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A re-evaluation of the relativistic redshift on frequency standards at NIST, Boulder, Colorado, USA*

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Abstract

We re-evaluated the relativistic redshift correction applicable to the frequency standards at the National Institute of Standards and Technology (NIST) in Boulder, Colorado, USA, based on a precise GPS survey of three benchmarks on the roof of the building where these standards had been previously housed, and on global and regional geoid models supported by data from the GRACE and GOCE missions, including EGM2008, USGG2009, and USGG2012. We also evaluated the redshift offset based on the published NAVD88 geopotential number of the leveling benchmark Q407 located on the side of Building 1 at NIST, Boulder, Colorado, USA, after estimating the bias of the NAVD88 datum at our specific location. Based on these results, our current best estimate of the relativistic redshift correction, if frequency standards were located at the height of the leveling benchmark Q407 outside the second floor of Building 1, with respect to the EGM2008 geoid whose potential has been estimated to be $W_0 = 62\ 636\ 855.69\ \text{m}^2\ \text{s}^{-2}$, is equal to $(-1798.50\pm0.06)\times10^{-16}$. The corresponding value, with respect to an equipotential surface defined by the International Astronomical Union's (IAU) adopted value of $W_0 = 62\ 636\ 856.0\ m^2\ s^{-2}$, is $(-1798.53 \pm 0.06) \times 10^{-16}$. These values are comparable to the value of $(-1798.70 \pm 0.30) \times 10^{-16}$, estimated by Pavlis and Weiss in 2003, with respect to an equipotential surface defined by $W_0 = 62\ 636\ 856.88\ \mathrm{m}^2\ \mathrm{s}^{-2}$. The minus sign implies that clocks run faster in the laboratory in Boulder than a corresponding clock located on the geoid.

Keywords: frequency standard, general relativity, relativistic frequency redshift, gravity potential

(Some figures may appear in colour only in the online journal)

1. Introduction

Primary frequency standards that realize the definition of the second based on the Cesium (Cs) atom are used to steer International Atomic Time (TAI). Their frequency needs to be adjusted for effects described by the theory of relativity to that at which these would operate if located on the geoid. Best standards for the current definition of the second are approaching uncertainties of one part in 10^{16} . However, optical

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frequency standards are now approaching uncertainties of one part in 10^{18} and are expected to lead to a new definition of the second. Their performance will require centimeter-level or better orthometric height accuracy in order to appropriately calculate the redshift frequency correction necessary for their use in TAI, commensurate with their frequency accuracy.

We begin by discussing how these optical frequency standards may be of benefit to geodesy. Following the original ideas that Bjerhammar presented more than thirty years ago [1], we discuss the prospects of using interconnected highly accurate frequency standards for the direct determination of gravity potential differences. These may provide in the not-too-distant

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future an alternative approach for the establishment of vertical datums and for the monitoring of their temporal changes, as well as for the independent verification of the accuracy of global and regional/local geoid models, if combined with accurate positioning from a Global Navigation Satellite System (GNSS). We then present our analysis and discuss the results that we obtained using three different methods for the estimation of the relativistic redshift correction applicable to a point that can be connected via local surveys to the current location of the frequency standards at NIST in Boulder, Colorado, USA. Two of these methods involved the use of geoid models, while the third method involved the independent result from spirit leveling and gravimetry. We conclude with our current best estimate of the redshift correction and of its uncertainty, considering the main error sources contributing to the total error budget. We compare our current estimates to those published by Pavlis and Weiss in 2003 [2], using the data and models that were available at that time. We note that our results apply to the orthometric height of a marker labeled Q407 outside the second floor of Building 1 at NIST, Boulder. Most of the frequency standards have been moved since 2013 to the new Building 81. These results will have to be adjusted to the appropriate orthometric height of any standard in order to be used.

2. Use of atomic clocks in geodesy

Currently, the accuracy and stability of optical clocks is competitive with state-of-the-art geoid determination accuracy over areas covered with dense and accurate gravity data [3, 4]. Optical clock absolute accuracy is approaching 1×10^{-18} [5–9], corresponding to about 0.9 cm of orthometric height difference accuracy. However, these clocks today are laboratory devices and cannot be taken into the field, although transportable devices are already being reported with accuracies on the order of 10^{-17} [10, 11]. In the future, such highly accurate and stable frequency standards may support the establishment and inter-connection of vertical datums, and, in combination with accurate GNSS-based positioning, the independent verification and validation of the accuracy of global and regional/local geoid models. In order to achieve this, however, it will be necessary to overcome the challenges of frequency transfer that could maintain that accuracy, either via fiber, or via free space. An experiment published in August of 2016 [12] describes a comparison of optical clocks over a 15 km distance, and demonstrates that the height difference was determined with an uncertainty of 5 cm. Frequency transfer via fiber has been shown to be extremely accurate in a number of experiments, reaching 1 part in 10²⁰ over 1400 km [13, 14]. Free-space frequency transfer has seen research progress approaching 2×10^{-19} [15, 16] and may lead toward transfer across the Earth, or perhaps via satellite, although much research remains to be done to enable free-space frequency transfer of such accuracy over such distances on a routine basis.

The proper inter-comparison of frequencies of optical clocks requires accounting for the differential gravity potential between them. The use of optical clocks for the generation of TAI, which will follow a redefinition of the second based on an optical transition, will further require reducing their frequencies to those corresponding to an equipotential surface defined by a 'standard' (conventional) value of gravity potential. With the absolute accuracies of optical clocks approaching 1×10^{-18} this in turn requires the determination of the absolute gravity potential at clock locations to $\pm 0.1 \text{ m}^2 \text{ s}^{-2}$ or better, corresponding to $\pm 0.01 \text{ m}$ or better of orthometric height accuracy. Geodetic determination of gravity potential differences can be performed either using spirit leveling combined with gravimetry, or using gravimetric techniques to estimate the gravity potential at sites with precisely determined geocentric coordinates [17, 18]. The former is a differential technique capable of providing only gravity potential differences between points that are not separated by ocean. Although spirit leveling and gravimetry can deliver sub-millimeter accuracy over short distances, this technique is susceptible to systematic errors that accumulate over large distances [4, 17]. The latter can deliver point-wise absolute gravity potential values, W, but relies critically on the accuracy of the determination of the geometric (ellipsoidal) height of the site, h, on the availability of an accurate reference gravitational model, as well as of detailed and accurate gravity (and elevation) observations, especially over a 'cap' centered at the site in question. Currently, the best results that this technique can deliver correspond to gravity potential uncertainties of ± 0.2 to ± 0.3 m² s⁻² [4], corresponding to clock fractional frequency uncertainties of $2-3 \times 10^{-18}$. With the possible exception of sites located over exceptionally smooth regions of the gravity field and supported by extremely high accuracy vertical positioning, it is difficult to envision absolute gravity potential determinations to $\pm 0.1 \text{ m}^2\text{s}^{-2}$ or better, achievable anywhere routinely, anytime in the near future.

Given the above, it appears that in order to reap the benefits of the exquisite accuracy and stability of optical frequency standards in geodetic applications, it would be necessary to establish cost-effective and efficient techniques to transfer their frequencies over both continental and inter-continental distances. If this can be accomplished, a network of sites, well distributed over the continents, where such inter-connected accurate clocks are operating continuously could realize a 'World Vertical Datum'. At the 10^{-18} level, a challenge in using these clocks to realize a 'World Vertical Datum' will be to distinguish true geopotential changes from subtle changes in the physical realization of the clocks. Continuous monitoring of the temporally varying gravity potential differences between such 'fiducial' sites could offer a point-wise counterpart of the information that is deduced today from satellite missions like the Gravity Recovery And Climate Experiment (GRACE) with (approximately) monthly temporal resolution. Although not absolutely necessary, it would be very beneficial if these fiducial sites realizing the World Vertical Datum were 'co-located' with sites that participate in the realization of the International Terrestrial Reference Frame (ITRF) [19], whose geocentric positions and motions are accurately determined and monitored. Development of portable clocks with commensurate accuracy and stability would allow the densification of such continental networks, provided again that cost-effective and efficient techniques to transfer frequency without degrading the clock's accuracy become available.

Of course, considerable expense would be involved with development and maintenance of such a network of clocks and with the associated frequency transfer systems. Geodetic applications requiring accurate point-wise realization and continuous temporal monitoring of variations of equipotential surfaces, i.e. vertical datum realization and monitoring, would be the main beneficiaries of such capabilities and infrastructure. Currently, only satellite missions like GRACE can address some aspects of these applications, but with limited spatial resolution, due to the attenuation of the gravitational signal with altitude. Point-wise determination of gravity potential differences and of their temporal variations, which would be readily available from frequency difference monitoring of accurate frequency standards, is at present impossible using space techniques, and prohibitively expensive using terrestrial methods. The capabilities provided by a network of these new clocks could conceivably support the temporal monitoring of subsurface density variations arising from variations in water or hydrocarbon deposits, with higher spatial resolution than that supported by space techniques (see also [20, 21]). Note that other scientific applications motivate the exploration of clock networks, such as optical time metrology, tests of fundamental physics, and time/frequency dissemination for radio astronomy; therefore such clock networks could be multi-tasked to significant geodetic benefit. Whether these, and possibly other future applications justify the investment required to develop and maintain the necessary technological infrastructure remains to be seen.

It is also conceivable that a network of these new clocks would be placed in orbit around the Earth [22], for which the following applications have been suggested:

- i. Distribution of time and frequency on earth and in space from an earth-orbiting 'master clock'.
- ii. Mapping of the earth's gravity field by frequency comparison of terrestrial clocks with the master clock. The terrestrial clocks are transported over land or sea to cover areas of interest. This method will complement terrestrial clock–clock comparisons using optical fibers.
- iii. Precision spacecraft navigation using the master clock signals.
- iv. Space-based Very Long Baseline Interferometry (VLBI).
- v. Metrological applications, e.g. space gravitational-wave interferometers.
- vi. High precision tests of gravitational physics.

3. Theory

The objective here is to calculate the fractional frequency offset correction necessary to reduce the frequency of an oscillator located at NIST in Boulder, Colorado, USA, to that at which it would operate if located on the geoid. This frequency offset is caused by the gravity potential difference between the oscillator's location in Boulder and an arbitrary location on the geoid. The relativistic redshift results in a clock running faster at higher elevations on the Earth, in agreement with the theories of special and general relativity. The general relativistic effect is proportional to the gravitational potential generated by the Earth's mass, the geopotential. In relativity, the geopotential is defined by a convention that assigns to it a negative value, approaching zero as a particle moves towards infinity away from the attracting body. Geodesy uses a sign convention for the geopotential opposite to that used in relativity theory. In this paper we will use the geodetic convention, whereby geopotential values are positive.

A second effect in relativity enters, the so-called secondorder Doppler shift of special relativity, in which a clock runs slower as it moves faster, relative to a clock at rest with the observer. The rotation of the Earth, therefore, gives rise to a centripetal potential that also changes the clock's frequency. We differentiate between the potential due to *gravitation* and that due to *gravity*: the former arises from the presence of attracting masses only, the latter contains in addition the centripetal potential due to the Earth's rotation ([23], section 2-1). It is the gravity potential that we need to consider here, therefore the term 'gravitational red shift' is somewhat misleading and has been avoided herein.

A primary frequency standard that contributes to International Atomic Time (TAI) must be corrected to run at the frequency clocks would run on the Earth's geoid, a surface of constant gravity potential that approximates mean sea level in a well-defined way. It is therefore necessary to determine the difference in gravity potential $(W_0 - W_P)$, between the geoid (0) and the location of a primary frequency standard (P), in order to correct for this frequency offset, with the correction given by [1]

$$(f_0 - f_P)/f_0 = \Delta f/f_0 = (W_P - W_0)/c^2,$$
 (1)

where *c* denotes the speed of light. Note that if the point *P* is above the geoid (as is the case for the points in question in Boulder, Colorado), we generally have $W_P < W_0$ in the geodetic convention for the sign of the geopotential. Hence, Δf is negative in this case, and the (additive) frequency correction would make a clock in Boulder run slower, in order to match the frequency of a corresponding clock located on the geoid. We emphasize that this definition of Δf is for the *correction* that needs to be added to the clock's frequency, not for the offset of the rate of the clock itself. Hence, since the clocks run faster at the higher location, the correction that must be added is negative.

The geopotential number $C = W_0 - W_P$ is given by ([23], page 56)

$$C = W_0 - W_P = \int_{H=0}^{H=H_P} g \cdot \mathrm{d}H, \qquad (2)$$

where *g* is the magnitude of the gravity acceleration vector, and d*H* is the length increment along the positive upward plumb line. The path-independent line integral in equation (2) starts from a reference equipotential surface whose gravity potential is W_0 (on which every point has orthometric height, *H*, equal to zero) and ends at the station location where $W = W_P$ and $H = H_P$. Although the reference equipotential surface can be defined unambiguously through a prescribed value of W_0 , such a definition has limited practical value for the physical realization of this surface, since absolute potentials cannot be measured. In theory, any equipotential surface of the gravity field is a suitable reference surface for orthometric heights worldwide. However, the human conception of 'heights' and historic practices make it convenient for such a reference surface (vertical datum) to be 'close' to the mean sea surface (MSS). Historically, the vertical datum of a country or a set of countries has been realized by prescribing a certain value to the orthometric height or the geopotential number of some tide gauge station(s). The geopotential numbers and orthometric heights of other points could then be determined by use of spirit leveling and gravity measurements, through the evaluation of a discrete counterpart (summation) of equation (2) ([23], chapter 4).

The presence of a quasi-stationary (i.e. non-vanishing through averaging over long time periods) component within the Dynamic Ocean Topography (DOT) results in departures of the MSS from an equipotential surface ranging geographically between -2 m and +1 m, approximately. Due to these departures (and in some cases due to additional considerations related to mapping applications), different vertical datums refer to different equipotential surfaces. Therefore, given a datum-dependent value of the geopotential number C, the determination of $\Delta f/f_0$ with respect to a unique equipotential surface requires the estimation of that datum's offset from that unique equipotential surface. A unique equipotential surface-the geoid-that closely approximates (in some prescribed fashion) the MSS has to be defined and realized through the operational development of models [24, 25]. There exist global geoid models, developed through the combination of satellite tracking data, surface and airborne gravimetry, and satellite altimetry, as well as regional/local geoid models. The latter are usually developed by incorporating regionally and/or locally available detailed land, marine, and airborne gravimetric data and very high resolution digital topographic model information, into the gravimetric information provided by a global geoid model that is used as a reference. A comprehensive presentation of the theory and the state-of-the-art methodologies used for global and regional/local geoid determination can be found in [26].

It is useful to recall here that since on the surface of the Earth $g \approx 9.8 \text{ ms}^{-2}$, one centimeter of orthometric height change corresponds to a fractional frequency offset of $\Delta f/f_0 \approx 1.1 \times 10^{-18}$. Therefore, a frequency standard with an accuracy of 1 \times 10⁻¹⁸ requires the determination of its orthometric height to an accuracy of 0.9 cm or better, in order for the relativistic frequency offset correction to not degrade the accuracy of the frequency standard in its contribution to TAI. Moreover, in order for such frequency standards located on different continents to be inter-comparable, such an orthometric height should refer to an equipotential surface of the Earth's gravity field that is uniformly accurate globally to 0.9 cm or better. This represents a very stringent requirement for orthometric height determination with respect to a unique global vertical datum, and presently challenges even the best geodetic results. However, as we discussed previously in section 2, viewed from the opposite perspective, this challenge offers the opportunity to establish a unique global vertical datum by determining the geopotential differences between ultra-precise frequency standards from measurements of the relativistic red shift frequency offsets between them.

In the next section we describe the specific data and the computational methods and models that we used for our estimation of the fractional frequency correction $\Delta f/f_0$ for the frequency standards at NIST in Boulder, Colorado.

4. Computational aspects

4.1. Data used

During the summer of 2011, the National Geodetic Survey (NGS) performed precise GPS positioning of three points on the roof of the NIST Building 1, which housed the frequency standards at that time. These points are designated NIST2, NIST4, and NIST5. The distances between these points are: NIST2—NIST4 = 52.705 m, NIST2—NIST5 = 53.871 m, and NIST4—NIST5 = 11.142 m. In addition to the GPS positioning of these three points, NGS also performed Order 1/Class 2 spirit leveling to connect the roof points with the NAVD88 leveling benchmark Q407 [27], located on the side of NIST Building 1 at the level of the second floor, as compared to the roof above the fourth floor. See figure 1 for a graphic view of the relationships among these points. One can see the location of the leveling benchmark Q407 on the side of the building; we determine here the gravity potential and the fractional frequency correction for the vertical level of this benchmark. We obtained from NGS the GPS network final adjustment report, which provided the coordinates of these points with respect to both the NAD 83 and the IGS08 reference frames. We also obtained the information pertaining to the leveling lines connecting the roof points with the Q407 benchmark. The formal errors and closure on the leveling were both 2 mm, hence these errors do not contribute significantly to the final uncertainty we report.

We first transformed the NAD 83 roof point coordinates provided by NGS to the corresponding WGS 84(G1762) coordinates [28], using the transformation formulas and transformation parameters given in ([28], section 7.3.1 and table 7.1). The resulting WGS 84(G1762) coordinates differed little from the corresponding IGS08 coordinates. In particular, the maximum absolute difference in ellipsoidal heights was 0.7 mm. Table 1 summarizes the positioning information of the points used in the computations. For the leveling benchmark Q407 we do not have precise positioning information; we only have its orthometric height, and its geopotential number, both with respect to the North American Vertical Datum 1988 (NAVD88).

4.2. Computational methods

In order to estimate the fractional frequency correction $\Delta f/f_0$ for the frequency standards at NIST in Boulder, Colorado, to adjust their rates to what they would have on the geoid, we determined the geopotential numbers $C_i = W_0 - W_{P_i}$



Figure 1. Building 1 on the campus of NIST in Boulder, Colorado, USA, showing the location of the four benchmarks used in this work. Imagery © 2016 Google.

Table 1. Positioning information of the points used	d in the computation	18.
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	Latitude	Longitude	Ellipsoidal height (h_P)		Orthometric height (H_P)
	WGS 84 (G1762)			$\Delta H = H_P - H_{Q407}$	NAVD88
Point 'P'	(Decimal degree)		(m)	(m)	(m)
NIST2 NIST4 NIST5 Q407	39.995 358 482 39.994 982 525 39.994 921 299	254.737 520 351 254.737 143 850 254.737 247 176	1643.762 1643.778 1643.775	9.737 9.732 9.730 0.000	1660.366 1660.361 1660.359 1650.629

corresponding to the three precisely positioned benchmarks P_i (i = 1, 2, 3) on the roof of Building 1, where frequency standards had been housed. From these, we then computed the corresponding geopotential numbers at the level (elevation) of the second floor of NIST Building 1, i.e. the same as the level of the Q407 leveling benchmark. For these computations we used:

- I. The global high resolution Earth Gravitational Model 2008 (EGM2008) [3].
- II. Two regional high resolution geoid models developed by the US National Geodetic Survey (NGS): the USGG2009 [29], and the USGG2012 that is accessible from www. ngs.noaa.gov/GEOID/USGG2012/
- III. The geopotential number published by NGS for the leveling benchmark designated Q407 [27]. This geopotential number is obtained from spirit leveling and gravimetry, as these measurements were incorporated into NAVD88.

The first two methods share some long wavelength errors, but the third method is *independent* of the other two. However, implementation of the third method requires the estimation of the offset of NAVD88 with respect to an ideal global geoid surface at the location of the frequency standards. We will discuss in detail how we estimated that offset.

A global Earth Gravitational Model, available in the form of spherical harmonic coefficients, permits the computation of the geopotential numbers to be performed in two ways, as we discuss next.

4.2.1. Method I(a). Following the formulation in ([2], section 2.3), the gravity potential W at a point P is the sum of the gravitational potential V and the centripetal potential Φ ,

$$W_P = V(r_P, \theta_P, \lambda_P) + \Phi(r_P, \theta_P), \qquad (3)$$

where $(r_P, \theta_P, \lambda_P)$ are the geocentric radius, geocentric co-latitude (90° minus geocentric latitude), and longitude respectively of the point *P*. These geocentric coordinates are computed from the geodetic WGS 84(G1762) coordinates given in table 1, and the defining semi-major axis *a* and flattening *f* of the WGS 84 reference ellipsoid [28],

$$a = 6378 \ 137.0 \ \mathrm{m}$$
 (4*a*)

Then,

$$f = 1/298.257\ 223\ 563. \tag{4b}$$

$$V(r_P, \theta_P, \lambda_P) = \frac{\mathrm{GM}}{r_P} \left[1 + \sum_{n=2}^{\infty} \left(\frac{a}{r_P} \right)^n \sum_{m=-n}^n \bar{C}_{nm} \cdot \bar{Y}_{nm}(\theta_P, \lambda_P) \right]$$
(5)

and

$$\Phi(r_P, \theta_P) = \frac{1}{2}\omega^2 r_P^2 \sin^2 \theta_P.$$
(6)

The notation used here is explained in detail in ([2], section 2.3). For the geocentric gravitational constant GM and the (mean) rotational speed of the Earth ω we used here the values:

$$GM = 3.986\ 004\ 418 \times 10^{14}\ m^3\ s^{-2} \tag{7a}$$

$$\omega = 7292 \ 115 \times 10^{-11} \ \text{rad s}^{-1}, \tag{7b}$$

which are consistent with the WGS 84 geodetic reference system [28]. The original EGM2008 spherical harmonic potential coefficients are consistent with the scaling values of GM = 3.986 004 415 × 10¹⁴ m³ s⁻² and a = 6378 136.3 m (see also ([3], appendix A)). These were properly re-scaled based on the invariance of the geopotential, as explained in ([33], page 7–79, equation 7.3.5.3-1), to render them consistent with the WGS 84 geodetic reference system's values of GM and a. This re-scaling has already been applied in the computation of the 1×1 arc-minute geoid undulation grid used in Method I(b) below. The geoid undulation values computed by Method I(b) and Method II are consistent with the WGS 84 reference values. After evaluating the gravity potential, W_P , at a point P, in order to evaluate the geopotential number, $C = W_0 - W_P$, of that point we need the value of the gravity potential on the geoid, W_0 . Here we used the value:

$$W_0 = 62\ 636\ 855.69\ \mathrm{m}^2\ \mathrm{s}^{-2}.$$
 (8)

This value is consistent with the $-41 \,\mathrm{cm}$ constant (zero degree) shift that must be applied to height anomalies computed from the EGM2008 model with respect to an 'ideal' mean-Earth ellipsoid (whose semi-major axis remains unspecified), in order to refer them to the specific reference ellipsoid used in the WGS 84 reference system ([3], section 6; [33], section 11.2). This W_0 value was also independently estimated previously as described in [30]. As it is also explained in ([3], section 6), the $-41 \,\mathrm{cm}$ zero degree height anomaly corresponds to -46.3 cm zero degree geoid undulation, due to the fact that the height anomaly to geoid undulation conversion terms do not average to zero globally. After the development of EGM2008, satellite altimetry was used to estimate the $-41 \,\mathrm{cm}$ height anomaly shift that is necessary in order to reference EGM2008 height anomalies computed without a zero degree term, to the specific ellipsoid of the WGS 84 geodetic reference system. This process followed essentially the concept described in [25]. The above value for W_0 differs from the value $W_0 = 62\ 636\ 856.0\ m^2\ s^{-2}$ that is implied by the speed of light value $c = 299\ 792\ 458\ {\rm m\ s^{-2}}$ and the $L_G = W_0/c^2 = 6.969\ 290\ 134\ imes\ 10^{-10}$ value that was adopted as a defining constant in the 2000 Resolution B1.9 of the International Astronomical Union (IAU) (see also ([31], page 2700)). We should note here that the W_0 value adopted by the IAU is a value derived from the defining constants L_G and c, and thus has zero uncertainty. In contrast, the W_0 value of equation (8), which represents the gravity potential on the geoid, is a numerical computation result subject, on the one hand to the uncertainty with which the oceanic geoid surface can be mapped, based on satellite altimetry data and a model of the quasi-stationary dynamic ocean topography over a specific time period, and on the other hand, to the uncertainty with which the (absolute) gravity potential over that geoid surface can be calculated, from an imperfect global gravitational model. While the uncertainty of the determination of the W_0 value of equation (8) is estimated to be $\pm 0.5 \text{ m}^2 \text{ s}^{-2}$ [30], this uncertainty does not impact the uncertainty of the computed fractional frequency correction, which depends only on the uncertainty of the computed (absolute) gravity potential at the location of interest. In sections 5 and 6 we present our final results, with respect to both the W_0 value of equation (8) and to the IAU adopted W_0 value.

The formulation presented above was used to compute the gravity potential values implied by EGM2008, to degree 2190 and order 2159, for the three points NIST2, NIST4, and NIST5 that are located on the roof of Building 1, where frequency standards used to be housed. We will be determining the fractional frequency correction for a clock at the orthometric height of the point Q407, on the side of NIST Building 1. Since we will be using the three points on the roof of Building 1 as part of this computation, we define fictitious points beneath each of these roof points at the height of Q407. We call these fictitious points NIST2*, NIST4*, and NIST5*, located directly below the corresponding roof points. We used the following formulas to compute the corresponding gravity potential values for the three fictitious points NIST2*, NIST4*, and NIST5*, and NIST5*,

$$\Delta H_{P^*P} = H_P - H_{P^*} = H_P - H_{Q407} \tag{9a}$$

$$W_{P^*} = W_P + g \Delta H_{P^*P}. \tag{9b}$$

In equation (9), the subscript *P* denotes each of the roof points NIST2, NIST4, and NIST5, while P* denotes each of the corresponding three fictitious points NIST2*, NIST4*, and NIST5* located directly below the roof points, at the level of the leveling benchmark Q407. In principle, the fractional frequency correction values computed for the three points NIST2*, NIST4*, and NIST5* should be nearly identical, as these three points are located on the same vertical level, and are in close horizontal proximity, so that geoid undulation differences between these three points are negligible. Therefore, any differences in the computed fractional frequency correction values are likely to be due to uncertainties of the local GPS survey. H_{O407} denotes the orthometric height of point Q407. The value of the gravity acceleration, g, used in equation (9b) was obtained from NGS's Data Sheet for point Q407, where it is given to be equal to $g = 9.796\ 022\ \mathrm{ms}^{-2}$. The height differences, ΔH_{P^*P} , for the three roof points are given in the 5th column of table 1.

Computational method and model used	Fractional frequency correction $\Delta f/f_0$ (×10 ⁻¹⁶)				
	Q407 ^a	NIST2 ^b	NIST4 ^b	NIST5 ^b	3-point average
I(a) (EGM2008)		-1798.503	-1798.522	-1798.523	-1798.516
I(b) (EGM2008)		-1798.508	-1798.528	-1798.529	-1798.522
II (USGG2009)		-1798.511	-1798.530	-1798.531	-1798.524
II (USGG2012)		-1798.525	-1798.544	-1798.545	-1798.538
III (NAVD88)	-1798.454				

Table 2. Fractional frequency correction results obtained for the four points using different computational methods and models (see text for details). Values refer to $W_0 = 62\ 636\ 855.69\ m^2\ s^{-2}$.

^a After accounting for an estimated 72 cm distortion of the NAVD88 datum at this location.

^b Evaluated at the level of leveling benchmark point Q407.

4.2.2. Method *I*(b). We computed the EGM2008 tide-free geoid undulation values, N_{P_i} , at the three locations NIST2, NIST4 and NIST5, using bi-cubic spline interpolation from the 1 × 1 arc-minute grid that is available at http://earth-info.nga.mil/GandG/wgs84/gravitymod/egm2008/egm08_wgs84. html. This grid is referenced to the WGS 84 geodetic reference system. We then computed the orthometric heights of the three roof points using

$$H_{P_i} = h_{P_i} - N_{P_i}, (10)$$

where h_{P_i} are the WGS 84 (G1762) ellipsoidal heights of the three roof points. We then reduced these orthometric heights to the level of the benchmark Q407, using again the height differences, ΔH_{P^*P} , that are given in the 5th column of table 1. Having determined these orthometric heights indirectly from the corresponding ellipsoidal heights and the EGM2008 geoid undulations, we then used Helmert's equation ([23], equation 4-26) to compute the geopotential numbers of the three fictitious points NIST2*, NIST4* and NIST5* by

$$C = H \left(g + 0.0424H \right). \tag{11}$$

Notice that in equation (11) *C* is given in 'geopotential units' or g.p.u. (1 g.p.u. = 10 m² s⁻²), g in Gal (1 Gal = 10^{-2} ms⁻²), and *H* in kilometers. In equation (11), we used again the value g = 9.796 022 ms⁻², for all three points.

We note here that for the case of EGM2008, which is available both in terms of spherical harmonic coefficients and in terms of a grid of geoid undulations, the results from methods I(a) and I(b) should in principal be identical. Any difference between these results should be within the accuracy of any approximations involved in equation (11) (see also ([23], page 169)), and also any residual inconsistency between the value used for the potential on the geoid W_0 , and the rounded to the nearest centimeter value of -41 cm offset used to shift the EGM2008 height anomalies computed with respect to an 'ideal' mean-Earth ellipsoid to the specific reference ellipsoid used in the WGS 84 reference system. As we show later (see also table 2), the difference of the fractional frequency correction results from these two computational methods using EGM2008 does not exceed 6 \times 10⁻¹⁹, for any of the three points considered here. This corresponds to about 5.5 mm and is well within the consistency expected between the two methods, which is not expected to be better than $\pm 1 \,\text{cm}$. Therefore, the 6 $\times 10^{-19}$ maximum difference result obtained here is also a good internal consistency check of the two methods that we used. We should also note here that according to [31], the formulation upon which equation (1) itself is based is accurate to about 5 parts in 10^{19} .

We performed all EGM2008 geopotential and geoid computations that are reported in this paper in the tide-free system (also known as non-tidal), as far as the permanent tide [32] is concerned. This produces results that are consistent both with the USGG2009 and USGG2012 regional geoid models, and with the NAVD88 dynamic and orthometric height information that is used in this paper.

For geoid models that are only available in terms of a grid of geoid undulations, Method I(a) cannot be applied, and therefore Method I(b) is the method used, as we discuss next.

4.2.3. Method II. We used Helmert's equation (11), with orthometric heights computed from equation (10), using here two regional geoid models developed and released by NGS: the USGG2009 [29], and the USGG2012 (www.ngs.noaa.gov/GEOID/USGG2012/). In both cases, we computed the geoid undulation at each roof point, using the interactive evaluator that is available on-line from NGS for each model (www.ngs.noaa.gov/cgi-bin/GEOID_STUFF/usgg2009_prompt1.prl and www.ngs.noaa.gov/cgi-bin/GEOID_STUFF/usgg2012_prompt1.prl). As in method I(b) above, we then reduced these orthometric heights to the level of the benchmark Q407, using again the height differences, ΔH_{P^*P} , that are given in the 5th column of table 1. Apart from the specific geoid undulation grid used and its associated interpolation algorithm, Method II is identical to Method I(b) in all other respects.

Both the USGG2009 and the USGG2012 geoid undulation grids possess negligible offsets from the 1×1 arc-minute EGM2008 geoid undulation grid that is referenced to the WGS 84 geodetic reference system, which was used in Method I(b) above. Over the entire domain of definition of the two NGS geoid undulation grids that extends from 24 °N to 58 °N and from 230 °E to 300 °E (8574241 values in total), the average difference USGG2009 minus EGM2008 is 0.005 cm and the average difference USGG2012 minus EGM2008 is -0.044 cm. The corresponding standard deviation differences are 1.7 cm and 3.5 cm, respectively. These comparison results indicate that the USGG2009 geoid is only marginally different from the EGM2008 geoid. The larger differences between the EGM2008 and the USGG2012 geoids may reflect in part the contribution of the gravimetric information from the Gravity and steady state Ocean Circulation Explorer (GOCE) mission [36] in the geoid determination, which was used in USGG2012, but was not available when the EGM2008 model was developed.

4.2.4. Method III. The NGS Data Sheet [27] for the leveling benchmark Q407 provides the dynamic height, H^{dyn} , which is defined in ([23], section 4-2) as

$$H^{\rm dyn} = \frac{C}{\gamma_{45^\circ}}.$$
 (12)

This dynamic height and the associated geopotential number are both given in the NGS Data Sheet with respect to NAVD88. The same Data Sheet specifies the value of normal gravity at 45° latitude (γ_{45°) that was used to compute the dynamic height from the NAVD88 geopotential number to be $\gamma_{45^\circ} = 9.806 \ 199 \ ms^{-2}$. From the value $H_{Q407}^{dyn} = 1649.034 \ m$ provided in the NGS Data Sheet, we computed using equation (12) the NAVD88 geopotential number of the Q407 benchmark to be

$$C_{\text{O407}}^{\text{NAVD88}} = 16\ 170.756\ \text{m}^2\ \text{s}^{-2}.$$
 (13)

The geopotential number obtained from the NGS Data Sheet for point Q407 is of limited use if the NAVD88 datum is significantly distorted with respect to the 'true' equipotential surface that represents the global geoid. Such a global geoid surface can be approximated most effectively, albeit with limited resolution, using global gravitational models whose long wavelength component relies primarily on satellite data for its determination. Indications of the presence of a significant tilt in the NAVD88 datum were already identified during the development and evaluation of the EGM96 global gravitational model [33]. The presence of such a tilt was unambiguously verified as soon as the first global gravitational models from the GRACE satellite-to-satellite tracking mission [34] became available. In the following section we present the approach that we followed in order to estimate the NAVD88 reference surface offset from the EGM2008 geoid at the location of interest. The appropriate implementation of Method III depends critically on the accuracy of the determination of this offset.

4.3. NAVD88 distortion determination

We need to estimate the correction that should be applied to the orthometric height of point Q407, in order to account for distortions present in NAVD88. To this end, in April 2014, we acquired the 'GPS on Bench Marks (GPSBM)' dataset that NGS used in the development of the GEOID12A hybrid geoid model, from the publicly accessible website at www.ngs.noaa. gov/GEOID/GEOID12A/GPSonBM12A.shtml. This dataset contains 23961 points over the Contiguous United States (CONUS), supplemented with 737 points from the OPUS database, over the same region. For each point these files contain a record with its ellipsoidal coordinates with respect to the NAD 83(2011) reference frame, its orthometric height with respect to the NAVD88 datum, error information pertaining to the ellipsoidal coordinates, and four rejection flags pertaining to the ellipsoidal height, the orthometric height, the GEOID09 geoid undulation, and a flag raised by the State Advisor when the information pertaining to the specific point is deemed questionable (e.g. a point residing in a region of known subsidence). We edited the CONUS file, rejecting one point that resides in Canada and 911 points where at least one of the rejection flags was raised. This left us with 23960 CONUS points. We edited similarly the OPUS file, rejecting 253 points where at least one of the rejection flags was raised, thus finally retaining 484 OPUS points. We then formed a combined CONUS plus OPUS point file containing 24444 points in total. Using again the transformation formulas and transformation parameters given in ([28], section 7.3.1 and table 7.1), we first transformed the NAD 83(2011) ellipsoidal coordinates of these 24444 points, to the corresponding WGS 84 (G1762) coordinates. For each of these points, P_i , we then computed the quantity

$$\Delta H_{P_i} = H_{P_i}^{\text{NAVD88}} - \left(h_{P_i}^{\text{WGS84}} - N_{P_i}^{\text{EGM2008}}\right), \qquad (14)$$

where $N_{P_i}^{\text{EGM2008}}$ is the value of the EGM2008 geoid undulation at the point P_i , with respect to the WGS 84 geodetic reference system. Our intent here is to extract from the quantities ΔH_{P_i} an estimate of the distortions present in the NAVD88 datum, which we could then use to correct the $H_{P_i}^{\text{NAVD88}}$ orthometric heights, and in particular the orthometric height of point Q407. If all the quantities on the right hand side of equation (14) were errorless, and the NAVD88 datum coincided with the EGM2008 geoid surface, then all ΔH_{P_i} would be zero. Since the distortions that we seek will be attributed *entirely* to NAVD88, and will be used to correct the orthometric heights $H_{P_i}^{\text{NAVD88}}$, our estimation approach should be able to safely assume any errors in the quantities $(h_{P_i}^{\text{WGS84}} - N_{P_i}^{\text{EGM2008}})$ as being negligible. Over long wavelengths (e.g. 500 km and longer), any errors in $h_{P_i}^{\text{WGS84}}$, which is obtained from GPS positioning, are not expected to be significant. Over the same spectral range, that corresponds to maximum spherical harmonic degree 80, $N_{P_i}^{\text{EGM2008}}$ is dominated by the GRACE information (see ([3], figure 7(b))) and has a cumulative geoid undulation error of about 1 cm. Accordingly, the long wavelength (broad) features of the NAVD88 distortions can be estimated accurately, since over these wavelengths, errors in $h_{P_i}^{\text{WGS84}}$ and $N_{P_i}^{\text{EGM2008}}$ can be effectively neglected. Figure 2 presents the quantities ΔH_{P_i} geographically.

Although the distribution of the available points P_i is far from uniform, and has large gaps in some western and southern States, a long wavelength trend that is present in the quantities ΔH_{P_i} emerges from the inspection of figure 2 (see also ([29], figure 2), which illustrates the trend present in quantities opposite to our ΔH_{P_i}). This trend is dominated by a tilt from north-west, where ΔH_{P_i} approximately equals 1.3 m, to south-east, where it equals approximately -0.2m. In order to represent this trend parametrically as a function of location, we used the *trend2d* function from the Generic Mapping Tools (GMT) [35], and estimated using least squares the parameters defining a 10-parameter surface that fits the ΔH_{P_i} quantities. We then computed the



Figure 2. Geographic distribution of the quantities ΔH_{P_i} (in meters) at 24444 locations over the Contiguous Unites States (CONUS) (see text for details).



Figure 3. Residuals $v(\Delta H_{P_i})$ (in millimeters) that remain after removing the 10-parameter fitted surface from the original ΔH_{P_i} quantities (see text for details).

residuals $v(\Delta H_{P_i})$ that remain after removing the fitted surface from the original ΔH_{P_i} quantities. These residuals are shown in figure 3. The standard deviation of the residuals in figure 3 is approximately ± 5 cm, which is comparable to the expected combined random error component associated with the three independent quantities $H_{P_i}^{\text{NAVD88}}$, $h_{P_i}^{\text{WGS84}}$, and $N_{P_i}^{\text{EGM2008}}$ entering in equation (14). This also indicates that

the 10-parameter surface used here does not over-fit the ΔH_{P_i} quantities.

After examining these residuals, we decided to exclude from the analysis all the points whose residuals exceeded in absolute value 15 cm, a threshold that corresponds approximately to three times the standard deviation of the 10-parameter surface fit (\pm 5 cm). The locations of the 341 points that



Figure 4. Locations of 341 points whose residuals (in millimeters) after the 10-parameter surface fit to the quantities ΔH_{P_i} exceed in absolute value 15 cm.



Figure 5. The 10-parameter surface fitted to the 24103 ΔH_{P_i} quantities that passed editing (unit is meter).

were thus excluded is shown in figure 4. In general, the residuals of these excluded points represented also outliers when compared to the residuals of their neighboring points.

We repeated the 10-parameter surface fit to ΔH_{P_i} , using now only the 24103 points that passed the editing described above. Figure 5 shows the 10-parameter surface obtained from this fit, sampled on a regular 30 arc-minute grid in latitude and longitude over CONUS. This rather smooth surface predicts a north-west to south-east tilt, with values ranging from 1.22 m to -0.09 m, with a mean value of 0.64 m and a standard deviation of ± 0.26 m.

The residuals $v(\Delta H_{P_i})$ that remain after removing this surface from the 24103 ΔH_{P_i} quantities are shown in figure 6. Their standard deviation is approximately ± 4.4 cm. Figure 6 shows that there is considerable 'structure' in these residuals (notice that the color-bar scale in figure 6 is different than



Figure 6. Residuals $v(\Delta H_{P_i})$ (in millimeters) that remain after removing the 10-parameter fitted surface of figure 5 from the 24103 ΔH_{P_i} quantities.

the corresponding scales of figures 3 and 4), with clusters of positive and negative values existing in some areas. Further examination and scrutiny of these residuals in order to possibly understand better their origin is outside the scope of this investigation.

We should point out here that by making the parametric form of the surface to be fitted to the quantities ΔH_{P_i} complex enough, one could conceivably drive the residuals $v(\Delta H_{P_i})$ to exceedingly small values. In that fashion, the entire magnitude of the discrepancies ΔH_{P_i} would be attributed to errors (distortions) of the NAVD88 datum, effectively implying that both the ellipsoidal heights $h_{P_i}^{\text{WGS84}}$ and the EGM2008 geoid undulations $N_{P_i}^{\text{EGM2008}}$ are entirely errorless, which of course is not true. The true distortions of the NAVD88 datum and their exact frequency content are unknown. However, the justification that we provided earlier supports well the argument that at least the long wavelength component of the distortions of the NAVD88 datum can be estimated using our approach with sufficient confidence. This implies that the parametric form of the surface to be fitted should be kept simple enough, yet capable of representing the long wavelength features of the NAVD88 datum distortions. In this regard, figure 5 indicates that the 10-parameter surface is a reasonable choice, which captures the major trend in the ΔH_{P_i} values. As expected, the residuals $v(\Delta H_{P_i})$ shown in figure 6 do not exhibit any long wavelength systematic trend, since such a trend has already been removed from the residuals shown in figure 2.

It is important to recognize here that the technique described above for the estimation of the NAVD88 distortions does require that both the ellipsoidal heights $h_{P_i}^{\text{WGS84}}$ and the EGM2008 geoid undulations $N_{P_i}^{\text{EGM2008}}$ are given with respect to a common geodetic reference frame and reference ellipsoid,

the WGS 84 having been chosen here to serve this purpose. We computed and plotted the differences between the ellipsoidal heights $h_{P_i}^{\text{WGS84}}$ and corresponding ellipsoidal heights $h_{P_i}^{\text{NAD83}}$ given with respect to NAD 83. These results (not shown here) indicated that the differences of the ellipsoidal heights given with respect to the two reference frames WGS 84 and NAD 83 are of similar magnitude, overall shape, and spectral content as the ΔH_{P_i} quantities shown in figures 2 and 5. Therefore, improper referencing of the ellipsoidal heights would impact significantly the estimation of the NAVD88 distortions, and would produce erroneous results.

Using the parameters defining the surface shown in figure 5, we evaluated the estimated magnitude of the NAVD88 distortion at the latitude and longitude locations of the three points NIST2, NIST4, and NIST5. As expected, due to the smoothness of the fitted surface and the proximity of the three points, the three values that we obtained were essentially identical, equal to

$$d = 0.72 \text{ m.}$$
 (15)

According to our sign convention, this value of *d* implies that at our location at NIST, the NAVD88 datum surface is located 72 cm *below* the EGM2008 geoid surface. For comparison, in 2003, Pavlis and Weiss [2] estimated this offset to be 30 cm, using the EGM96 gravitational model [33] and the 'GPS on Bench Marks (GPSBM)' data that were available at that time. Notice that in [2], *d* is defined to have the opposite sign than our corresponding ΔH_{P_i} defined in equation (14). In the present sign convention, obtaining the orthometric height of point Q407, corrected for the NAVD88 datum distortion, requires subtracting the value of d = 0.72 m from H_{Q407}^{NAVD88} . Accordingly, we computed using Helmert's equation

$$dC = d(g + 0.0424d), \tag{16}$$

and found $dC = 7.053 \text{ m}^2 \text{ s}^{-2}$, which implies due to equation (13),

$$C_{\rm Q407} = C_{\rm Q407}^{\rm NAVD88} - dC = 16\ 163.703\ {\rm m}^2\ {\rm s}^{-2}.$$
 (17)

This value of the geopotential number of point Q407 is compatible with the values obtained from methods I(a), I(b) and II, to the extent that our estimation of the distortion of NAVD88 at NIST is accurate.

Finally, we note here that equation (14) yields $\Delta H_{\text{NIST2}} = 0.67 \text{ m}$ and $\Delta H_{\text{NIST4}} = \Delta H_{\text{NIST5}} = 0.65 \text{ m}$. As expected, these values are close to the d = 0.72 m estimate that we obtained from the parametric surface that we fitted to all the data available over CONUS.

5. Results

From the geopotential number, C_P , of a point P, we can then compute the fractional frequency correction at that point, based on equations (1) and (2), as:

$$\Delta f/f_0 = -C_P/c^2. \tag{18}$$

For the speed of light, c, we used the value

$$c = 299\ 792\ 458\ \mathrm{m\ s}^{-2}.$$
 (19)

The results of our computations, using the different methods described above, are summarized in table 2.

The last column of table 2 lists the arithmetic averages of the values obtained for the three points NIST2, NIST4, and NIST5, evaluated at the level of leveling benchmark Q407, using each computational method and model. Based on these results, we have to estimate a single value for the fractional frequency correction at NIST in Boulder, Colorado. Towards this goal, averaging the results obtained for the three points using each specific method and model, is a rather reasonable approach. However, averaging results obtained from all the different methods and models presented in table 2 does not necessarily lead to an optimal, or even a well-justified answer, since methods I(a), I(b), and II, and the models used in their implementation share common errors. Firstly, as we already mentioned in section 4.2, the computational methods I(a) and I(b) are equivalent and were implemented primarily as a consistency check. Of these two methods, I(a) should be more rigorous theoretically, since it does not involve any of the assumptions leading to the simple form of Helmert's equation (see also ([23], section 4-4) for details). We therefore consider the results from method I(a) in favor of those from I(b) in any subsequent averaging. Secondly, the results from the use of USGG2009 in method II are essentially identical to those of EGM2008, when using the same computational method I(b). This should be expected since the regional model USGG2009 [29], used essentially the same data as EGM2008, and as noted earlier only marginally differs from EGM2008. USGG2012 offers a more appropriate choice for method II, since this regional model has also benefited from gravimetric information from the GOCE mission [36]. Accordingly, from

the results computed on the basis of geoid models, in the following we consider only the value $-1798.516 \times 10^{-16}$ from method I(a) and the value $-1798.538 \times 10^{-16}$ from method II, using the USGG2012 model. The difference between these two values arises from an equivalent geoid height difference USGG2012 minus EGM2008 of approximately -15 mm, which in part may reflect the effect of the GOCE gravimetric information on the EGM2008 geoid over this region.

The NAVD88 result from method III is in principle independent from the results from the previous methods, which are based on geoid models. This independence however is compromised to some degree, due to the estimation of the long wavelength distortions of the NAVD88 datum, which depends on the long wavelength component of the geoid models. The very long wavelength nature of these distortions, in conjunction with the extremely high accuracy of the GRACE-implied geoid information at these wavelengths, arguably support the assessment of independence of this result from the other.

Accordingly, averaging the result -1798.516×10^{-16} from method I(a) with the result from method III yields $-1798.485 \times 10^{-16}$, while averaging the result $-1798.538 \times 10^{-16}$ from method II (using USGG2012) with the result from method III yields $-1798.496 \times 10^{-16}$. Considering the advantage of the GOCE information in the latter result, we offer the value of -1798.50×10^{-16} as our current best estimate of the fractional frequency offset correction at the level of the Q407 benchmark, outside of the second floor of Building 1, NIST, Boulder, Colorado, USA. This value is consistent with the value of $W_0 = 62\ 636\ 855.69\ m^2\ s^{-2}$ for the potential on the geoid, as this was estimated using the EGM2008 geoid model and the DNSC08B Mean Sea Surface (see [3] for details). With respect to an equipotential surface defined by the IAU adopted value of $W_0 = 62\ 636\ 856.0\ \mathrm{m}^2\ \mathrm{s}^{-2}$, our estimate would become -1798.53×10^{-16} . At our location, the latter surface resides approximately 0.032 m below the former. The NAVD88 datum surface would therefore be 0.688 m below the equipotential surface defined by the IAU adopted value of W_0 .

Equally important to the estimated value of the fractional frequency offset correction at NIST in Boulder, Colorado, is an estimate of its uncertainty. Of the models and data used in the previous sections to determine the value of the fractional frequency offset correction, only the EGM2008 model is accompanied by error estimates ([3], section 5). From the gridded values of the estimated commission (propagated) error of the EGM2008 geoid undulations that are available http://earth-info.nga.mil/GandG/wgs84/gravitymod/ from egm2008/egm08_error.html we obtained for the three points NIST2, NIST4, and NIST5 the same value of ± 7.9 cm. This value translates to an error of $\pm 0.086 \times 10^{-16}$ for the fractional frequency offset correction at NIST, in Boulder, Colorado. The EGM2008 geoid undulation error at NIST may be somewhat pessimistic, as indicated by the \pm 4.4 cm standard deviation of the residuals shown in figure 6. We estimate that a more representative value for the uncertainty of the relative frequency offset at NIST, in Boulder, Colorado is probably around $\pm 0.06 \times 10^{-16}$. We note that in an experiment reported in August 2016 [37], describing the

inter-comparison of optical clocks over a 1415 km distance, the claimed uncertainty for the similarly computed gravity potential difference correction corresponds to an orthometric height uncertainty of about 4 cm. This uncertainty corresponds to a differential gravity potential determination, where common (geographically correlated) gravity model errors cancel out, therefore, as expected, it is slightly smaller than our estimated uncertainty of ~6 cm, which refers to an absolute gravity potential determination. Thereby our current best estimate of the relativistic redshift correction for frequency standards at the second floor of Building 1, NIST, Boulder, Colorado, USA, with respect to the EGM2008 geoid whose potential was estimated to be $W_0 = 62\ 636\ 855.69\ m^2\ s^{-2}$, is equal to $(-1798.50 \pm 0.06) \times 10^{-16}$. The corresponding relativistic redshift correction value, with respect to an equipotential surface defined by the IAU adopted value of $W_0 = 62\ 636\ 856.0\ {\rm m}^2\ {\rm s}^{-2}$ is $(-1798.53\pm 0.06)\times 10^{-16}$. For comparison, using the data and models available in 2003, Pavlis and Weiss had estimated this value to be equal to $(-1798.70 \pm 0.30) \times 10^{-16}$, with respect to an equipotential surface defined by $W_0 = 62\ 636\ 856.88\ \mathrm{m}^2\ \mathrm{s}^{-2}$. Notice that the difference between these estimates is well within their respective uncertainty estimates.

6. Conclusions

We have used precise positioning results from a GPS and leveling survey of three benchmarks on the roof of Building 1 that had previously been housing the frequency standards at NIST in Boulder, Colorado, USA, to re-evaluate the relativistic redshift correction required to reduce their frequency to that at which these would run if located on the geoid. For our computations we used the global gravitational model EGM2008 and the regional geoid models USGG2009, and USGG2012. EGM2008 and USGG2009 are supported by data from the GRACE mission, while USGG2012 incorporated in addition data from the GOCE mission. We also evaluated the redshift offset correction based on the published NAVD88 geopotential number of the leveling benchmark Q407, after estimating the bias of the NAVD88 datum at our specific location. Based on these results, our current best estimate of the relativistic redshift correction needed if there was a frequency standard at NIST at the elevation of Q407, in Boulder, Colorado, USA, with respect to the EGM2008 geoid whose potential was estimated to be $W_0 = 62\ 636\ 855.69\ \mathrm{m}^2\ \mathrm{s}^{-2}$, is equal to $(-1798.50 \pm 0.06) \times 10^{-16}$. The corresponding value, with respect to an equipotential surface defined by the IAU adopted $W_0 = 62\ 636\ 856.0\ \mathrm{m}^2\ \mathrm{s}^{-2}$ value is $(-1798.53 \pm 0.06) \times 10^{-16}$. These values are comparable to the value of $(-1798.70 \pm 0.30) \times 10^{-16}$, estimated by Pavlis and Weiss in 2003. The minus sign implies that clocks run faster in the laboratory in Boulder than a corresponding clock located on the geoid. Since most of the frequency standards at NIST, Boulder, are at different levels in a different building, this will need to be adjusted to apply correctly.

At present, the accuracy and stability of optical clocks is competitive with state-of-the-art geoid determination

accuracy. In the not-too-distant future however, highly accurate and stable clocks may support the establishment and inter-connection of vertical datums, and, in combination with accurate GNSS-based positioning, the independent verification and validation of the accuracy of global and local geoid models, using techniques that follow Bjerhammar's original ideas from the mid 1980s [1]. To accomplish this, the challenges of frequency transfer, either via fiber, or preferably via free space, must be overcome.

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