

## Pulsed force sequences for fast phase-insensitive quantum gates in trapped ions

A M Steane<sup>1</sup>, G Imreh<sup>1</sup>, J P Home<sup>1</sup> and D Leibfried<sup>2</sup>

<sup>1</sup> Clarendon Laboratory, Department of Physics, University of Oxford, Parks Road, Oxford OX1 3PU, UK

<sup>2</sup> NIST Boulder, Time and Frequency Division, Boulder, Colorado 80305, USA

Received 20 December 2013, revised 19 March 2014

Accepted for publication 24 March 2014

Published 28 May 2014

*New Journal of Physics* **16** (2014) 053049

doi:[10.1088/1367-2630/16/5/053049](https://doi.org/10.1088/1367-2630/16/5/053049)

### Abstract

We show how to create quantum gates of arbitrary speed between trapped ions using a laser walking wave, with complete insensitivity to the drift of the optical phase, and requiring cooling only to the Lamb–Dicke regime. We present pulse sequences that satisfy the requirements and are easy to produce in the laboratory.

Keywords: ion trap, quantum gate, quantum computing, fast gate, pulse sequence, fault tolerant

A variety of proposals exist for quantum logic gates between ions trapped in high vacuum [1–14], several of which have been experimentally implemented [15–26]. For quantum computing, the primary goal is fast, precise quantum logic. Methods are sought that are insensitive to those experimental parameters  $\mathcal{P}_h$  which are, in practice, harder to control [27–31]. Such methods still require accurate parameter settings, but only for  $\mathcal{P}_e$  parameters whose precise control is available in the laboratory. Examples of  $\mathcal{P}_h$  are ion temperature  $T$  and optical phases  $\phi_o$ ; examples of  $\mathcal{P}_e$  are rf frequencies and phases. An optical phase is the phase of a travelling- or standing-wave of light which is sensitive to nm-scale changes in optical path lengths. Single-qubit gate methods, such as stimulated Raman transitions, can use co-propagating laser beams to avoid sensitivity to the slow drift of  $\phi_o$ , and this is extremely important for the realization of the high degree of precision which is needed for quantum computing. Similar insensitivity is available in some two-qubit gate methods, but at a cost in speed. We show how to construct



Content from this work may be used under the terms of the [Creative Commons Attribution 3.0 licence](https://creativecommons.org/licenses/by/3.0/). Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.

simple laser pulse sequences which allow arbitrarily fast gates robust against  $T$  and having complete insensitivity to the value of the optical phase  $\phi_o$  when the gate is applied.

The issues of gate speed and sensitivity to  $\phi_o$  have been addressed separately in recent work. Methods broadly similar to those presented here have been used to perform gates on ions in linear chains where the objective is spatial selection of a subset of the ions rather than gate speed [32, 33]. Using the general concept of forced displacements in phase space [4, 5, 17], García-Ripoll *et al* [6, 7] showed how to find the time-dependence  $f(t)$  of a spin-dependent force which would allow a two-qubit gate of any speed in a given trap. The issue of optical phases arises when the force is produced by a laser standing wave: if  $\phi_o$  were not controlled then the ions would experience the wrong  $f(t)$  and the gate would not work in general. For ‘slow’ gates, on the other hand, i.e. with  $\omega_c \tau \gg 2\pi$ , where  $\omega_c$  is the centre of mass (COM) mode frequency, and  $\tau$  the gate time, one may use the simple laser pulse described in [17] and below, resulting in an entangling gate insensitive to  $\phi_o$ . The method of Mølmer and Sørensen involving spin flips [3, 4], as originally discovered, was sensitive to  $\phi_o$  even in the slow regime, and this was an important limitation on its practical realizability. However, it was shown how to choose laser beam propagation directions so that insensitivity to  $\phi_o$  is possible for a slow implementation of the gate on a pair of ions [27]; see also [25]. Here we achieve both high speed and insensitivity to  $\phi_o$ .

The gate methods mentioned above work by driving the system around a closed loop in phase space, using an oscillating force whose magnitude or direction depends on the qubit state. The closure and the loop area can depend on the initial phase of the force because for fast gates with  $\omega_c \tau \lesssim 2\pi$ , the rotating wave approximation is not valid. A given oscillation  $\sin(\omega t + \phi_o)$  appears as an oscillation at two frequencies  $\omega_0 \pm \omega$  in an interaction picture. The phase  $\phi_o$  is half the phase difference between these two oscillations at  $t = 0$ , and it has significant physical consequences. Similar considerations arise in fast manipulation of spins by Rabi flopping.

A summary of our analysis is as follows. We consider a pair of ions subject to a laser walking wave, of phase  $\phi_o$  at some arbitrary position and time, which produces a spin-dependent oscillating force through the ac Stark shift. We extract the dependence of the dynamics on  $\phi_o$ , and obtain four complex number conditions which suffice to ensure that all loops in the motional phase space are closed for any value of  $\phi_o$ . We then extract a further two complex number conditions which ensure that loop areas are independent of  $\phi_o$ , and another condition which ensures that single-qubit rotations due to ac Stark shifts also vanish for all  $\phi_o$ . Next, we search for pulse sequences that satisfy all the conditions. We show that some remarkably simple pulse sequences can succeed at fast speeds  $\omega_c \tau < 2\pi$ : for example, a symmetric five-pulse sequence of fixed frequency and phase origin (as could be produced by a single oscillator gated between zero and three output levels). Such a high speed requires high laser intensity, however, and this may be impractical. We also present a very simple symmetric four-pulse sequence, with all pulses at the same amplitude, which produces infidelity below  $10^{-4}$  (averaged over a uniform distribution of  $\phi_o$ ) at gate speed  $\omega_c \tau / 2\pi \simeq 1.76$ .

Consider a walking wave of light interacting with two ions in the same trap. The light field is produced by two laser beams of difference wavevector  $\mathbf{k}$  directed along  $x$ , difference frequency and phase  $\omega$ ,  $\phi_o$ . The internal qubit states of the ions will be labelled  $|\uparrow\rangle$  and  $|\downarrow\rangle$  and

referred to as ‘spin states’; in practice they are usually a pair of levels in the ground state hyperfine structure. We assume the light polarization is adjusted so that the ac Stark shift from either laser beam acting alone is the same for  $|\uparrow\rangle, |\downarrow\rangle$ . This is usually the case in experiments and serves to eliminate an error source (random qubit rotations from laser intensity noise). The laser–ion interaction Hamiltonian is then

$$H = \sum_{m_1, m_2} \sum_j C_{m_j} V_j \cos(k\hat{x}_j - \omega t - \phi_o - \varphi_{m_j}) |m\rangle \langle m|$$

where  $|m\rangle$  is shorthand for  $|m_1 m_2\rangle$ ,  $j = 0, 1$  counts the ions,  $m_j = \uparrow, \downarrow$  indicates the internal state, the  $C$  coefficients account for different coupling to different spin states,  $V_j(t)$  (real and positive) is proportional to the product of the two laser beam electric field amplitudes at ion  $j$  at time  $t$ , and  $\varphi_{m_j}$  depend on the light polarization.

In general, for each two-ion spin state, both the COM and stretch modes of motion of the ions are excited, by differing amounts. After making the Lamb–Dicke approximation  $|kq_j| \ll 1$ , where  $q_j = x_j - x_j^0$  are the excursions from equilibrium, we have

$$H \simeq \hbar \sum_m |m\rangle \langle m| \left[ \Omega_m^{\text{LS}} \cos(\omega t + \phi_o + \phi_m^{\text{LS}}) + \sum_{l=c}^s \Omega_{ml} \sin(\omega t + \phi_o + \phi_{ml}) k\hat{x}_l \right] \quad (1)$$

where  $\{\Omega(t), \phi\}$  gives the amplitudes and phases of the various contributions, and  $x_c \equiv (q_1 + q_2)/2$ ,  $x_s \equiv (q_1 - q_2)/2$  are the COM and stretch coordinates. The equations relating  $\Omega, \phi$  to  $C, V, \varphi, k(x_2^0 - x_1^0)$  are easily derived but lengthy to write down.

The first term in (1) is a time-dependent light shift (LS) (ac Stark shift) causing spin precession about  $z$ . Although for slow gates it can be negligible, it will be important here. The second term is a sum of time-dependent forces acting on the normal modes. It is well known that the effect of such a uniform force on quantum simple harmonic motion is simply to displace the motional state in its  $x$ - $p$  phase space [34, 35]. Let  $|\alpha, n\rangle$  be a Fock state displaced by  $\alpha$ , then the total evolution has the form

$$|m\rangle |0, n_c; 0, n_s\rangle \rightarrow e^{i\Phi_m} |m\rangle |\Delta\alpha_c^m, n_c; \Delta\alpha_s^m, n_s\rangle. \quad (2)$$

The phases  $\Phi_m$  have a contribution from the LS term, and a contribution proportional to the sum of the (signed) areas enclosed by the phase space orbits  $\alpha_c^m(t), \alpha_s^m(t)$ . The desired two-qubit phase gate is obtained when  $\Psi \equiv \Phi_{\uparrow\uparrow} + \Phi_{\downarrow\downarrow} - \Phi_{\uparrow\downarrow} - \Phi_{\downarrow\uparrow} = \pi$ .  $\Delta\alpha_{c,s}^m$  are the net displacements at the end of the gate operation. In an ideal case these would be zero so that the spin and motion are disentangled. We calculate the infidelity  $\epsilon = 1 - \langle |\psi\rangle | U_{\text{id}}^\dagger U_{\text{id}} |\psi\rangle |^2 \rangle$  where  $U_{\text{id}}$  is an ideal operation and the outer brackets represent averaging over an initial thermal state, and over

spin states<sup>3</sup>. For small errors we obtain

$$\epsilon \simeq \sum_{l,m} \frac{1 + 2\bar{n}_l}{4} |\Delta\alpha_l^m|^2 + \frac{\Delta\Psi^2}{9} + \frac{\Delta\theta_1^2 + \Delta\theta_2^2}{5}, \quad (3)$$

where  $\theta_{1,2} \equiv \left( (\Phi_{\uparrow\uparrow} - \Phi_{\downarrow\downarrow}) \pm (\Phi_{\uparrow\downarrow} - \Phi_{\downarrow\uparrow}) \right) / 2$  are the single-qubit rotation angles, and  $\Delta\Psi = \Psi - \pi$ ,  $\Delta\theta_j = \theta_j - \bar{\theta}_j$ .

For an oscillator with mass  $M$  and natural frequency  $\omega_0$ , the coherent state parameter in the laboratory frame is defined as  $\alpha = \exp(i\omega_0 t)(x + ip/M\omega_0)/2x_0$ , where  $x_0 = (\hbar/2M\omega_0)^{1/2}$ . The Argand diagram for  $\alpha$  corresponds to an  $x$ - $p$  phase space in which the motion can be conveniently described. For a uniform driving force  $f(t)$ , the evolution is given by [34, 35]

$$\Delta\alpha \equiv \alpha(t) - \alpha(0) = \frac{i}{2M\omega_0 x_0} \int_0^t e^{i\omega_0 t'} f(t') dt'. \quad (4)$$

Consider the force owing to any one of the terms in (1):

$$f(t) = 4M\omega_0 x_0 \Omega(t) \sin(\omega t + \phi),$$

where  $\Omega = -\eta_l \Omega_{ml} / 2$ ,  $\phi = \phi_o + \phi_{ml}$ , with the Lamb–Dicke parameter  $\eta_l = kx_{0l}$ . One finds

$$\Delta\alpha = e^{i\phi} \Delta\alpha^+ + e^{-i\phi} \Delta\alpha^- \quad (5)$$

where  $\Delta\alpha^\pm = \pm \int_0^t \Omega(t') \exp(i\delta^\pm t') dt'$  with  $\delta^\pm \equiv \omega_0 \pm \omega$ . Therefore  $\Delta\alpha$  describes an ellipse in the Argand diagram as  $\phi_o$  is varied. In order to guarantee that  $\Delta\alpha = 0 \forall \phi_o$ , it is sufficient and necessary that  $|\Delta\alpha^\pm| = 0$ . If  $\phi_o$  is uniformly distributed between 0 and  $2\pi$  then the mean value of  $|\Delta\alpha|^2$  is  $\langle |\Delta\alpha|^2 \rangle = |\Delta\alpha^+|^2 + |\Delta\alpha^-|^2$ .

When  $\Delta\alpha = 0$  the orbit is closed, and the phase  $\Phi$  acquired by the quantum state is twice the enclosed area,  $\Phi = \text{Im}[I]$  where

$$I = \int_{\text{path}} \alpha^* d\alpha = I^0 + e^{2i\phi} I^+ + e^{-2i\phi} I^- \quad (6)$$

with

$$I^0 = \int_0^t \Omega(t') (\Delta\alpha^{+*} e^{i\delta^+ t'} - \Delta\alpha^{-*} e^{-i\delta^- t'}) dt' \quad (7)$$

$$I^\pm = \pm \int_0^t \Omega(t') \Delta\alpha^{\mp*}(t') e^{i\delta^\pm t'} dt'. \quad (8)$$

In order that  $\Phi$  is independent of  $\phi_o$  it is sufficient and necessary that  $I^+ = I^{-*}$ . When  $\phi_o$  is uniformly distributed between 0 and  $2\pi$ , the variance of  $\Phi$  is  $\Delta\Phi^2 = |I^+ - I^{-*}|^2 / 2$ .

<sup>3</sup> We average over product spin states where each spin is uniformly distributed over the Bloch sphere. The numerical factors in front of the terms in (3) depend on the averaging and on correlations amongst  $\Phi_m$ . However, for solutions  $\epsilon = 0$  the values of these factors are irrelevant.

Now consider the  $\Omega_m^{\text{LS}}$  term in (1). The contribution to  $\Phi_m$  is  $\theta_m^{\text{LS}} \equiv \exp(i\phi_o)\theta_m^+ + \text{c.c.}$  where  $\theta_m^+ = -\int (\Omega_m^{\text{LS}}/2) \exp i(\omega t + \phi_m^{\text{LS}}) dt$ . This does not contribute to  $\Psi$  but produces single-qubit rotations. When  $\phi_o$  is uniformly distributed,  $\theta_m^{\text{LS}}$  has mean zero and variance  $2|\theta_m^+|^2$ .

Now consider a sequence of laser pulses, where in general the amplitude, frequency and relative phases of the pulses may differ, although later we will restrict to all  $\omega_n$  and  $\phi_n$  the same. The force on a given mode for a given spin state has the form

$$f(t) = \sum_{n=1}^N T((t-t_n)/\tau_n) f_n \sin(\omega_n t + \phi_n), \quad (9)$$

where the ‘top hat’ function  $T(x)$  is 1 for  $0 < x \leq 1$  and zero otherwise. Thus the  $n$ th pulse begins at  $t_n$  and has duration  $\tau_n$ . Then during any given pulse the change in  $\alpha$  is given by  $\Delta\alpha_n(t, \phi_o) = A_n^+(t) e^{i\phi_o} + A_n^-(t) e^{-i\phi_o}$  where

$$A_n^\pm(t) = \pm i\Omega_n e^{i(\delta_n^\pm t_n \pm \Delta\phi_n)} (1 - e^{i\delta_n^\pm(t-t_n)}) / \delta_n^\pm, \quad (10)$$

and  $\Delta\phi_n \equiv \phi_n - \phi_o$ .  $\Delta\alpha_n(t, \phi_o)$  describes a cycloid.

Let  $A_n^\pm \equiv A_n^\pm(\tau_n)$ . The orbit area calculation (6) gives

$$I^0 = \sum_n \alpha_n^{+*} A_n^+ + \alpha_n^{-*} A_n^- + B_n^0, \quad (11)$$

$$I^\pm = \sum_n \alpha_n^{\mp*} A_n^\pm + B_n^\pm \quad (12)$$

where

$$\alpha_n^\pm = \sum_{j=1}^{n-1} A_j^\pm, \quad (13)$$

$$B_n^0 = \Omega_n^2 \left[ \frac{2i\omega_0\tau_n}{\delta_n^+\delta_n^-} + \frac{C_n(\delta_n^+)}{\delta_n^+} + \frac{C_n(\delta_n^-)}{\delta_n^-} \right], \quad (14)$$

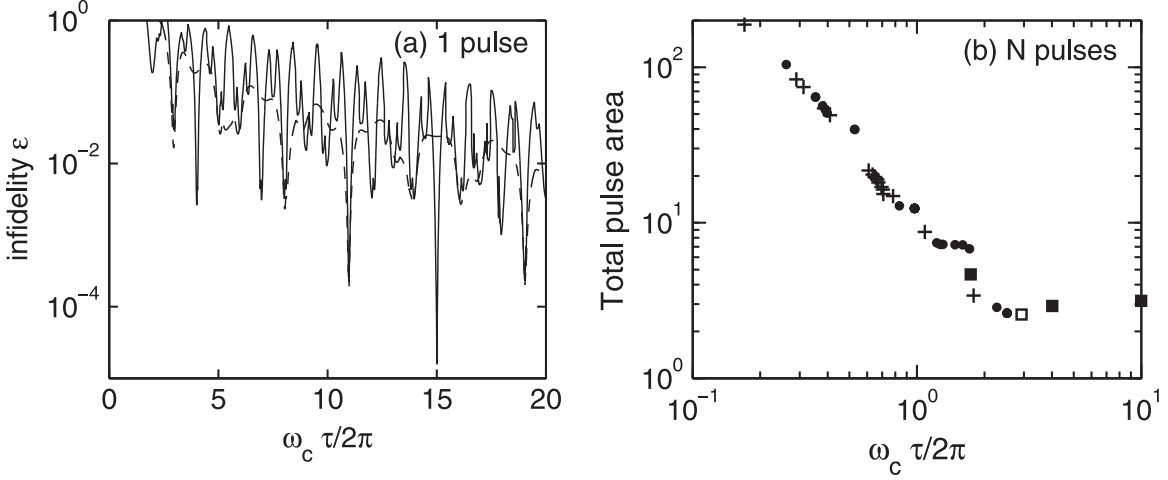
$$B_n^\pm = \Omega_n^2 e^{\pm 2i\tilde{\phi}_n} (C_n(\pm 2\omega_n) - C_n(\delta_n^\pm)) / \delta_n^\mp. \quad (15)$$

For brevity we introduced the circle function  $C_n(\omega) \equiv (1 - \exp(i\omega\tau_n))/\omega$ , and phase  $\tilde{\phi}_n = \omega_n t_n + \Delta\phi_n$ . The LS term gives

$$\theta_m^+ = \sum_n (-i\Omega_{m,n}^{\text{LS}}/2) \exp(i\tilde{\phi}_n) C_n(\omega_n). \quad (16)$$

We are interested in the case where  $\Delta\phi_n$  are well-defined but  $\phi_o$  is not. Our problem is to find a sequence of pulses such that  $\epsilon \ll 1$  when  $\phi_o$  is uncontrolled, with the total time  $\tau \equiv t_N + \tau_N - t_1$  small. Smaller pulse magnitudes and areas are preferred, to minimize the laser intensity and decoherence from photon scattering. Also the number of parameters describing the sequence should be small, to reduce the control problem.

To solve for general values of the coupling coefficients, it is sufficient to find a sequence producing



**Figure 1.** (a) Infidelity versus  $\omega_c \tau/2\pi$  for a gate using a single laser pulse. The case  $\eta_c = 0.1$ ,  $\{\bar{n}_c, \bar{n}_s\} = \{1, 1\}$  is shown, for  $C_\uparrow = -C_\downarrow$ ,  $V_1 = V_2$ ,  $k(x_2^0 - x_1^0) = 2\pi p$  (equal and opposite light shifts, balanced intensities, ion separation an integer number  $p$  of standing wave periods). At each  $\tau$ ,  $\omega$  is optimized. Full curve: single pulse, dashed curve: spin-echo sequence, with two pulses of duration  $\tau/2$  and  $\theta^+ \neq 0$ . (b) Near-exact solutions: total pulse area versus total gate time, for example pulse sequences. The examples shown are time-symmetric with fixed  $\omega_n = \omega$ ,  $\Delta\phi_n = 0$ . +, ·, □, ■:  $N = 6, 5, 4, 4$  pulses; ■ has all pulses of the same amplitude.

$$\Delta\alpha_c^+ = \Delta\alpha_c^- = \Delta\alpha_s^+ = \Delta\alpha_s^- = 0, \quad (17)$$

$$\theta^+ = 0, \quad (18)$$

$$I_c^+ = I_c^{-*}, \quad I_s^+ = I_s^{-*}, \quad (19)$$

for non-zero  $\Omega_{ml}$ , because orbits of different spin states only differ by an amplitude factor and phase origin. Therefore there are seven complex numbers that must be zero. By contrast, if we only needed a solution at one value of  $\phi_o$ , there would be only two complex number conditions:  $\Delta\alpha_c = 0$ ,  $\Delta\alpha_s = 0$ . One can drop the condition on the LS term  $\theta^+$  while doubling the total gate time by applying a given pulse sequence twice, with a spin-flip in between (spin-echo sequence). However  $\theta^+$  must not be ignored altogether because for fast gates the LS phases are greater than the orbit area phases by approximately  $1/\eta$ .

For each pulse, a change in  $\phi_o$  rotates and displaces the orbit along itself:  $\Delta\alpha_n(t, \phi_o) = \exp(-i\theta_d)(\Delta\alpha_n(t + t_d, 0) - \Delta\alpha_n(t_n + t_d, 0))$  where  $\theta_d = \phi_o\omega_0/\omega_n$ ,  $t_d = \phi_o/\omega_n$ . Therefore if it were possible to close both orbits  $\alpha_l$  with a single pulse, then the closure would be guaranteed for all  $\phi_o$ , and also the areas would be independent of  $\phi_o$ . However, because  $\omega_s/\omega_c = \sqrt{3}$  is irrational, this is not possible<sup>4</sup>. We show in figure 1(a) the best that can be done with a single pulse. There are two free parameters,  $\omega$ ,  $\tau$ . The minimum infidelity  $\min_\omega \epsilon$  was

<sup>4</sup> One can have  $\omega_s/\omega_c$  rational in an anharmonic trap, or in a trap with a potential ‘sculpted’ on the length scale of the inter-ion separation (e.g. into a double well). In the former case the mode frequencies and the ion separation become sensitive to stray dc electric fields.

calculated for durations in the range  $(1 - 20)/2\pi\omega_c$ , at  $\eta_c = 0.1$ ,  $\bar{n}_c = \bar{n}_s = 1$ . It is seen that high fidelity can be obtained when  $\omega_c\tau/2\pi$  is near to the denominator of a rational approximation to  $\sqrt{3}$  (i.e. the values 4,11,15), but  $\epsilon \leq 10^{-4}$  is not available for  $\omega_c\tau/2\pi < 15$ .

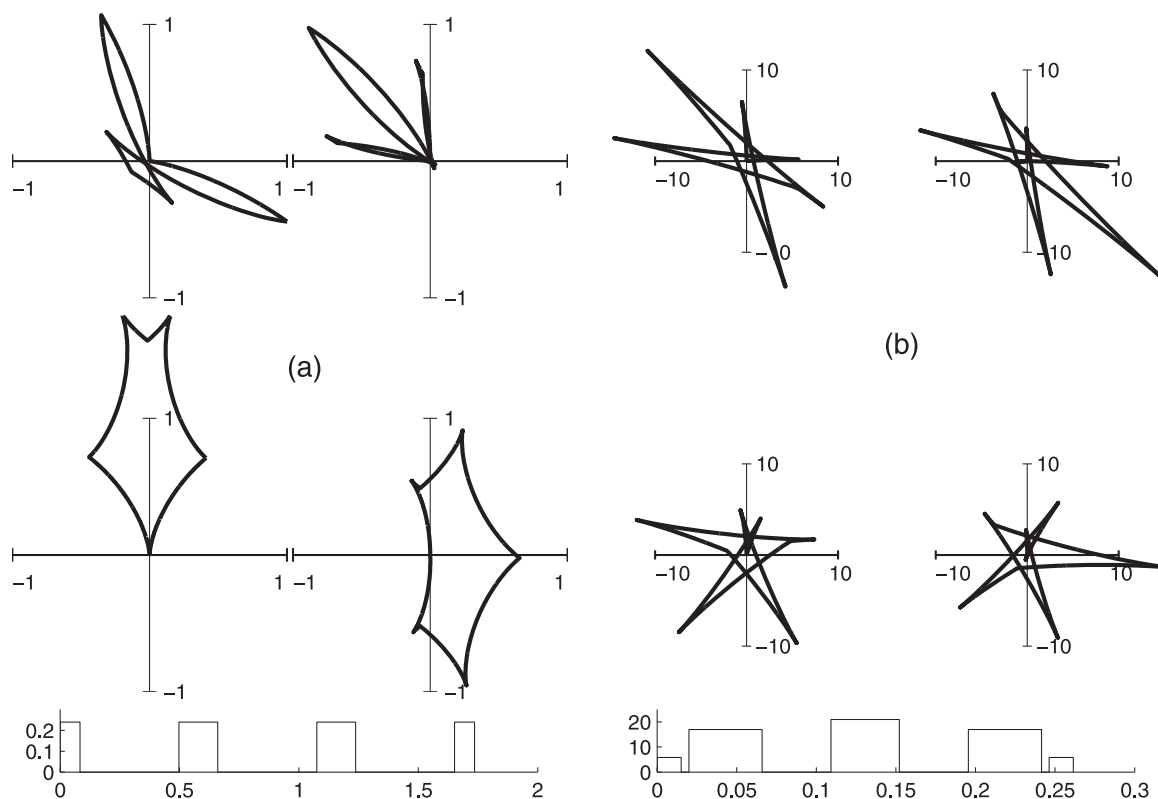
We performed a numerical search for fast pulse sequences that solve the problem. A sequence was deemed a ‘solution’<sup>5</sup> if  $\epsilon < 10^{-8}$  at  $\eta_c = 0.1$ ,  $\bar{n}_c = \bar{n}_s = 1$ , or if  $\epsilon < 3 \times 10^{-5}$  in the case of a pulse sequence with fewer than seven parameters. A sequence of  $N$  pulses has  $5N - 3$  parameters, since the start time and phase origin are arbitrary, and the absolute amplitude is fixed by the requirement  $\Psi = \pi$ . This suggests that solutions might be possible with few pulses. However to simplify the experimental requirements we assumed fixed  $\omega_n = \omega$  and restricted the values of  $\Delta\phi_n$ . For example, useful solutions were found with  $\Delta\phi_n$  restricted to multiples of  $\pi/2$ , and also with  $\Delta\phi_n = 0$ . For  $\Delta\phi_n = 0$  and fixed  $\omega_n$  the number of parameters is  $3N - 1$ . We found that by further restricting to time-symmetric sequences ( $\lceil 3N/2 \rceil$  parameters), the rapidity with which solutions were found increased. This is because the parameter space is smaller, but it shows that the symmetric space contains a good density of solutions. Another possibility is to remove the gaps between the pulses, so that the sequence describes a single shaped pulse, with  $2N$  parameters if all sections of the pulse have the same frequency and phase origin. We find there are solutions at  $\omega_c\tau/2\pi \simeq 2.93$  for a shaped pulse with three sections, and symmetric shaped pulses with five sections can give faster solutions (see for example figure 3).

More generally, one may parametrize essentially any functional form (e.g. Gaussian, Blackman, etc) in terms of some modest numbers of parameters, and then seek solutions in the parameter space. In order to compute the effect of some such more general pulse shape, one may treat it for example by Fourier analysis, or simply model it numerically as a continuous sequence of a large number of short pulses, and then the equations of this paper apply.

Here we report examples using small numbers of square pulses. To find a solution, we searched among values of  $\omega$ ,  $t_n$ ,  $\tau_n$  from a random starting point, for each case solving a set of linear equations for the  $N - 1$  amplitudes  $\Omega_{n>1}$ , and using the Nelder–Mead simplex method and simulated annealing to find a minimum of  $\epsilon$ . The linear equations were a subset of (18), (17). We found that  $N = 4$  can give some quite fast pulse sequences ( $\tau \simeq 4\pi/\omega_c$ ) at  $\epsilon = O(10^{-5})$ , see figure 2, and  $N = 5$  was sufficient and necessary to get fast solutions ( $\tau < 2\pi/\omega_c$ ,  $\epsilon < 10^{-8}$ ). In figure 1(b) we show the total pulse area  $\int \Omega(t) dt$  versus length of the pulse sequence, for sequences of minimal area at given  $\tau$ . The area is important because the unwanted photon scattering is proportional to it. With a single (slow) pulse the area is approximately  $\pi$ . For  $\omega_c\tau/2\pi < 2$  we find the same  $\tau^{-3/2}$  scaling law as was described in [7].

We studied the effect of experimental imprecision by evaluating  $\epsilon$  as a function of all the parameter values, for small departures from the optimum, for each of several interesting pulse sequences. The terms in (16) contribute single-qubit phases that are each of order  $\pi/(N\eta)$ . They cancel when the frequency and timing are accurate, but in the presence of timing errors these are the main source of infidelity. However their influence can be reduced by employing a spin-echo, and as long as any timing error is constant, the value of  $\theta_m^+$  can be adjusted to high accuracy in

<sup>5</sup> When  $\epsilon \ll 10^{-4}$  in (3), the fidelity will be limited in practice by other considerations, such as breakdown of the Lamb–Dicke approximation or laser intensity noise.



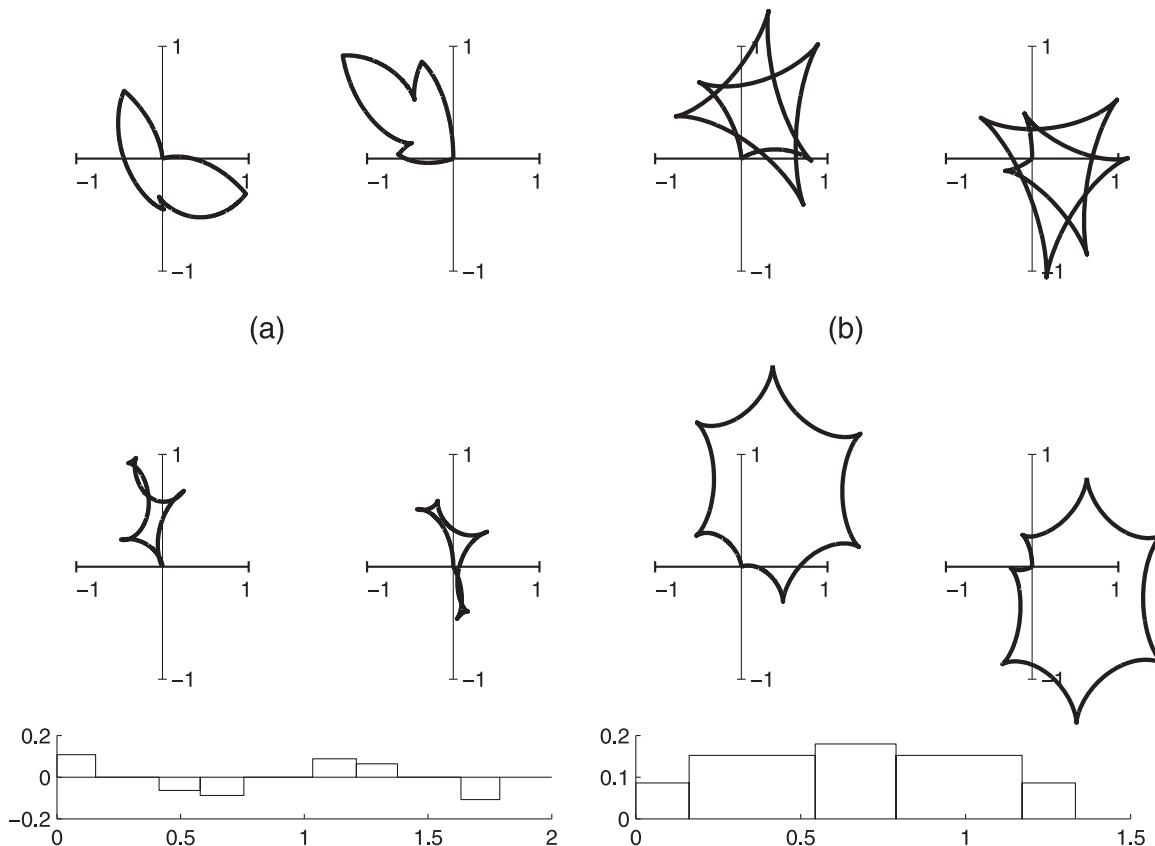
**Figure 2.** Behaviour of two example symmetric pulse sequences: (a) moderately fast and simple, (b) very fast. The orbit  $\alpha$  is shown for COM (top) and stretch (bottom) modes, for two values of  $\phi_0 = 0$  (left) and  $\pi/2$  (right), with pulse amplitude versus  $t$  at the bottom, in units of the COM period,  $2\pi/\omega_c$ . Case (a) has  $\omega \simeq 4.0376\omega_c$ , pulse durations and gaps  $\tau_1, t_2 - \tau_1, \tau_2, \dots \simeq \{0.524696, 2.60288, 1.02264, 2.60407\}/\omega_c$ . Case (b) has  $\omega \simeq 20.4761\omega_c$ ,  $\tau_1, t_2 - \tau_1, \tau_2, \dots, \tau_3 = \{0.0953071, 0.0305622, 0.288972, 0.272151, 0.269998\}/\omega_c$ , relative pulse amplitudes  $\Omega_{1,2,3} = \{1, 2.91057, 3.59685\}$ .

practice by tweaking the pulse frequency  $\omega$ . Once this is done, we found that the dependence of infidelity  $\epsilon$  on the fractional inaccuracy  $\sigma = \Delta p/p$  of other parameters  $p$  (such as pulse duration or height) is of order  $\epsilon \sim 10\sigma^2$ . For example, to achieve  $\epsilon \leq 10^{-3}$  would typically require pulse areas accurate to one percent. The method does not require that the pulses be square; in practice one would seek solutions for whatever shape is conveniently produced in the laboratory; the examples given in this paper can serve as starting points of such a search.

For a fast gate the orbits have to be large to enable the required phase difference to be acquired rapidly (cf figure 2(b)). Eventually the motion goes outside the Lamb–Dicke regime and then equations (1)–(10) must be replaced by a more general analysis or numerical integration.

A fast gate with low photon scattering leads to high laser power requirements, and this may in practice limit what speed one would aim to achieve. As an example case, consider the pulse sequence in figure 3(b), applied to a pair of calcium ions in a trap with COM frequency





**Figure 3.** Behaviour of two further examples, both moderately fast and simple. (a) Has a small total pulse area, (b) illustrates a single shaped pulse. Case (a) has  $\omega \simeq 1.36603\omega_c$ , pulse durations and gaps  $\tau_1, t_2 - \tau_1, \tau_2, \dots \simeq [0.984464, 1.6124, 1.04219, 0.00028, 1.10990, 1.7475]/\omega_c$ ; pulse amplitudes  $\Omega_{1,2,3} = \{0.6786, -0.4002, -0.5528\}$ . Case (b) has  $\omega \simeq 2.60258\omega_c$ , segment durations  $\tau_{1,2,3} = \{1.0168, 2.3997, 1.5416\}/\omega_c$ , amplitudes  $\Omega_{1,2,3} = \{0.5415, 0.9561, 1.1280\}$ .

$\omega_c/2\pi = 2$  MHz, giving a COM Lamb–Dicke parameter  $\eta = 0.12$  for the Raman transition at 397 nm laser wavelength. With 350 mW power in a beam waist of  $17 \mu\text{m}$ , the Rabi frequency on the  $S_{1/2}, F = 4, M_F = 4 \leftrightarrow P_{3/2}, F = 5, M = 5$  transition is 20 GHz. At a detuning of 3 THz the effective Rabi frequency for the ‘stretched state’ Raman transition ( $F = 4, M = 4 \leftrightarrow F = 3, M = 3$  in  $S_{1/2}$ ) is 15 MHz on the carrier, and therefore 1.8 MHz on the first sideband. The gate time is 1.33 oscillation periods in the trap, which is  $0.67 \mu\text{s}$ . The total Raman scattering rate from both ions during the gate is  $\sim 750 \text{ s}^{-1}$ , so the number of scattered photons is  $5 \times 10^{-4}$ . The infidelity including the effect of Rayleigh scattering is approximately twice this. The sequence shown in figure 3(a) has a total pulse area smaller by a factor of two and hence requires approximately four times less laser power, at the same scattering error, to obtain a gate time of  $0.9 \mu\text{s}$  in the same trap. Overall, we have achieved a simple, practical solution which offers to increase gate speed by an order of magnitude, while maintaining insensitivity to optical phase drift, with the cost in photon scattering as expected for this general type of gate.

The gate method we have analyzed has the disadvantage that the qubits must be stored, for the duration of the gate, in levels whose energy difference has a first-order Zeeman effect. Supposing the qubits are ordinarily in ‘clock’ states, this can easily be achieved by fast microwave pulses before and after the gate, but it is natural to enquire whether this aspect can be avoided altogether, for example by exploiting the Mølmer–Sørensen gate [3, 4]. The latter does not require a first-order Zeeman effect. However, for running-wave-driven Mølmer–Sørensen gates, carrier transitions are only detuned on the order of the motional frequencies and cannot be neglected for the timescales anticipated in our method. The carrier terms do not commute with the sideband terms under the Mølmer–Sørensen interaction, therefore it is not straightforward to integrate the equations of motion [4]. The situation is different for Mølmer–Sørensen gates induced by standing waves or microwave near-fields where the fields can be designed so that the carrier term, which is proportional to the field strength, vanishes to first order and also commutes with the sideband terms [13, 23, 36]. Still, to produce standing wave fields with precise indexing to the position of single ions and high enough microwave gradients to drive sidebands with Rabi-frequencies sufficient for our method is very challenging with existing technology. It would be interesting to discover whether the simpler strategy of allowing a Zeeman effect during the gate would in fact be acceptable in the lab, because the gate is fast enough to make the accumulated phase owing to a magnetic field fluctuation sufficiently small.

## Acknowledgments

We thank D M Lucas, D J Szwer and C Ballance for helpful discussions. This work was supported by the National Security Agency (NSA) and Disruptive Technology Office (DTO) (W911NF-05-1-0297), EPSRC (QIP IRC) and the Royal Society.

## References

- [1] Cirac J I and Zoller P 1995 Quantum computations with cold trapped ions *Phys. Rev. Lett.* **74** 4091–4
- [2] Cirac J I and Zoller P 2000 A scalable quantum computer with ions in an array of microtraps *Nature* **404** 579–81
- [3] Sørensen A and Mølmer K 1999 Quantum computation with ions in thermal motion *Phys. Rev. Lett.* **82** 1971–4
- [4] Sørensen A and Mølmer K 2000 Entanglement and quantum computation with ions in thermal motion *Phys. Rev. A* **62** 022311
- [5] Milburn G J, Schneider S and James D F 2000 Ion trap quantum computing with warm ions *Fortschr. Physik* **48** 801–10
- [6] Garcia-Ripoll J J, Zoller P and Cirac J I 2003 Speed optimized two-qubit gates with laser coherent control techniques for ion trap quantum computing *Phys. Rev. Lett.* **91** 157901
- [7] Garcia-Ripoll J J, Zoller P and Cirac J I 2005 Coherent control of trapped ions using off-resonant lasers *Phys. Rev. A* **71** 062309
- [8] Staunum P, Drewsen M and Moelmer K 2004 Geometric quantum gate for trapped ions based on optical dipole forces induced by gaussian laser beams *Phys. Rev. A* **70** 052327
- [9] Duan L-M, Blinov B B, Moehring D L and Monroe C 2004 Scalable trapped ion quantum computation with a probabilistic ion-photon mapping *Quant. Inf. Comput.* **4** 165–73

- [10] Beige A 2003 Dissipation-assisted quantum gates with cold trapped ions *Phys. Rev. A* **67** 020301
- [11] Beige A 2004 Ion-trap quantum computing in the presence of cooling *Phys. Rev. A* **69** 012303
- [12] Jonathan D, Plenio M B and Knight P L 2000 Fast quantum gates for cold trapped ions *Phys. Rev. A* **62** 042307
- [13] Ospelkaus C, Langer C E, Amini J M, Brown K R, Leibfried D and Wineland D J 2008 Trapped-ion quantum logic gates based on oscillating magnetic fields *Phys. Rev. Lett.* **101** 090502
- [14] Bentley C D B, Carvalho A R R, Kielpinski D and Hope J J 2013 Fast gates for ion traps by splitting laser pulses *New J. Phys.* **15** 043006
- [15] Turchette Q A, Wood C S, King B E, Myatt C J, Leibfried D, Itano W M, Monroe C and Wineland D J 1998 Deterministic entanglement of two trapped ions *Phys. Rev. Lett.* **81** 3631–4
- [16] Sackett C A *et al* 2000 Experimental entanglement of four particles *Nature* **404** 256
- [17] Leibfried D *et al* 2003 Experimental demonstration of a robust, high-fidelity geometric two ion-qubit phase gate *Nature* **422** 412–5
- [18] Schmidt-Kaler F, Häffner H, Riebe M, Gulde S, Lancaster G P T, Deuschle T, Becher C, Roos C F, Eschner J and Blatt R 2003 Realization of the Cirac-Zoller controlled-not quantum gate *Nature* **422** 408–11
- [19] Häffner H *et al* 2005 Scalable multiparticle entanglement of trapped ions *Nature* **438** 643
- [20] Leibfried D *et al* 2005 Creation of a six-atom ‘Schrödinger cat’ state *Nature* **438** 639–42
- [21] Home J P, McDonnell M J, Lucas D M, Imreh G, Keitch B C, Szwer D J, Thomas N R, Webster S C, Stacey D N and Steane A M 2006 Deterministic entanglement and tomography of ion–spin qubits *New J. Phys.* **8** 188
- [22] Brickman K-A, Haljan P C, Lee P J, Acton M, Deslauriers L and Monroe C 2005 Implementation of Grover’s quantum search algorithm in a scalable system *Phys. Rev. A* **72** 050306
- [23] Ospelkaus C, Colombe Y, Brown K R, Amini J M, Leibfried D, Wineland D J and Warring U 2011 Microwave quantum logic gates for trapped ions *Nature* **476** 181–4
- [24] Lanyon B P, Jurcevic P, Zwerger M, Hempel C, Martinez E A, Dür W, Briegel H J, Blatt R and Roos C F Nov 2013 Measurement-based quantum computation with trapped ions *Phys. Rev. Lett.* **111** 210501
- [25] Hayes D, Clark S M, Debnath S, Hucul D, Inlek I V, Lee K W, Quraishi Q and Monroe C Jul 2012 Coherent error suppression in multiqubit entangling gates *Phys. Rev. Lett.* **109** 020503
- [26] Mizrahi J, Senko C, Neyenhuis B, Johnson K G, Campbell W C, Conover C W S and Monroe C 2013 Ultrafast spin-motion entanglement and interferometry with a single atom *Phys. Rev. Lett.* **110** 203001
- [27] Haljan P C, Brickman K-A, Deslauriers L, Lee P J and Monroe C 2005 Spin-dependent forces on trapped ions for phase-stable quantum gates and entangled states of spin and motion *Phys. Rev. Lett.* **94** 153602
- [28] Zhu S-L, Monroe C and Duan L-M 2006 Arbitrary-speed quantum gates within large ion crystals through minimum control of laser beams *Europhys. Lett.* **73** 485–91
- [29] Roos C 2008 Ion trap quantum gates with amplitude-modulated laser beams *New J. Phys.* **10** 013002
- [30] Bermudez A, Schmidt P O, Plenio M B and Retzker A 2012 Robust trapped-ion quantum logic gates by continuous dynamical decoupling *Phys. Rev. A* **85** 040302
- [31] Lemmer A, Bermudez A and Plenio M B 2013 Driven geometric phase gates with trapped ions *New J. Phys.* **15** 083001
- [32] Lin G-D, Zhu S-L, Islam R, Kim K, Chang M-S, Korenblit S, Monroe C and Duan L-M 2009 Large-scale quantum computation in an anharmonic linear ion trap *Europhys. Lett.* **86** 60004
- [33] Choi T, Debnath S, Manning T A, Figgatt C, Gong Z-X, Duan L-M and Monroe C 2014 Optimal quantum control of multi-mode couplings between trapped ion qubits for scalable entanglement *Phys. Rev. Lett.* **112** 190502
- [34] Carruthers P and Nieto M M 1965 Coherent states and the forced quantum oscillator *Am. J. Phys.* **7** 537–44
- [35] Walls D F and Milburn G J 1994 *Quantum Optics* (Berlin: Springer)
- [36] Allcock D T C, Harty T P, Ballance C J, Keitch B C, Linke N M, Stacey D N and Lucas D M 2013 A microfabricated ion trap with integrated microwave circuitry *Appl. Phys. Lett.* **102** 044103