted giving a slope of 6-db attenuation per octave with increasing frequency. For the conditions of the experiment described in paper 1, the slope of 6 db/octave would imply noise of a thermal origin.

The present authors feel, however, that this portion of data could better be fitted by two straight line segments rather than by one. In order to test this possibility 78 points were read from Fig. 12 for processing by digital computer. Since the original positions of the plotted points for constructing Fig. 12 were fairly evident, these were the points scaled for analysis.

This spectrum, presented as a phase fluctuation spectrum (in db) in Fig. 12 of paper 1, was converted to a frequency fluctuation spectrum by adding 20 log F in order to facilitate comparison with the data published by the present authors and is plotted in this form in Fig. 1. This presentation appears more suggestive of two regimes of behavior than the original plot, with the dividing point at about 4.6 cps. A horizontal line on this plot would be consistent with noise of a thermal origin.

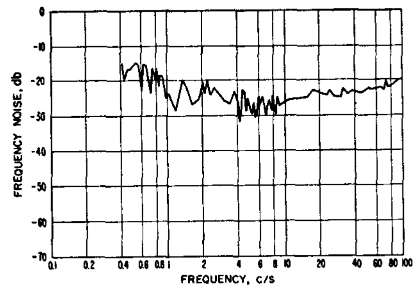

Fig. 1—Crystal-oscillator phase fluctuation noise taken from Fig. 12, paper 1, and replotted as frequency fluctuation noise.

A least squares fit was then made to logarithms of both sides of the equation, $G(F)=AF^{\alpha}$, to determine the value of α which best fit the portion of the data analyzed. This portion excluded the section with frequencies lower than 0.4 cps in order to avoid the region of the curve in the vicinity of f_k, where a departure from a straight line is expected to occur, as is seen in Fig. 11 of paper 1. For the calculation, the program was made to give approximately equal weights to equal frequency intervals. For example, a point at 100 cps was given 10 times the weight of a point at 10 cps. The values of α obtained using this weighting, however, are not greatly different, in the first two cases below, from those obtained using points equally weighted as scaled from Fig. 12. The results were as follows.

* Received August 12, 1963.

[1] L. R. Malling, "Phase-stable oscillators for space communications, including the relationship between the phase noise, the spectrum, the short-term stability, and the Q of the oscillator," PROC. IRE, vol. 50, pp. 1656–1664; July, 1962.

[2] W. R. Atkinson, L. Fey and J. Newman, "Spectrum analysis of extremely low-frequency variations of quartz oscillators," PROC. IEEE, vol. 51, p. 379; February, 1963.

Frequency Range, cps	α
0.4 to 4.6	−0.846
4.6 to 100	+0.442
0.4 to 100	+0.279

Thus, according to this analysis, the frequency fluctuations of the crystal oscillator used in the experiment would, at the lower portion of the frequency interval examined, increase with decreasing frequency as $G_f(F) \sim 1/F^{0.85}$.

This is the behavior noted by the present authors, who found the exponent α to vary from about -0.9 to -1.4 for the oscillators which they studied.[2] Above 4.6 cps the present analysis resulted in a positive α of 0.442, denoting increasing spectral density with increasing frequency, in the range of a few cycles per second to 100 cycles per second, a trend which has also been reported.[2]

Inasmuch as the fitting of two straight lines rather than one to the data gave twice as many parameters that could be optimized for the purpose of improving the goodness of fit, it is not surprising that the average square residual deviation was reduced. No statistical tests were made to see if the observed reduction was significantly larger than the reduction that might be expected to result from the two additional parameters. Rather than to rely on elaborate statistical tests of our own limited data in the 4-cps region and on the data of paper 1, to decide on the universality of the two regimes of spectral behavior, the authors would prefer to examine more data from different laboratories using different instruments for spectral analysis.

In the authors' opinion the origin of frequency fluctuations in oscillators remains sufficiently uncertain to justify the publication of further measurements. The theory of flicker fluctuations in oscillators is particularly obscure and needs clarification.

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Author's Reply[3]

It is encouraging to note the interest that has been elicited by the spectral density curve presented in my paper.[1] I think that the additional information provided by Atkinson, Fey and Newman[2] (referred to as paper 2) further clarifies the origin of oscillator phase noise. However, before discussing this, I should like to reiterate the objectives of my paper which are essentially directed at systems behavior rather than at component performance.

The sensitivity and, hence, the range of a deep-space receiver is a function of kTB where T is the thermal-noise temperature and B is the noise bandwidth. It would seem at first that the range in a phase-coherent system could be indefinitely extended by compressing the bandwidth. However, as indicated in my paper,[1] and particularly in Figures 11 and 12, as the bandwidth is compressed the phase-noise of the local oscillator steadily increases. The ultimate range is thus limited by the noise temperature of the system components and the stability of the local oscillator. If the stability of the local oscillator can be expressed in terms of thermal noise, we have then a unified concept of systems performance in relation to range in terms of thermal noise, bearing in mind, of course, differences in spectral responses.

While there has been an industry-wide attack on the reduction of receiver noise-temperatures with numerous and well-documented papers on low noise antennas, masers and parametric amplifiers, for example, very little, if any, data have appeared on oscillator phase-noise, and that which have appeared have not been readily applicable to systems design use. Deep-space components must frequently be held to tolerances of less than 0.1 db to ensure range. This is particularly true of those components located at the input end of the receiver. As the local oscillator is one of the sensitive receiver inputs, its performance specifications must be held quite rigid. Hence one of the major purposes of my paper was to define the problem, establish suitable phase-noise criteria, propose techniques for evaluation and, in addition, provide actual measured systems data to act as guideposts. In addition, if oscillator phase-noise can be related to thermal noise then a language can be readily established for interpreting oscillator stability. Experience with ultra-stable oscillators in coherent systems has shown that phase-noise is primarily a function of crystal-Q and the design of the electronic networks. The difference in stability between two similar oscillators has been shown to be, in almost all cases, due to electronic circuit design.

The foundation of the thermal noise approach is established in Section III of my paper and further expanded in Sections IV and V. The spectral-density curve of Figure 12 was inserted to indicate the nature of phase-noise as measured at the output terminals of a phase-coherent receiver. The close adherence of the curve to a 6-db/octave slope does, however, to a first-order bear out the original premise that the noise originates in the oscillator as white noise having a flat spectrum.

The frequency spectrum covered by Figure 12 is measured for a system bandwidth of $2B_L = 2.5$ cps and is seen to enter the ultra-low-frequency region, a region of particular interest for the authors of paper 2. Generally speaking, a bandwidth of 2.5 cps is lower than could be generally accommodated for space communications due to the correspondingly low information rate achievable. However for the Venus radar experiment commented on in my paper, a $2B_L = 5$ cps was employed. As will be noted in Figure 12 and as commented on by the authors of paper 2, there is an increase in spectral density as the frequency drops below 2 or 3 cps; this had been noted and several explanations suggested. However, it was not possible in the scope of the original paper to expand on this matter. The possibility of the presence of 1/f frequency instability noise from the crystal has been raised in paper 2. 1/f noise is generally recognized to have a slope of 3 db/octave amplitude wise. If we now make the assumption that 1/f crystal phase noise adds directly to the thermal phase noise at the same rate of 3 db/octave, then at frequencies above f_k in Figure 12 the slope would be increased to 9 db/octave and below f_k decreased to 3 db/octave. The problem then arises as to where the 1/f crystal frequency instability noise sets in. The authors of paper 2 favor 2 cps. Hence if we reconstruct Figure 12 with a slope of 6 db/octave above 2 cps, a slope of 9 db/octave from 0.4 cps to 2 cps and a slope of 3 db/octave below 0.4 cps, we arrive at the dotted curve shown in the accompanying figure. This new dotted curve does appear to be a closer match to the measured data. f_k is moved closer to 0.4 cps. As for $2B_L = 2.5$ cps this is a more realistic figure, $f_k \approx B_L/\pi$. I do not think the departure from 6 db/octave at frequencies above 40 cps too significant in this particular case. The spectral components at 60 cps are over 50 db down from the lower frequency components, and instrument noise for this data is becoming a prominent factor at -60 db. I would suggest that the instrumentation accuracy over-all is not better than ± 2 db.

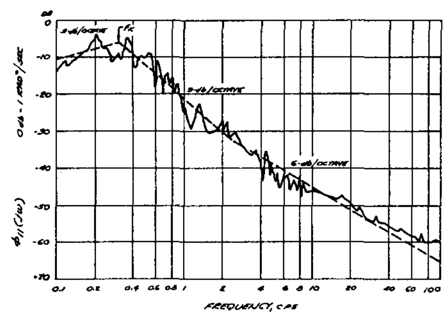

Fig. 2.

Inspection of many records of phase-noise (phase amplitude vs time), as measured at the output terminals of a closed-loop receiver with a meter having dc response, indicates the presence almost invariably of the largest components in the f_k region, and in fact the bandwidth can frequently be estimated from the appearance of the noise. This would appear to indicate then that no significant components are arising below the f_k region that deviate from the proposed laws. Hence extrapolation of 1/f crystal noise at a 3-db/octave rate into the ultra-low-frequency region, *i.e.*, below 1 cps, appears to be justified. As the flicker noise and thermal noise are noncoherent, we might perhaps expect the ± 3-db variations in spectral amplitude which appear in Figure 12. I would also mention that similar characteristics have been obtained both for vacuum tube oscillators and solid-state oscillators. Further data in this area from other workers in the field would be, as stated by the authors of paper 2, of extreme interest.

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[3] Received August 28, 1963.