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Comments on the validity of the unified classical path theory of stark broadening

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Abstract. In a recent paper by Frisch and Brissaud certain aspects of the unified classical path theory for Stark broadening are criticized; since we feel that their results could be misleading, we offer some comments on their paper.

The soluble model discussed in § 3 of Frisch and Brissaud (1971) assumes that a radiating hydrogen atom interacts with a gas of perturbing electrons via *unshielded* coulomb fields. Positive charges were not included and the electron–electron coulomb repulsion was ignored; these are the interactions which produce shielding in realistic systems. Their model should not be confused with the familiar 'electron gas' or 'single component' model of plasma which is electrically neutral and which represents correlations by Debye shielding or correlation function methods. The unified theory assumes that the binary radiator–perturber interactions are statistically independent. This is an extremely bad approximation for unshielded coulomb fields (eg the two-body correlation function diverges) hence it is not surprising that the unified theory breaks down for such a model. This breakdown manifests itself in the familiar logarithmic divergence at large impact parameters which occurs for unshielded coulomb fields. Since the Frisch–Brissaud model cannot represent any physically realistic systems, this failure of the unified theory is of no consequence.

In § 4 Frisch and Brissaud present an argument for a more general case. They employ a parameter $\Delta\omega_{qs}$ which is the average Stark splitting of Ly α (ie the splitting due to a Holtsmark electric field strength) and a parameter A which is approximately equal to the impact width of a single Stark component. They assert that deviations from static behaviour will narrow a line hence the impact width must always be smaller than the average Stark splitting; consequently they find that the true line centre intensity for a line normalized to π must always satisfy $I(0) \geqslant \pi/\Delta\omega_{qs}$. They further argue that the line centre intensity in the unified theory is $I_{UCP}(0) = A^{-1}$ (also normalized to π). Thus they conclude that the unified theory is invalid when $A^{-1} < \pi/\Delta\omega_{qs}$ (this inequality is just the cube root of their equation (18)).

First, if their estimate of $I(0)$ in equation (17) is based on a Lorentz profile normalized to π, $I(\omega) = \gamma/(\omega^2+\gamma^2)$, it should read $I(0) \geqslant 1/\Delta\omega_{qs}$ rather than $I(0) \geqslant \pi/\Delta\omega_{qs}$. Second, one can evaluate the impact width for the unshifted component of Ly α by simply evaluating the matrix element $\langle 201|\mathscr{L}(0)|201\rangle$ where $|201\rangle$ is the parabolic state $|nqm\rangle$ which represents the unshifted component (see equations (26), (27) and

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(18) of Vidal *et al* (1971)) this impact width is $\Delta\omega_I = A/3$. Further, since the line centre intensity is determined largely by the unshifted component, and since the unshifted component makes up $\frac{2}{3}$ of the Ly α line intensity, the line centre intensity in the unified theory (normalized to π) should be $I_{UCP}(0) = 2/A$. This result is not rigorously correct since the shifted components also contribute a small amount to the line centre but the estimate $2/A$ agrees with our computer results to better than 10% for the cases of interest in this paper.

We thus find that the condition $I_{UCP}(0) \geqslant I(0)$ is equivalent to the inequality $A < 2\Delta\omega_{qs}$ and not $A < \Delta\omega_{qs}/\pi$ which they used. Further, when one evaluates the conditions under which this inequality is violated, one must not replace the constant $C = (\frac{1}{2} - \lg y_{min})$ by 7 as they do (see their equation (15)); indeed, at the densities where $A < 2\Delta\omega_{qs}$ is violated, C is the order of unity not 7. Correcting these errors, we find that the condition $I_{UCP}(0) \geqslant I(0)$ is violated at a temperature $T = 10^4$ for densities $n_e > 10^{20}$ not for $n_e \simeq 10^{17}$ as they claim. The unified theory does indeed break down for such densities, to find out why, we consider the following argument.

We will take $C = 1$ since that is its approximate value when $A \simeq 2\Delta\omega_{qs}$. We next introduce the thermal de Broglie wavelength $\lambda = h/(2\pi mkT)^{1/2}$ and the average particle spacing $r_a = n_e^{-1/3}$. In terms of these parameters we find

$$A = (18\hbar/m_e)(\lambda/r_a^3) \tag{1}$$

$$\Delta\omega_{qs} = (3\hbar/m_e)(4\pi/3)^{2/3} r_a^{-2} \tag{2}$$

hence $A \leqslant 2\Delta\omega_{qs}$ is equivalent to

$$2(\lambda/r_a) \leqslant 1. \tag{3}$$

When this inequality breaks down, the de Broglie wavelength is on the order of or larger than the average interparticle distance; for such a case the binary collision treatment breaks down and the concept of a classical particle becomes meaningless. The unified theory is certainly invalid for such cases.

Since the above argument applies only to Ly α, we would like to note that the Frisch and Brissaud validity criterion $\Delta\omega_I < \Delta\omega_{qs}$ appears to be equivalent to the usual condition (see Smith *et al* 1969) that strong collisions do not overlap in time. The latter condition may be written (see § (1) of Smith *et al*) $\nu_s \leqslant \Delta\omega_c$ where ν_s is the strong collision frequency and $\Delta\omega_c$ is the Weisskopf frequency or the inverse of a strong collision duration time. Since $\nu_s \simeq \Delta\omega_I$ if we put $C = 1$ in the latter and since $\Delta\omega_c \sim \Delta\omega_{qs}$ when $\Delta\omega_I \sim \Delta\omega_{qs}$, it appears that $\Delta\omega_I \lesssim \Delta\omega_{qs}$ is equivalent to $\nu_s \lesssim \Delta\omega_c$.

In conclusion, we agree with Frisch and Brissaud that the unified theory breaks down for unshielded coulomb fields, but such a model is unrealistic and hence not very illuminating. We also agree that the unified theory for Ly α breaks down when $I_{UCP}(0) < I(0)$ because classical mechanics is invalid. We do not agree with the numerical results of Frisch and Brissaud which put this breakdown at $T = 10^4$, $n_e = 10^{17}$; our calculations give $n_e > 10^{20}$ for that temperature. We would also like to point out that the condition $\Delta\omega_I \lesssim \Delta\omega_{qs}$ used by Frisch and Brissaud appears to be equivalent to the usual requirement that strong collisions must not overlap in time.

We do not wish to leave the impression that the only approximation made by the unified theory is the assumption that strong collisions are non-overlapping. The current versions of the unified theory usually ignore time ordering, dynamic ion effects, the effect of static ion fields on the electron time development operator and quantum effects resulting from close electron–atom encounters to name only the most important approximations. We generally expect these approximations to have a 10% or less effect on hydrogen lines although there are cases where larger discrepancies are observed in the very line centre (see Wiese *et al* 1972).

References

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