Three-Corner Hat Analysis of the Stability of UTC and Various UTC(*k*)s

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BIOGRAPHY

Since September 2007 Dr. Parker has been a contractor for the Time and Frequency Division of the National Institute of Standards and Technology, NIST. From June 1994 to September 2007 he was the leader of the Atomic Standards Group in the NIST Time and Frequency Division. His areas of expertise are time transfer, time scales, and primary frequency standards.

ABSTRACT

The stabilities of some local UTC time scales have recently improved dramatically with the use of cesium or rubidium fountains as nearly always present frequency references. Along with time scales using maser ensembles, there are now a number of other very stable scales with comparable frequency stability. Improved frequency stability means better performance at serving as a local real-time representation of UTC. It is now possible to perform a meaningful three-corner hat Allan deviation analysis to determine the individual stabilities of the best local UTC(*k*)'s, as well as UTC itself. Included in this analysis using data from Circular T are the local time scales at the US Naval Observatory, UTC(USNO), the National Institute of Standards and Technology, UTC(NIST), the Paris Observatory, UTC(OP), and in Germany, UTC(PTB). All local UTC(*k*)'s are of course steered to UTC, which artificially improves the apparent long-term stability (hundreds of days). Therefore, in addition to the UTC(*k*) time scales, the stabilities of two local free-running atomic time scales, TA(*k*)'s, have also been analyzed along with TAI. Results of these evaluations are presented. Fractional frequency stabilities are in the range of $2x10^{-15}$ to $3x10^{-16}$ over various values of tau. The improved long-term stability achieved with nearly continuously operating fountains is clearly evident.

INTRODUCTION

In recent years the operation of Cs and Rb fountains has become sufficiently robust that they can now run almost continuously. As a result several laboratories now use a fountain (or fountains) as the primary frequency reference in the generation of their local time scales [1]. Local time scales, UTC(*k*)s, are local realizations of Coordinated Universal Time, UTC. Previously the best local time scales were obtained with maser ensembles, and UTC(USNO) at the US Naval Observatory in Washington DC was the most stable local time scale. Its large number of clocks make up a significant fraction (25 to 30 %) of UTC, and consequently there is significant correlation between the two scales. UTC(NIST) at the National Institute of Standards and Technology in Boulder Colorado is generated with a small maser ensemble and has quite often been the next most stable scale, but generally is not as stable as UTC(USNO). Consequently, there were never three independent time scales (including UTC) of comparable high stability that could be used to perform a meaningful three-corner hat analysis [2] to determine individual scale stabilities.

The Physikalisch-Technische Bundesanstalt, PTB, in Germany, the Observatoire de Paris, OP, in France and the Institute of Metrology of Time and Space, SU, in Russia are all now using Cs fountains as frequency references for their local time scales. The USNO has several operational Rb fountains and may also be using them as the main frequency reference. As a result there are now several very stable local time scales that can be used to perform a meaningful three-corner hat Allan deviation analysis.

Figure 1 shows a plot of UTC-UTC(USNO) and UTC-UTC(PTB) taken from Circular T [3] since Modified Julian Date, MJD, 55004 (a period of about 4.4 years). The improvement in the stability of UTC(PTB) after MJD 55400 is clearly evident. This is approximately when PTB started to use their fountains in the generation of UTC(PTB) [1]. Since MJD 55400 UTC(PTB) has been comparable in stability to UTC(USNO).

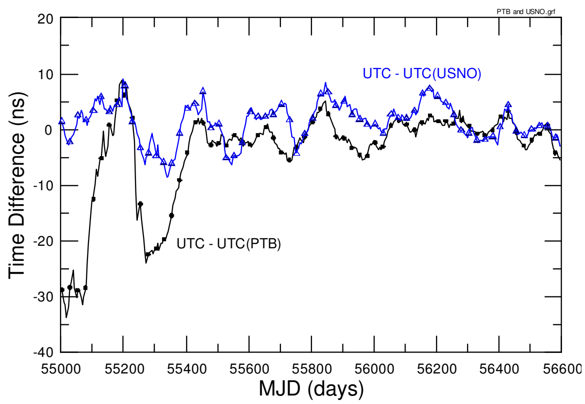

Figure 1. UTC-UTC(USNO) and UTC-UTC(PTB) showing improvement in UTC(PTB) since about MJD 55400 with the use of a Cs fountain as a frequency reference.

Figure 2 shows a plot of UTC-UTC(USNO) and UTC-UTC(OP) taken from Circular T since MJD 56004 (a period of about 1.6 years). The improvement in the stability of UTC(OP) around MJD 56229 is clearly evident after OP started to use a Cs fountain as a frequency reference.

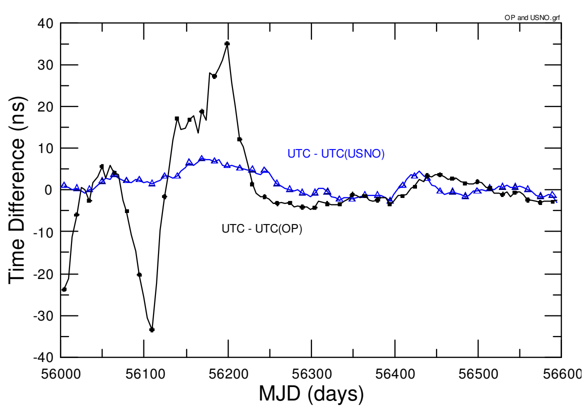

Figure 2. UTC-UTC(USNO) and UTC-UTC(OP) showing improvement in UTC(OP) since about MJD 56229 with the use of a Cs fountain as a frequency reference.

STABILITY ANALYSIS

The stability analysis in this paper is performed using data from Circular T (data on a five-day interval) and the Stable32 analysis software [4]. This software does not provide confidence limits for the three-corner hat analysis, so it is useful to examine a normal Total Allan deviation plot, with confidence limits, for a pair of time scales. This is shown in Fig. 3 for UTC(USNO)-UTC(PTB). (Total Allan deviation [5] is essentially the

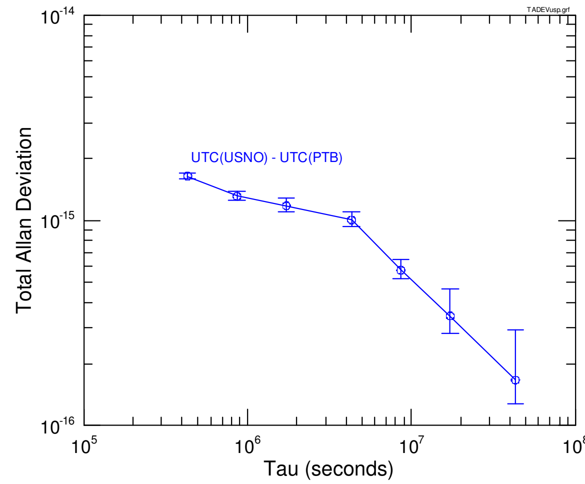

Figure 3. Total deviation of UTC(USNO – UTC(PTB), showing confidence limits, for the period MJD 55404 to 56594.

same as the Allan deviation, but has better confidence limits. A comparison of the two for small values of tau gives nearly identical results.) In a Total deviation three-corner hat analysis, with the same number of data points, the confidence limits will be approximately two times larger [6]. However, the exact ratio depends on the relative stabilities of each of the time scales being analyzed. Typically, the Total deviation values at the first 4 or 5 smallest tau values will have reasonable confidence limits for this 3.3 year data interval.

The decrease in the values of the Total deviation in Fig. 3 at large tau values is artificial and is due to the fact that both UTC(USNO) and UTC(PTB) are steered to UTC. They are effectively phase locked to UTC with a very long time constant on the order of months.

We will first examine UTC, UTC(PTB), UTC(USNO) and UTC(NIST) since this gives the longest time interval. Figure 4 shows the results of the three-corner hat analysis for UTC, UTC(PTB) and UTC(NIST) for the interval MJD 55404 to 56594, a period of about 3.3 years. The three missing points in the PTB analysis in Fig. 4 are due to negative variances.

Though both NIST and PTB contribute to UTC, there is very little correlation between the local scales and UTC. The clocks that make up UTC(NIST) contribute only a few percent to UTC, and for PTB the fountains that are references for UTC(PTB) do not contribute at all as clocks in UTC. As can be seen all three scales have similar stabilities out to about 20 days (1.728×10^6 seconds). Beyond that, UTC(PTB) appears to have the best stability, but since the level is much lower than the other two scales, the confidence limits are quite large and there are missing points. UTC is intermediate in stability

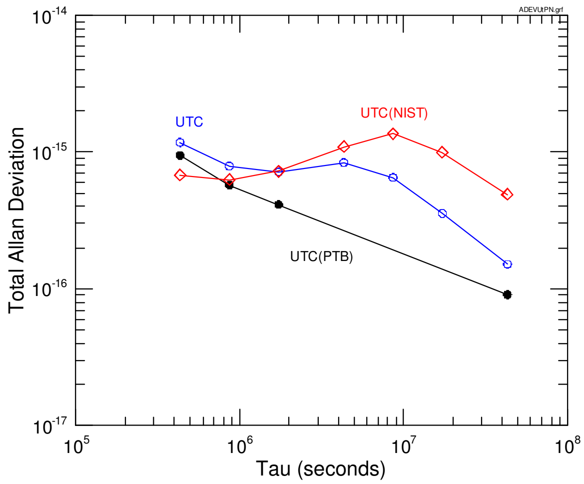

Figure 4. Total deviation from the three-corner hat analysis for UTC, UTC(PTB) and UTC(NIST) for the period MJD 55404 to 56594.

and UTC(NIST) has the highest level. UTC(NIST) has very good short-term stability, but being a small scale it is vulnerable in the longer term to disturbances in only a few masers. Again, be aware that the local time scales are steered to UTC in the long term, and this artificially reduces the Total deviation values at large tau. In any case, the benefit from having a fountain as a frequency reference is very clear.

Figure 5 shows the results of the three-corner hat analysis for UTC(USNO), UTC(PTB) and UTC(NIST) for the same interval as in Fig. 4. The missing point in the PTB analysis is due to a negative variance. Note the change in the range of the vertical scale. In this case there is absolutely no correlation in the short term between these local scales since all three are based on independent sets of clocks. Again UTC(NIST) is the most stable in the short term, but not so good in the long term. UTC(USNO) is seen to have a stability similar to that of UTC in Fig. 4. UTC(PTB) again is seen to perform very well, and is competitive with the maser ensembles. All four scales in Figs. 4 and 5 have stabilities down into the mid 10^{-16} range for some tau values less than 10^{7} seconds.

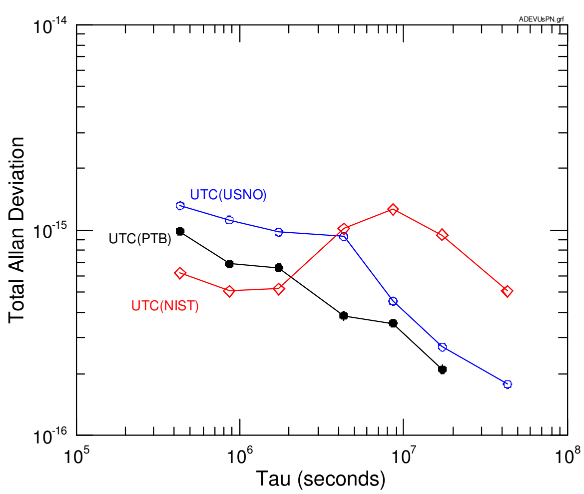

Figure 5. Total deviation from the three-corner hat analysis for UTC(USNO), UTC(PTB) and UTC(NIST) for the period MJD 55404 to 56594.

UTC(OP) will now be include in the analysis. This gives a better balance in the stabilities of the scales, but limits the time interval to just one year. Figure 6 shows the results of the three-corner hat analysis for UTC, UTC(OP) and UTC(PTB) for the interval MJD 56229 to 56594.

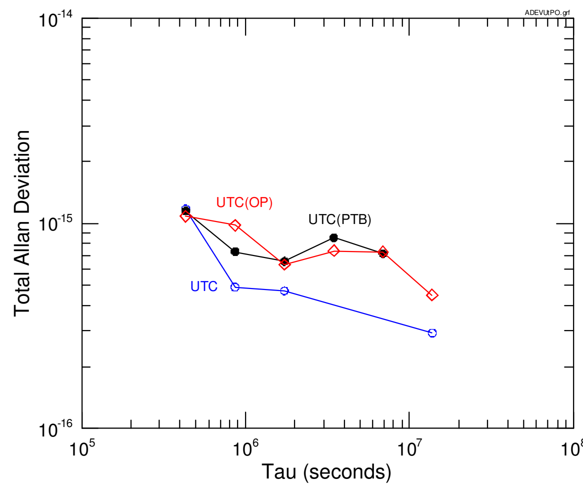

Figure 6. Total deviation from the three-corner hat analysis for UTC, UTC(PTB) and UTC(OP) for the period MJD 56229 to 56594.

UTC has two missing points due to a negative variance and UTC(PTB) has one. Otherwise all three scales show similar stabilities, with UTC being slightly more stable for this MJD interval. Note that for this shorter interval the confidence limits will be approximately 2 times larger than for the data in Figs. 4 and 5.

To avoid any correlation in the short term the three local time scales UTC(USNO), UTC(PTB) and UTC(OP) are examined and the results are shown in Fig. 7. Two points are missing from UTC(USNO) due to negative variances. Otherwise the scales are all similar in stability, with values reaching into the mid to upper 10^{-16} range for tau larger than about 10^{6} seconds.

The problem of artificially lowered Total deviation values at large tau can, in principle, be reduced by examining the stabilities of local atomic time scales, TA(*k*), and International Atomic Time, TAI. Local TA(*k*)s are free running and only loosely coupled to TAI. Except for leap seconds, TAI and UTC are the same, thus their frequency

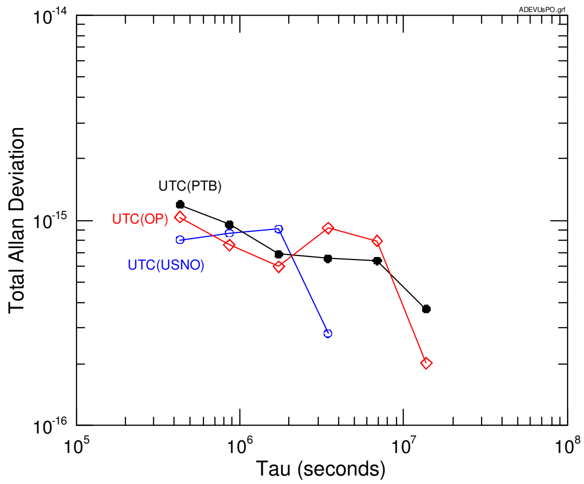

Figure 7. Total deviation from the three-corner hat analysis for UTC(USNO), UTC(PTB) and UTC(OP) for the period MJD 56229 to 56594.

stabilities are the same. At NIST, the frequency structure of TA(NIST), (also known as AT1), is similar to UTC(NIST) except there is large frequency offset and there are no frequency steps inserted as with UTC(NIST) to keep it in phase with UTC. At PTB the relationship between TA(PTB) and UTC(PTB) is more complicated. There is no TA(OP). Figure 8 shows the results of the three-corner hat analysis between TAI, TA(PTB) and TA (NIST) for the interval MJD 55754 to 56594 (a period of about 2.3 years). The data for the improved TA(PTB) is not available for as long a period as the improved UTC(PTB). As can be seen the values of the Total deviations in the long term are not nearly as low as for the UTC(*k*)s since the local TA(*k*)s are not directly steered to TAI.

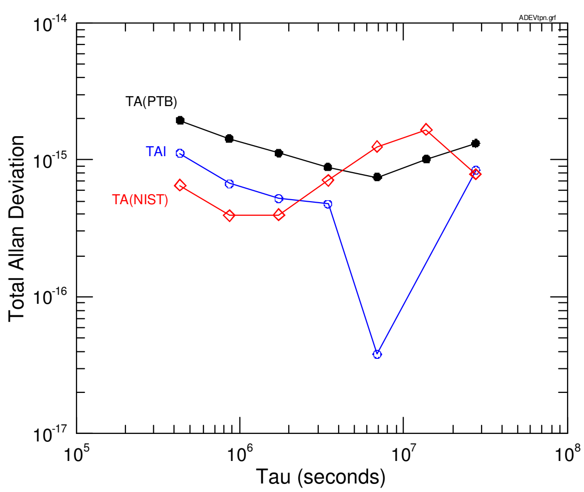

Figure 8. Total deviation from the three-corner hat analysis for TAI, TA(PTB) and TA(NIST) for the period MJD 55754 to 56594.

The Cs fountains at PTB do contribute to the frequency accuracy of TAI, and therefore TAI is not completely independent of the PTB fountains. However, there are four other Cs fountains that regularly contribute to TAI and therefore reduce the weight of the PTB fountains in TAI. Furthermore, the frequency steering of TAI to the SI second (the fountains) now occurs very infrequently (the last steer was on MJD 56199). Thus the correlations between the PTB fountains and TAI are fairly weak.

The frequency drift rates of the masers at NIST are determined from measurements against Cs fountains that contribute to TAI and therefore TA(NIST) is also not completely independent of TAI. However, the correlation is very weak.

The long-term data in Fig. 8 is also influenced by the fact that for about the first 400 days of the 2.3 years of data used in Fig.8 TAI was steered fairly strongly in order to take out a frequency offset from the SI second. As more unperturbed data is accumulated, the long-term stability of TA(PTB) should become more representative of what is achieved with a fountain-based atomic time scale.

The short-term stabilities of TAI and TA(NIST) are similar to the short-term stabilities of UTC and UTC(NIST) respectively. However, the short-term instability of TA(PTB) is significantly higher than UTC(PTB) because the relationship between TA(PTB) and UTC(PTB) is considerably more complicated and sometimes involves a second maser.

SUMMARY

Now that there are several local times scales, as well as UTC and TAI, that are very stable, uncorrelated and comparable in magnitude, it is possible to perform a meaningful three-corner hat analysis to determine the individual stabilities among the most stable time scales. At PTB and OP the improved frequency stabilities are due to the use of Cs fountains as frequency references for the local time scales. It has been shown that UTC, UTC(USNO), UTC(PTB), UTC(OP) and UTC(NIST) all have stabilities that reach down into the mid 10^{-16} range. At present, the quality of this analysis is somewhat limited by a relatively small amount of data, but this will improve with time.

ACKNOWLEDGMENTS

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