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Upper Limit on the Gravitational Flux Reaching the Earth from the Crab Pulsar

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A 30-m laser interferometer has been used in a search for gravitational radiation from the Crab pulsar. The minimum detectable signal would be produced by an incident gravitational flux of 10^9 ergs/sec cm^2 and we find no effect at this level.

The possibility that pulsars may emit gravitational radiation has prompted us to search for the strain fluctuations that these waves would induce in the earth. For reasons to be outlined below, we feel that the radiation from the Crab pulsar, NP0532, presents the most favorable experimental situation, and we present here our upper bound on the energy flux reaching the earth from the Crab.

Landau and Lifshitz[1] have shown that a rotating object will emit gravitational radiation according to the formula

$$\dot{E} = \frac{G}{45c^5} \dddot{D}_{\alpha\beta}{}^2 ,$$

where D is the quadrupole tensor, G is the gravitational constant, and c is the speed of light. Thus if a pulsar has a varying component of its quadrupole moment tensor perpendicular to the axis of rotation, it will emit gravitational radiation at twice its rotational frequency. The radiated power will be proportional to the square of the perpendicular component of the quadrupole moment tensor and to the sixth power of the rotation frequency.

Estimates of the gravitationally radiated power from the Crab pulsar vary widely. Ostriker and Gunn[2] estimate a gravitationally radiated power of 10^{38} ergs/sec, while Melosh[3] estimates that the power is 8×10^{33} ergs/sec or less. The gravitational flux at the earth is therefore anywhere from 2×10^{-11} ergs/sec cm^2 to about 10^{-7} ergs/sec cm^2.

The interaction of this gravitational energy with the earth has been treated by Weber,[4] Dyson,[5] and Misner *et al.*[6] The calculations are difficult in general and become especially so in the case of the Crab pulsar because the absorption cross section at high frequencies depends on local terrestrial inhomogeneities. If we assume that a calculation such as Dyson's[5] contains the essential physics of the problem, we can relate the displacement induced in the earth's surface, Δy, to the incident gravitational flux, E. We obtain

$$(\Delta y)^2 = \frac{16\pi G S^2}{\omega^4 c^3} E , \qquad (1)$$

where S is the seismic velocity which is on the order of 5 km/sec. For the Crab pulsar, using the Ostriker-Gunn estimate,

$$y < 5 \times 10^{-22} \text{ cm.}$$

It is important to note that, other things being equal, the earth displacement is proportional to the square of the rotational frequency so that the short-period pulsars are more likely to be detected than the longer-period ones.

In an attempt to detect this pulsar-induced displacement we have used a 30-m laser strainmeter located in an unworked gold mine near Boulder, Colorado.[7] The interferometer is mounted on piers and is oriented along an axis 7° west of north. The interferometer is illuminated with a 3.39-μm helium-neon laser; the laser's frequency is locked to one of the interferometer transmission maxima by means of a first-derivative servo lock. The lock has a unity gain point near 1 kHz and sufficient dynamic range to follow strain fluctuations of 1×10^{-7} $(\Delta L/L)$. Thus the system transfer function is unity for all signals of interest in the current work. (See Fig. 1.) The frequency of the laser locked to the long path is heterodyned against a second helium-neon laser stabilized using saturated absorption in methane.[8] The beat frequency is extracted for further processing. As we have shown[7]

$$(\Delta L/L)=1.13\times10^{-14}(\Delta f_{\text{beat}}),$$

where $(\Delta L/L)$ is the strain change in the long path and Δf_{beat} is the corresponding fluctuation in the beat frequency. The sensitivity of the system is limited primarily by fluctuations in the frequency of the methane stabilized device, since the earth noise is very low at twice the Crab pulsar frequency.

Methane-stabilized devices are being studied by Barger and Hall,[8] and by Hellwig *et al.*[9] Both groups determine the stability of the devices by monitoring the fluctuations in the beat frequency between two independent devices. In a 1-sec averaging time both groups report fractional fluctuations of parts in 10^{13}. Since the fractional fluctuation of the frequency of the methane-stabilized device enters linearly in the present work, we would expect that the methane device contributes an equivalent strain noise of about 10^{-26} $(\Delta L/L)^2/\text{Hz}$. The actual system noise is 8×10^{-28} $(\Delta L/L)^2/\text{Hz}$, about a factor of 12 better than this. Most of this improvement is probably due to the quieter environment in the mine.

A second major problem when working near 60 Hz is pickup from power lines. We have taken extreme precautions to minimize this pickup. The residual power-line pickup results in a strain-equivalent noise amplitude of 1.4×10^{-15} $(\Delta L/L)$, a factor of 8 less than the system noise. In addition the power-line frequency is 0.4 Hz away from the mean pulsar frequency. It is therefore not a severe problem.

The beat frequency contains all the information about the long-path motion, but because of the wide bandwidth of the servo system it is not practical to record it directly. Rather we extract the frequency region of interest and heterodyne it down to a more convenient frequency region near dc. This has the advantage of reducing the rate at which information need be recorded at the expense of information about strain fluctuations at frequencies removed from the region of interest. The details of the system are as follows:

The beat frequency is converted to a voltage us-

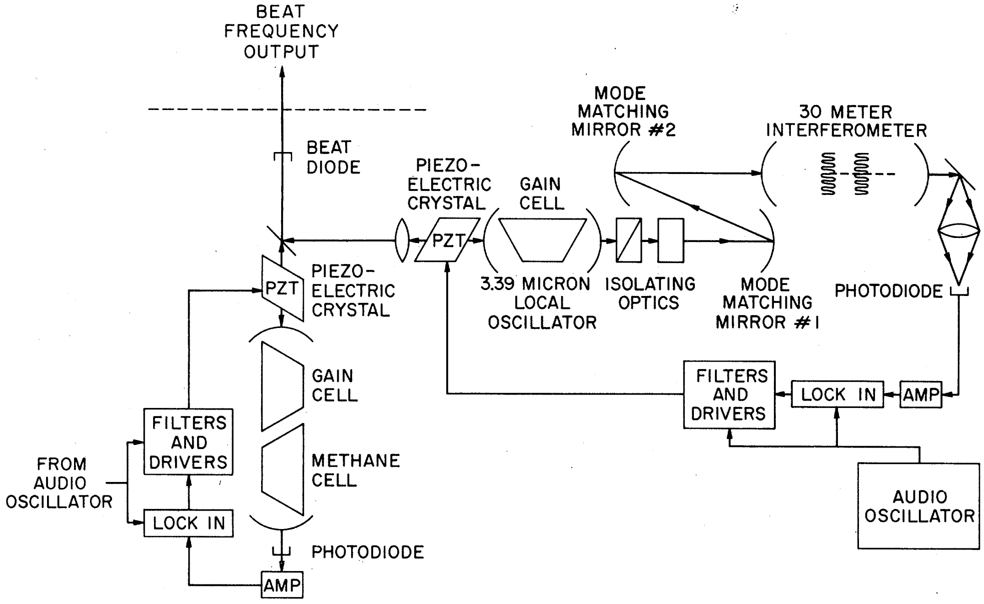

FIG 1. Block diagram of the 30-m laser strainmeter.

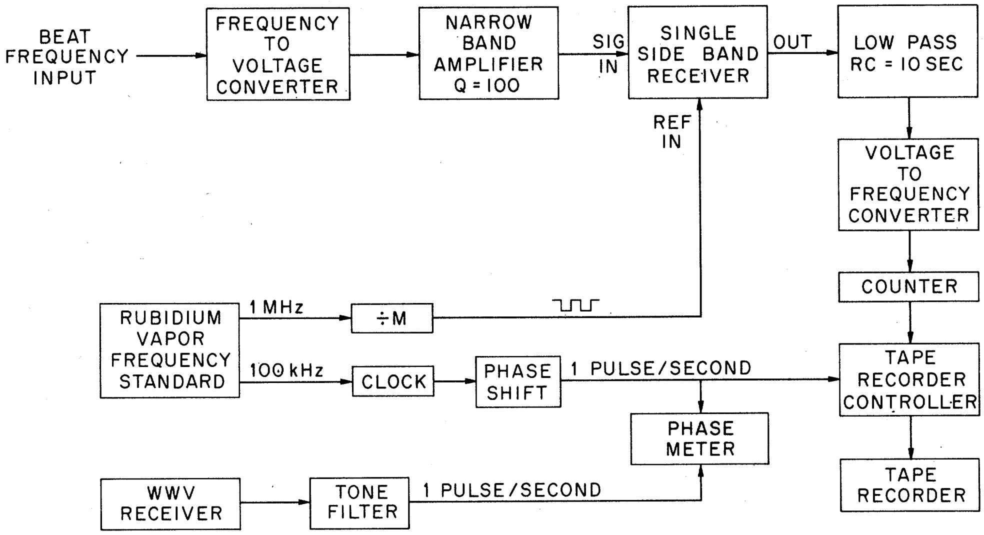

FIG. 2. Block diagram of system for extracting pulsar signal. The integer M is adjusted to heterodyne the mean pulsar frequency down to approximately 0.1 Hz. In the 60.4-Hz experiment M was 16 540.

ing a frequency-to-voltage converter and the voltage is heterodyned in a single sideband receiver against a frequency synthesized from a rubidium vapor frequency standard. The second beat frequency, which is on the order of 0.1 Hz, is digitized and recorded. (See Fig. 2.) This "second i-f" amplifier has an effective Q of about 300 at a frequency near the average pulsar frequency.

It is *essential* to recognize that a simple Fourier transform of the data, using either digital or analog techniques, does not represent the optimum method of extracting the pulsar-coherent signal. The instantaneous pulsar frequency is affected by Doppler shifts, pulsar slowdown, and, depending on the orientation of the antenna relative to the local meridian, the amplitude modulation discussed by Tyson and Douglass.[10]

We have overcome these difficulties by constructing a digital tracking filter which computes this instantaneous pulsar frequency using the data of Nelson *et al.*[11,12] The instantaneous Doppler shifts are computed using the JPL EPHEMERIS DE-69[13] and the usual pulsar coordinates.[11]

We have performed the cross correlation between the beat frequency and the expected signal for a period of approximately 1.5×10^{6} sec at both 60.4 and 30.2 Hz. During these periods the instrumental noise power over the regions of interest was approximately white and was about 8×10^{-28} $(\Delta L/L)^2$/Hz. In neither case do we find a statistically significant pulsar-coherent strain signal. We conclude that if a pulsar-coherent strain signal is present, its amplitude is less than 3×10^{-17} $(\Delta L/L)$, a limit consistent with our wide-band noise measurement and integration bandwidth.

Using Eq. (1) we conclude that the gravitational flux striking the earth from the Crab pulsar is less than 10^{9} ergs/sec cm^2. This limit is not astrophysically significant in the light of the previous discussion.

We are aware of only one other published experiment similar to the work reported here. Wiggins and Press[14] used the Large Aperture Seismic Array in Montana in an unsuccessful search for gravitational flux from several pulsars near 1 Hz. The minimum detectable motion in their work was 10^{-9} cm, four orders of magnitude larger than the current work.

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Radiation from a Free Electron Interacting with a Circularly Polarized Laser Pulse*

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The radiation from an electron driven by an intense circularly polarized electromagnetic wave of finite duration and smoothly varying intensity is analyzed. The effect of radiative reaction on the electron orbit is accounted for self-consistently. Expressions are derived for the total scattering cross section, the angular and spectral distributions, and the polarization tensor of the scattered radiation. The results are relativistically correct, and exhibit the dependence of the characteristics of the scattered radiation on the intensity profile of the incident radiation.

In the present communication we analyze the properties of the radiation emitted by a free electron interacting with an intense circularly polarized electromagnetic wave of finite duration. First we derive relativistically correct expressions for the total cross section associated with the scattering of the incident wave by the moving electron. We then calculate the angular and spectral distributions of the scattered radiation and discuss the polarization of its spectral components.

Sarachik and Schappert[1] have dealt with essentially all of these characteristics of the scattered radiation for the case of an elliptically polarized incident wave. Although radiation damping is neglected in their calculations, they point out that for a sufficiently long pulse it may induce a significant modification of the electron motion and consequently of the emitted radiation. Several authors[2-5] have discussed such a modification of the particle motion. For a linearly polarized incident wave, Sen Gupta[5] has employed an expansion technique valid for pulses of short duration to calculate the modified electron orbit and the associated change of the spectrum of the scattered radiation.

The analysis we are presenting is applicable to pulses of arbitrary duration. We assume the pulse of the incident electromagnetic radiation to be characterized by the vector potential

$$\vec{A}(\eta) = \mathrm{Re}[A_0(\eta) e^{-i\eta}(\hat{e}_1 + i\lambda\hat{e}_2)], \qquad (1)$$

where λ denotes the helicity, the (real) amplitude $A_0(\eta)$ is a smooth function of the phase

$$\eta = \omega_0 t - k_0 z, \qquad (2)$$

and where the propagation vector $\vec{k}_0 = k_0\hat{e}_3$ lies along the z axis of the Cartesian coordinate system (x, y, z). The amplitude $A_0(\eta)$ is different from zero only in the given interval $0 \leq \eta \leq \eta_f$. Since the electric field associated with the vector potential (1) is

$$\vec{E}(\eta) = -\frac{1}{c}\frac{\partial \vec{A}(\eta)}{\partial t} = -\frac{\omega}{c}\frac{d\vec{A}(\eta)}{d\eta},$$

it follows that