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Low-noise, high resolution microwave synthesis for atomic frequency standards

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The performance of newer generations of ultra high stability atomic frequency standards based on laser cooled atoms and ions appears to be limited by the local oscillator and the microwave synthesis chain. To improve this performance, the design considerations of microwave synthesizers for Cs hyperfine clock transition at 9.192 GHz that emphasize high resolution, low phase modulation (PM) and amplitude modulation (AM) noise, low spurs near the carrier, and very high phase stability with respect to environmental effects have been studied. Typical performance achieved are: internal fractional frequency stability of 1.5×10^{-15} at 10 s and 1×10^{-18} at 1 day; fractional frequency step of 2×10^{-17} and temperature coefficient less than 0.1-0.5 ps/K. Our design is quite general and enables generation of microwave outputs corresponding to several different atomic transitions. The phase locking of the synthesizer to a high stability external frequency reference based on a cavity stabilized oscillator has been studied.

[Keywords: Atomic frequency standards, Microwave synthesis, Synthesizer, Cavity stabilized oscillator**]**

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1 Introduction

Remarkable progress has been made in the last decade on the development of Cesium frequency standards, especially using laser cooled atoms[1-3]. Accuracy figures near 1×10^{-15} have already been achieved using laser cooled Cs fountain frequency standards[2,3]. Projected accuracies approaching 10^{-16} are expected for the future generation space-borne Cs clocks[4,5]. In addition, the potential stabilities of these standards, based on purely quantum mechanical considerations are expected to be at the level of 10^{-14} $\tau^{-½}$, where τ is the sampling time. These accuracy and stability requirements place stringent demands on the performance of the local oscillator and the microwave synthesizer used to interrogate the atoms. Several schemes of microwave synthesis have been recently described[6-8]. In particular, our earlier synthesizer[8] has been used fairly extensively in the Cs fountains standards in several time and frequency standard laboratories[9].

This paper briefly reviews the primary requirements in a synthesized microwave output for atom interrogation. This is followed by the description of a synthesis scheme that generally meets the above requirements. Although the description is specifically for a Cs synthesis, the scheme is general enough that it can be easily extended to generate microwave frequencies needed for other atomic standards.

2 Required Specifications for the Synthesized Output

In this section, various requirements for the synthesized microwave output have been presented. The synthesized output must be at a frequency of about 9.19263 GHz at a power output between -5 dBm and 0 dBm. The various other requirements are:

2.1 Overall tunability and resolution

The microwave output needs to be tunable over a range of about ± 500 kHz about the nominal value. This is wide enough range to scan the entire span of Rabi resonances. An estimate of the required resolution can be had from the fact that in a terrestrial Cs fountain the central Ramsey fringe is generally about 1 Hz wide and it is expected to approach 0.1 Hz for a space clock. Assuming that for the purpose of locking, this width can be resolved to 1 ppm (a line-splitting factor of about 10^{-6}) gives the needed resolution of the output as about 0.1-1.0 μHz. In terms of normalized frequency this means a resolution of about 10^{-17} to 10^{-16}.

2.2 Modulation

The standard technique of interrogation in slow beam and Cs fountain frequency standards is a square-wave frequency modulation[10] (SWFM) . This has been shown to result in an improvement of the short term stability by nearly a factor of two over the conventional sine wave phase modulation[11]. The interrogation frequency is switched alternately between two values, spaced equally about the central Ramsey fringe with a period T_c (≈1 to 2 s). The periodicity, T_c, includes the up-going and down-coming Ramsey interaction times (≈10 to 15 ms), the transit time between the two Ramsey interactions (≈0.5 to 1 s), and the dead time that includes the times to prepare and launch cold atom and detect them after the interrogation. As a consequence of such long period modulation, the settling time for the final microwave output because of phase lock loops (PLL) in the synthesizer is no longer an issue.

For space borne cold Cs beam frequency standards, the transit time between Ramsey interrogations can be as long as 10 to 15 s. Accelerations of the spacecraft during this period could result in fluctuations in the transit time and consequently the Ramsey fringe width. Using SWFM in this case, results in corresponding frequency fluctuations. An effective way of mitigating this problem is to use square wave phase modulation[12] (SWPM) . SWPM has been tried out in terrestrial Cs fountains at NIST and no significant frequency offset at the 10^{-15} level has been found[12] relative to SWFM.

2.3 Spectral purity

Spurious sidebands can cause significant frequency shifts in Cs atomic standards[13,14]. One of the effects of unbalanced sidebands on the interrogating signal is to cause the frequency servo to shift from the centre of the atomic resonance line[7]. Spurs that happen to overlap with the Zeeman or other structure of the atomic line can cause pulling by inducing unwanted transitions. It has been shown that for a typical Cs fountain configuration[7,13] the frequency shift by an unbalanced spur -40dBc within a kHz of the carrier is less than 10^{-15}.

The ultimate attainable frequency stability of a Cs fountain is dictated by the so-called quantum projection noise first discussed for trapped ions by Itano[15]. This limiting noise is proportional to $(Q_{at}^2.N_{at})^{-1/2}$, where, $Q_{at} = \nu_0/\Delta\nu$ is the Q of the atomic line and N_{at} is the number of detected atoms per second during the Ramsey interrogation. For a typical Cs fountain configuration this allows a potential fractional frequency stability[16] of a few $10^{-14}.\tau^{-1/2}$. The microwave interrogation signal, usually derived from state-of-the-art quartz oscillators by multiplication, has short-term frequency stability of $10^{-13}.\tau^{-1/2}$, which is worse than the above quantum projection noise limit. It has been shown that in such a situation the observed stability of the Cs fountain is degraded and appears as a sum of the quantum projection noise and the noise of the microwave output aliased at from Fourier frequencies which are multiples of the fountain cycle[17] frequency ($1/T_c$). This is the well known Dick Effect first discussed by John Dick[18]. The only way to mitigate Dick effect and operate in the quantum limited stability regime is to use a microwave source with short term stability less than $10^{-14}.\tau^{-1/2}$. This has recently been made possible by using cryogenic Sapphire loaded cavity oscillators[19].

2.4 Phase stability

The microwave synthesizer normally has an *rf* chain that connects the Cs output to one or more standard frequency outputs of 5/10/100 MHz which can be used to generate a local time scale or to compare with other frequency standards. In the event of ambient temperature fluctuations, phase instabilities in the *rf* chain can cause apparent errors in the standard frequency outputs. Internal phase stability of the synthesizer is usually expressed in terms of a temperature co-efficient of phase and the typical values[6,7] are 1 to 2 ps /K . Recently, using a compensation scheme Sen Gupta *et al.*[8] have obtained values of few hundred fs/K. For a standard laboratory environment this translates to an internal fractional frequency stability of a few parts in 10^{-15} at 10 s.

3 Architecture of Cesium Synthesizer

The design of the Cs synthesizer is an extension of the one described in our earlier paper[8]. The block schematic of the synthesizer is shown in Fig. 1. The basic local oscillator here is a 9.6 GHz ultra-high stability cavity oscillator[20,21]. The target for short term stability is 1 to 2×10^{-14} to operate in the quantum projection noise regime. This requirement is met at present only by the cryogenic Sapphire oscillator[21]. However, it is expected that with clever temperature

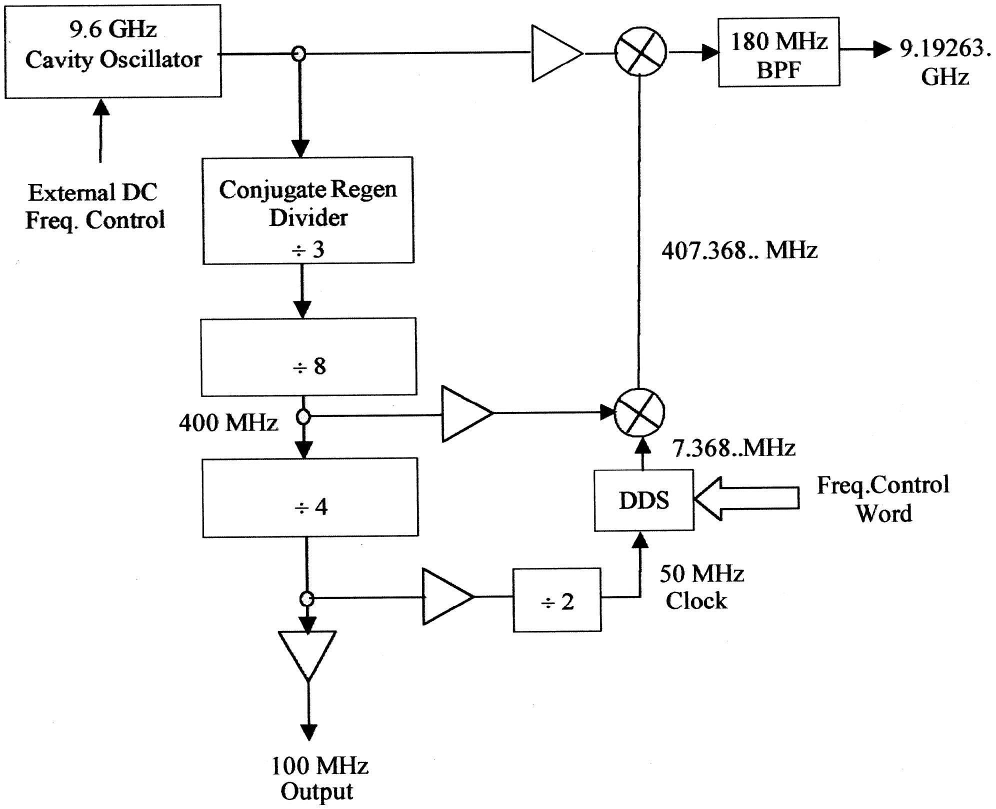

Fig. 1— Block diagram of the Cesium frequency synthesizer

compensation schemes, such stability of Sapphire loaded cavity oscillators or indeed air dielectric cavity oscillators may be possible[22,23]. The 9.6 GHz output is divided by three in a low-noise conjugate regenerative divider[24] to give 3.2 GHz, which is further divided by 8 and 4 using regenerative dividers to produce outputs at 400 MHz and 100 MHz respectively. The advantage of regenerative dividers over the conventional digital dividers arises from the fact that they introduce much lower noise[24, 25] and do not have problems of noise aliasing for large division ratios owing to the sinusoidal outputs[26]. It is possible to use a succession of the conventional divide-by-two regenerative dividers or achieve higher division ratios using conjugate regenerative dividers. The 100 MHz is buffered through isolation amplifiers and forms the output to the outside world for frequency comparisons.

It is convenient to obtain the fine resolution of the microwave output[27] using a Direct Digital Synthesizer (DDS). The DDS can provide a clean sine wave output with precise frequency setting under computer control. We used[8] a 48-bit DDS, which is clocked by a 50 MHz (obtained by dividing the 100 MHz output by 2) to produce a sine wave output over 0 to 20 MHz with a least count of 0.2 μHz. This meets the output resolution target of $\sim 10^{-17}$ for terrestrial and space Cs fountains adequately. The DDS output is set at 7.368 MHz and is mixed with the 400 MHz in an upper side band (USB) mixer to produce 407.368 MHz. The use of an USB mixer eliminates the need for a narrow band filter to attenuate the unwanted sideband at 392.63 MHz. The USB mixer operates with optimum efficiency over a range of ± 2 MHz of the nominal 7.368 MHz DDS output. This enables more than adequate tunability of the synthesizer output.

The 407.368 MHz signal is mixed with the 9.6 GHz to produce the final Cs output of 9.192 GHz to drive the hyperfine clock transition. A bandpass filter of 9.192 GHz with a width of 180 MHz is used to eliminate the unwanted 407 MHz sidebands. As stated earlier, the interrogation of the Cs central Ramsey line is usually carried out by square wave frequency modulation. The frequency of the synthesizer can be alternated between two values on either side of the central line by just switching the DDS frequency. The switching can be carried out quite simply, under computer control, by periodically rewriting frequency as the control word. Since there is no PLL between the DDS and the 9.192 GHz output, the settling time after frequency switching is just that of the DDS. This is within a few clock cycles (50 MHz) of the DDS chip and is essentially instantaneous. Square wave phase modulation (phase alternated between 0 and $\pi/2$), as would be needed in case of a space clock[12], can also be carried out quite simply with the DDS. There is a provision to switch the DDS output between a SINE and COSINE form by switching one bit. These two outputs are precisely $\pi/2$ apart in phase.

To be useful for precision standards work, the phase delay between the 9.19 GHz output and the 100 MHz, which is the primary user output for measurements should be stable with time and changing environmental parameters such as temperature and supply voltage. Since there are no highly tuned band pass filters or multipliers in the present design, the temporal variation of the internal phase delay is small. The main delay variation appears to arise from temperature changes. The uncompensated temperature coefficient of phase delay, which primarily arises from the digital dividers and the output amplifiers, is about 2 ps/K for the 100 MHz output. Since the electronics are heat sunk to the metal body of the unit, there is a single time constant of temperature variation for all parts of the electronics. This results in a very stable temperature coefficient. It is quite simple to significantly reduce the temperature coefficient using a dc voltage proportional to the temperature difference from room temperature to drive a RC phase shifter. The temperature sensor is heat sunk to the inside of the case containing the divider. The *R* in the phase shifter is the 50 Ω output impedance of the *rf* amplifier and the *C* is a varactor. Since the correction voltage is just the difference between the real temperature and the nominal room temperature set point, changing amplifier gain primarily changes the temperature coefficient and not the phase delay (varactor operating point). It then becomes a relatively simple matter to observe the phase of the synthesizer as the temperature is ramped up and down and adjust the gain to minimize the temperature coefficient. With appropriate scaling of the dc amplifier gain, a final temperature coefficient of about 0.1 ps/K was achieved for the synthesis from 9.2 GHz to 100 MHz. Typical results are shown in Fig. 2. Figure 3 shows the fractional frequency stability between the 9.192 GHz outputs of two synthesizers locked to the same 100 MHz reference.

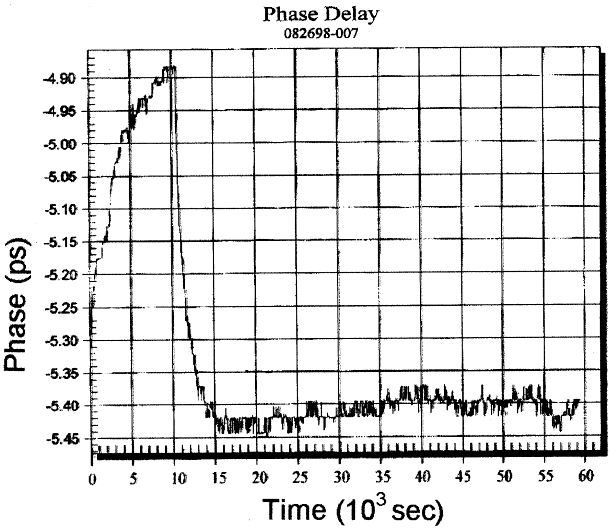

Fig. 2 —Compensated phase response of a prototype Cs synthesizer for a temperature step up and back of 3.5 K

4 General Synthesizer for Atomic Standards

A novel feature of our approach to the Cs frequency synthesis as described above, is that it is quite straightforward to produce several outputs applicable to other atomic frequency standards. Fig. 4 shows the basic divider unit. It consists of the 9.6 GHz oscillator as its high spectral purity local oscillator. This is divided to obtain the 3.2 GHz output, which is successively divided to 1.6 GHz, 400 MHz, 200 MHz, 100 MHz. Using this basic divider unit one can then go on to generate the various output frequencies as needed as described below.

4.1 Cs output

Figure 5 shows the scheme for generating the Cs output of 9.192 GHz. This has already been adequately discussed in the previous section.

4.2 Rubidium output

Figure 6 shows the scheme for generating the Rb output of 6.834 GHz. Here we can generate a 34 MHz output by mixing the roughly 9 MHz DDS signal with 25 MHz derived by dividing 100 MHz by 4, in an USB mixer. The use of a USB mixer eliminates the need for a narrow band filter to reduce the unwanted 9 MHz sidebands. The 34 MHz is mixed with 400 MHz in another USB mixer to produce 434 MHz. The 434 MHz is mixed with 6.4 GHz to give the needed 6.834

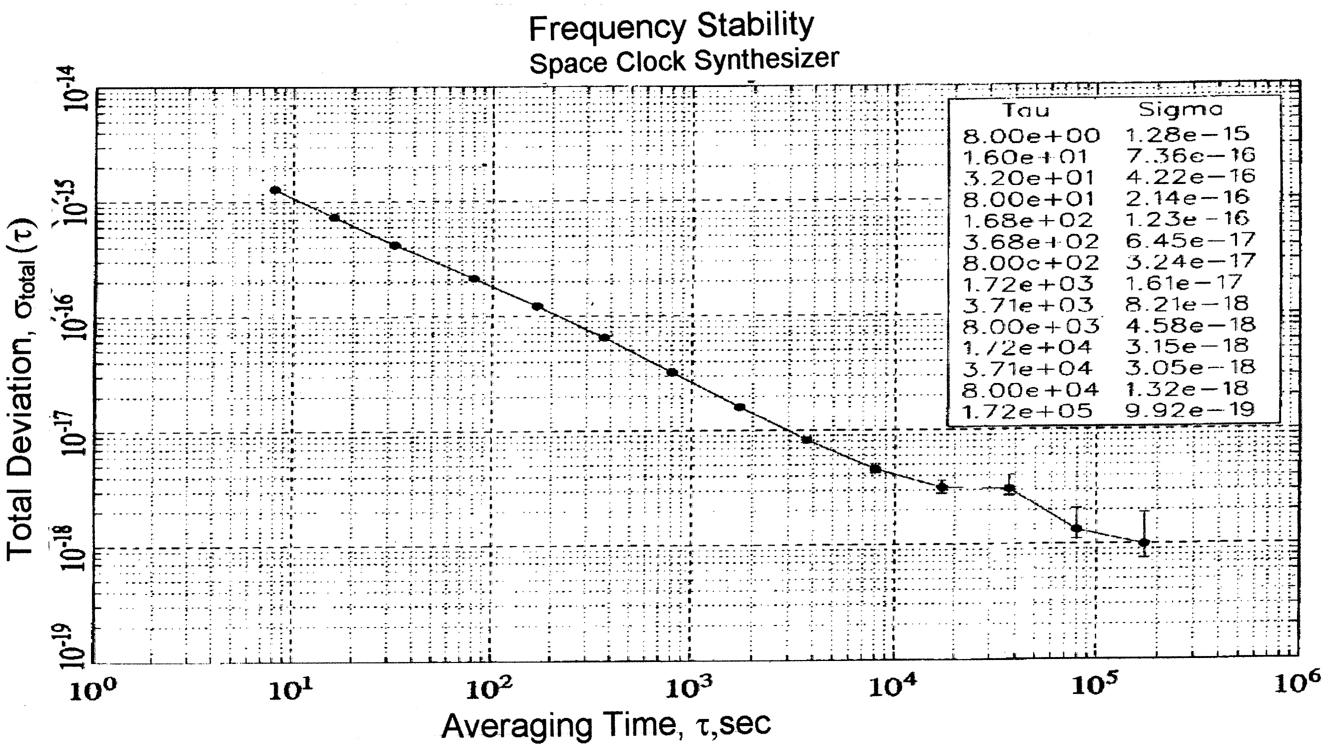

Fig. 3— Frequency stability for a pair of our Cs synthesizers computed using total deviation, an estimator of the allan deviation which has improved confidence at long term

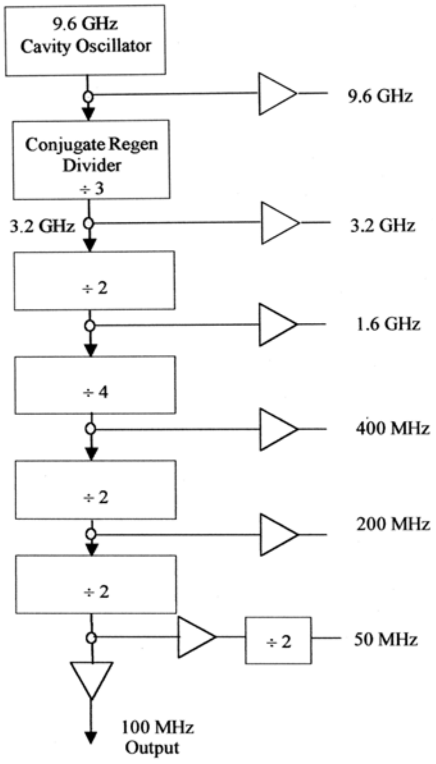

Fig. 4— Basic oscillator, phase lock loops and divider chains in the proposed synthesizer

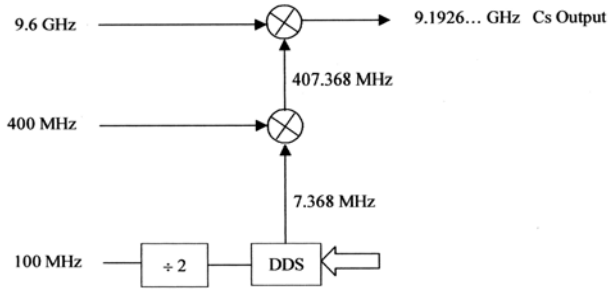

Fig. 5— Synthesis scheme for producing output frequency corresponding to cesium

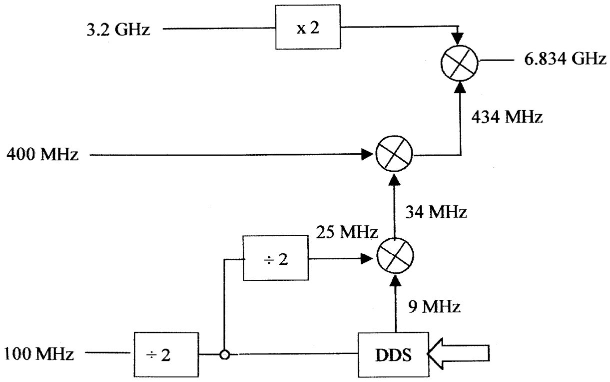

Fig. 6— Synthesis scheme for the generating output frequency corresponding to rubidium

4.3 Hydrogen-maser output

In an active H-maser the basic signal is generated at 1.420405 GHz. It is usual to phase lock a local oscillator to this signal using double super heterodyning. Thus, in the first stage one uses a local oscillator of 1.4 GHz to get an intermediate frequency of 20.405 MHz. This is then mixed with a synthesized 20.405 MHz to generate the error voltage used to lock the local oscillator. There can be further stages of heterodyning if desired. In our approach (Fig. 7) the generation of the 1.4 GHz would be obtained by mixing 1.6 GHz and 200 MHz. The 20.405 MHz would be generated by subtracting the roughly 4.595 MHz DDS from 25 MHz derived by dividing 100 MHz by 4, in a lower sideband mixer (LSB).

4.4 Trapped mercury ion (Hg^+) output

The hyperfine transition of Hg^+ is at 40.5073 GHz. The basic scheme for its synthesis is shown in Fig. 8. We could use a 25 MHz, obtained by dividing 100 MHz by 4, to mix with the DDS output of 1.825 MHz in an USB mixer to produce 26.825 MHz. The use of an USB eliminates the need for a narrow band filter to reduce the unwanted 1.825 MHz sidebands. The 26.825 MHz signal could then be successively combined with 100 MHz and 400 MHz. The resulting 526.825 MHz signal is combined with 9.6 GHz to give 10.126825 GHz. A band pass filter with a width of roughly 180 MHz is used to reduce the unwanted 526 MHz sidebands. The 10.126 GHz signal could then be multiplied by 4, using a step recovery diode (SRD) or active multiplier to generate the needed Hg^+ output. A waveguide filter could be used to reduce the unwanted harmonics.

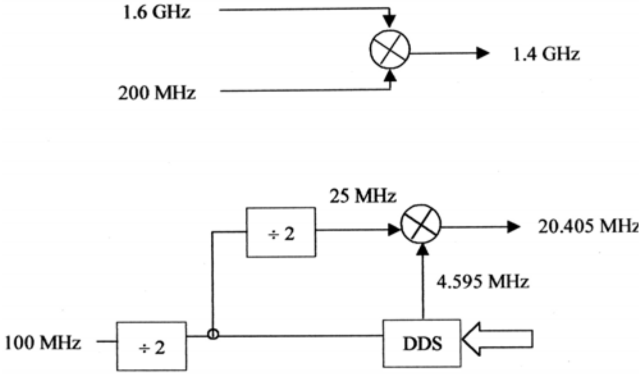

Fig. 7— Synthesis scheme to generate outputs corresponding to an active H-Maser

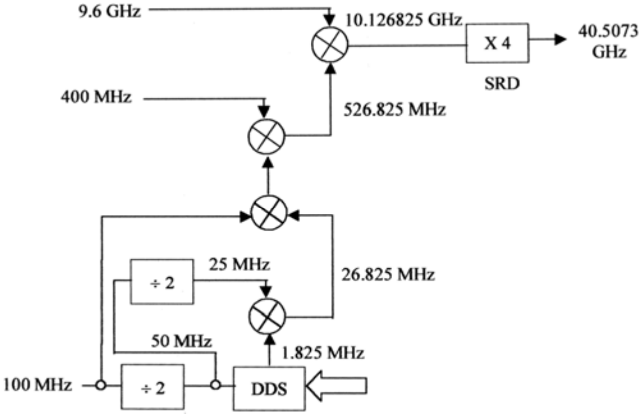

Fig. 8— Synthesis scheme for generating the output frequency corresponding to mercury ion

4.5 Femtosecond laser output

Mode-locked fs pulse laser provides ultra-short pulses (~30 fs) at the pulse repetition rate (PRR). This is typically 100 MHz to a few GHz. This modulation yields a comb of frequencies given by

$$\nu_o = n.\mathrm{PRR} + \nu_{\mathrm{offset},} \qquad \ldots(1)$$

where n is the harmonic number and ν_{offset}, is the offset of the comb from zero[28,29]. The shortness of the pulse requires phase coherence between all the comb lines, even to the optical region. Mode-locked fs lasers have been used to make frequency comparisons between the PRR or multiples of the PRR and optical frequencies with unprecedented precision and accuracy. A particularly attractive scheme allows one to lock ν_{offset} to a fixed value and then lock one of the comb lines to a stable optical frequency standard. The PRR then must be locked to the optical reference frequency. Such optical standards have the potential of achieving fractional frequency stabilities of order 1×10^{-15} at 1 s and to 10^{-18} at a day and beyond[30].

The frequency of the PRR is sensed by detecting a portion of the optical signal on an optical photo diode. The broadband noise in the optical detector primarily originates from shot noise and thermal noise in the detector. The detector noise adds white PM noise that is independent of the comb separation (until the separation approaches the bandwidth of the detector). This white PM noise significantly limits the short-term frequency stability of the detected PRR. One method to improve the white PM level and hence the short-term frequency stability of the readout signal is to detect at as high a multiple of the PRR as possible within the 3-dB bandwidth of the photo detector and use the low noise frequency synthesis techniques of Figs 3 and 4 to provide the link to a standard reference frequency. Readily available synthesizer frequencies are 10.0, 9.6, 9.2, 6.4, or 3.2 GHz. Detected frequencies as high as 40 GHz can be prescaled to the 10 GHz region using microwave regenerative dividers[25].

5 Conclusion

The requirements of a high precision synthesizer for the cold atom frequency standards based Cs hyperfine frequency of 9.192 GHz have been comprehensibly reviewed. A scheme for synthesizing the 9.192 GHz that satisfies all the requirements has been described. The additional advantage of the scheme is that it can be generalized to easily synthesize frequencies corresponding to the rubidium, 6.834 GHz; hydrogen, 1.4142 GHz; mercury ion, 40.5 GHz with a resolution of ~0.2 μHz and a range of several megahertz. The synthesizer may also prove useful for the microwave end of the femtosecond laser synthesis from the optical to the *rf*.

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