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NOTE

ELECTRON CORRELATIONS IN STARK BROADENING*

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Abstract—It is shown that the previous calculations of electron correlation effects in Stark broadening are in error because of an incorrect evaluation of the electric field autocorrelation function. The corrected result involves the screening of *both* fields in the autocorrelation function. For hydrogen, this divides the usual Debye impact parameter cutoff by $\sqrt{e} = 1.65$.

THE PURPOSE of this letter is to indicate a correction to previous calculations of Stark broadening. This correction arises from the use of a more accurate expression for the autocorrelation function of the electron microfields. Several authors[1–4] have shown that the contributions of the weak collisions of electrons to Stark broadening can be described in terms of this electric field autocorrelation function. The earlier calculations of the quantity $\langle \mathbf{E}(0)\mathbf{E}(t)\rangle$ neglected interactions entirely and introduced a cutoff in the impact parameter at the order of the Debye length;[5] more recent calculations only accounted for the initial correlations and neglected the correlations established during the time t.[1–4] However, since the correlation time is ω_p^{-1}, it might be expected that such a procedure would break down for $t > \omega_p^{-1}$. That this is the case can be shown by using a result obtained by ROSTOKER[6] and others through the methods of plasma kinetic theory which is suitable whenever the number of particles in a Debye sphere is large. This result is, in Fourier-transformed form (retaining, for convenience, only the real part),

$$G(\Delta\omega) = \mathrm{Re}\int_0^\infty \mathrm{d}t\, e^{-i\Delta\omega t}\langle \mathbf{E}(0)\mathbf{E}(t)\rangle$$

$$= \frac{(4\pi e)^2 n}{2(2\pi)^2}\int \mathrm{d}\mathbf{k}\frac{\mathbf{k}\mathbf{k}}{k^4}\int \mathrm{d}\mathbf{v}\frac{f_1(\mathbf{v})\delta(\Delta\omega-\mathbf{k}.\mathbf{v})}{|\varepsilon^+(\mathbf{k},\Delta\omega|^2}, \tag{1}$$

where

$$\varepsilon^+(\mathbf{k},\Delta\omega) = 1+\frac{\omega_p^2}{k^2}\int \mathrm{d}\mathbf{v}\frac{\mathbf{k}.[\partial f_1(\mathbf{v})/\partial\mathbf{v}]}{\Delta\omega-\mathbf{k}.\mathbf{v}+i\eta}. \tag{2}$$

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The quantities n, ω_p, $f_1(\mathbf{v})$, and η are the mean density, the plasma frequency $(4\pi ne^2/m)^{1/2}$, the one-particle distribution function, and the positive infinitesimal, respectively. The quantity $\varepsilon^+(\mathbf{k}, \Delta\omega)$ is the usual wave number- and frequency-dependent dielectric constant for a plasma.[7]

The quantity $G(\Delta\omega)$ can then be used to obtain the line shape. For hydrogen, $\Delta\omega$ represents the frequency separation from the line center. For isolated lines (non-degenerate), $\Delta\omega$ usually represents the splitting between levels.[8] For $\Delta\omega < \omega_p$, the dynamic dielectric constant can be approximated by the static dielectric constant, $\varepsilon(\mathbf{k}, 0) = 1+(k_D^2/k^2)$, where $k_D^2 = \rho_D^{-2} = 4\pi ne^2/kT$. In this case the result given by equation (1) is that for independent electrons with Debye screened fields. This result differs from that of LEWIS,[3] BARANGER,[2] and GRIEM *et al.*,[1] in that *both fields are screened instead of only one.* This use of double screened fields for hydrogen is equivalent to a reduction in the usual impact parameter cutoff[5] by a factor of $\sqrt{e} = 1.65$.

For $\Delta\omega > \omega_p$, we can approximate the dielectric constant by unity, which then yields essentially the Lewis result[3] giving a cutoff for hydrogen at $k = \Delta\omega/v$. Thus, in this limit the previous results are unchanged.

The remarkable feature of these low and high frequency approximations to $G(\Delta\omega)$ is that they are amazingly exact over essentially the entire regions, $\Delta\omega < \omega_p$ and $\Delta\omega > \omega_p$, respectively. This can be shown by noting that equation (1) can be transformed[9] into an integral which has been evaluated numerically by DAWSON and OBERMAN[10] in connection with an entirely different problem. An examination of their results (see Fig. 1 of Ref. 10) shows that the low-frequency approximation (double screening) can be used in the range $0 \leq \Delta\omega \leq 1.7\,\omega_p$ with less than a 4 per cent error. The high-frequency approximation can be used in the range $\Delta\omega \geq 1.7\,\omega_p$ with less than a 4 per cent error. Moreover, when $\Delta\omega < \omega_p$ or $\Delta\omega > 2.5\,\omega_p$, the error is less than 1 per cent. The double screening correction results in about a 20 per cent change in the function Φ_{ab} calculated by GRIEM *et al.*[5] for hydrogen.

The details of this calculation will be given in a later paper. However, we might note that the correction arises from the fact that the quantity $W(\mathbf{R}_i' t_1 ; \mathbf{R}_j t_2)$, as introduced by LEWIS,[3] was calculated incorrectly. This quantity is the probability of finding the ith electron at $\mathbf{R}_i'$ at t_1, and the jth electron at $\mathbf{R}_j$ at t_2. Lewis assumed that the particles travel straight line paths during the time $t = t_2 - t_1$. In fact, for $i \neq j$, this quantity obeys a more complicated equation of motion. If we introduce $\tilde{W}(\mathbf{R}_i', \mathbf{V}_i', t_1 ; \mathbf{R}_j, \mathbf{V}_j, t_2)$ which is the probability that the ith particle is at $\mathbf{R}_i'$ with velocity V_i' at t_1 and the jth particle is at $\mathbf{R}_j$ with velocity $\mathbf{V}_j$ at t_2, it can be shown[6,11,12] that $n^2\tilde{W}(i,j)+n\tilde{W}(i,i)$ approximately obeys the linearized Vlasov equation [redacted] and not the straight-line equation. The use of the linearized Vlasov equation is only valid for times short compared with the time t_D for 90° deflection of the electrons. However, this time is usually greater than (or comparable to) the time between strong collisions; thus the use of the linearized equation should not impose restrictions on the results obtained within the impact theory (provided of course there are many particles in the Debye sphere). We also note that the effect of the collective motion (plasma waves) is contained in $\varepsilon^+(\mathbf{k}, \omega)$.[13] In this case the integration over wave number completely washes out the effect of the plasma waves. In a future paper we will discuss situations in which the collective motions can play a significant role (e.g., the two-temperature plasma where the ion temperature is much lower than the electron temperature).

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