A Proposed Ranging System with Application to VLF Timing

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Abstract—In general, without a clock it is not possible to determine the range between a radio transmitting and receiving site by making only passive observations at the receiving site. However, if the medium is dispersive, signals transmitted simultaneously from the same site, at different carrier frequencies, will not arrive simultaneously at some distant point. Thus, the difference in arrival time is related to the observer's distance from the transmitter. This effect is considered in conjunction with the VLF two-frequency timing system.

I. Introduction

FOR ACCURATE time dissemination to some point at a distance r away from the master clock transmitting site, it is necessary to known the signal propagation delay over r; otherwise, the slave clock will be in error by an amount

$$t = v/r$$

where v is the signal velocity. In general, it is not possible to determine r by making only passive observations of the timing signal at the slave site. Thus, the user must rely upon some other system, such as the VLF OMEGA navigational system, to obtain his range from the master site. In general, these navigational systems involve measuring the difference in times of arrival of signals transmitted from several different locations It is the purpose of this paper to point out the possibility of obtaining r directly from VLF timing signals which are transmitted from a single location. This possibility arises because of the dispersive character of the propagation medium at VLF. Using available experimental evidence, a calculation is made which suggests the order of magnitude of timing accuracy which could be obtained using the proposed system. A more precise calculation must await further experimental evidence.

II. Description of the Basic Method

Consider two pulses transmitted simultaneously from the same location with carrier frequencies f_1 and f_2. If the medium is dispersive, the pulses will travel with different group velocities V_1 and V_2. An observer, at a distance r from the transmitter, will observe that one pulse arrives before the other by an amount of time

$$\Delta t_g = r\left[\frac{1}{V_1} - \frac{1}{V_2}\right]. \quad (1)$$

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If the observer measures Δt_g and knows V_1 and V_2, he may solve (1) for r.

Although there are many examples of dispersive media, the earth ionosphere waveguide has particular interest for VLF navigational and time dissemination systems because of the phase stability of signals propagated over great distances.[1],[2] It is difficult to transmit short rise time pulses in this frequency range because of restricted antenna bandwidths. Therefore, other types of signal modulation must be considered for making precise time of arrival measurements. A discussion of such a modulation scheme follows.

III. The VLF Two-Frequency Timing System

Watt *et al.*[3] and Morgan[4] have proposed a VLF time and frequency dissemination system which consists of two closely spaced CW signals at frequencies f_1 and f_2 where

$$\Delta f = f_2 - f_1.$$

In this system, the beat frequency Δf is used as a coarse time mark to identify a specific cycle of either carrier frequency. This removes ambiguities, and the more precise time-of-arrival measurements are made at the higher carrier frequency. The beat note will travel from transmitter to receiver with a group velocity which depends upon the phase velocity versus frequency characteristic of the propagation medium.

In general, a second pair of signals at different carrier frequencies, but with the same frequency separation as the first pair, would produce the same beat frequency traveling with a different group velocity. If both beat notes were simultaneously transmitted in phase from the same location, then the difference in arrival time Δt_B at the receiving station could be used to establish the distance between transmitter and receiver. The precision of measurement would depend on the user's knowledge of the group velocities and his ability to measure the difference in arrival time of the two beat notes. The standard deviation in the time of arrival of a beat note is given by Jespersen,[5]

$$\sigma(B) = \frac{\sigma(f)f}{(\Delta f)}(1 - 2\gamma\rho + \gamma^2)^{1/2} \quad (2)$$

where ρ is the correlation of the phase fluctuations on the two carriers, γ is the ratio of the standard deviation of the phase fluctuations on the higher frequency carrier to the lower, and $\sigma(f)$ is the standard deviation of the fluctuation in time of arrival on the lower frequency carrier at frequency f. If $\rho=0$ and $\gamma=1$, (2) shows that the fluctuation in the time of arrival of the beat note is a factor $\sqrt{2}f/\Delta f$ larger than the fluctuation $\sigma(f)$ on the carrier signal. Thus, it would be difficult to make a precise measurement of Δt_g, and therefore r, from measurements on the beat notes alone. However, it might be possible, using the coarse value of d obtained from Δt_g, to select a pair of cycles (one from either carrier of each of the two pairs) which left the transmitting location simultaneously. In this case, the difference in arrival time of, say, the positive-going zero-crossings of the selected pair of cycles could be measured with a precision which depended only on the jitter of the zero-crossings associated with each of the carriers and not on some additional multiplicative factor.

IV. Illustrative Example

Group velocity V_g is related to phase velocity V_p by

$$V_g = 1/\left[\frac{d}{d\omega}(\omega/V_p)\right]$$

where ω is the angular frequency. In a dispersive medium, V_p is a function of ω so that the group delay over a path r is given by

$$t_d = (r/V_p)\left(1 - \frac{\omega}{V_p}\frac{dV_p}{d\omega}\right). \quad (3)$$

At VLF, the phase velocity versus frequency dependence cannot be expressed as a simple function of the properties of the propagation medium. The most accurate results are obtained from numerical solutions of Maxwell's equations with some realistic model of the ionosphere. For the sake of illustration we will use a solution obtained by Wait[6] for a sharply bounded isotropic ionosphere at height h above the earth. In this analysis the propagation medium is treated as a waveguide and the phase velocity is obtained as a function of frequency for each waveguide mode. At distances greater than about 3000 km, the higher-order modes are greatly attenuated, during the day at least, so that mode one is dominant. The phase velocity for this mode is given by

$$V_p \cong c\left(1 - \frac{\lambda^2}{16h^2}\right)^{-1/2}(1 - h/2a) \quad (4)$$

where a is the earth's radius and λ is the free space wavelength.

After substituting expression (4) for V_p into (3), the group delay was computed over a 1000-km path, as a function of frequency. A reflection height of 70 km was used which corresponds to daytime conditions. The results of this calculation are shown in Fig. 1 along with the corresponding phase delay over the same length of path. Suppose we pick pairs of signals centered near 10 and 20 kHz. Then, from the figure, the difference in group delay Δt_B of the beat notes over the 1000-km path is about 5 μs. For $d=20\,000$ km (approximately one-half the earth's circumference),

$$\Delta t_B \sim 100\ \mu\text{s}.$$

This value of Δt_B establishes the largest Δf which we can use. That is, the period of the beat note of a particular pair of signals must be at least 100 μs; otherwise, it will no longer be clear which pair of beat notes should be used to obtain Δt_B. Thus,

$$\Delta f \leq \frac{1}{\Delta t_B} \leq 10\ \text{kHz} \quad (5)$$

for our example. In this illustration, we will assume $\Delta f=1$ kKz which satisfies (5).

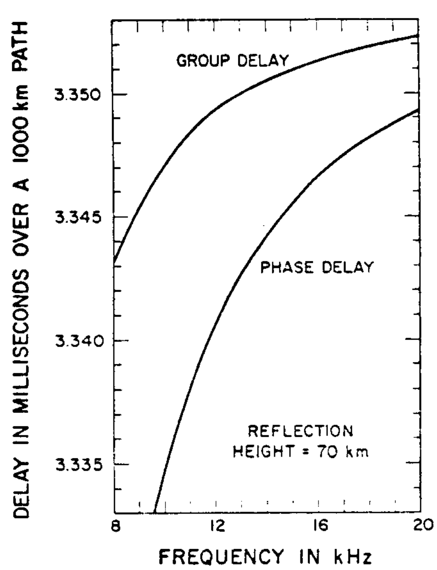

Fig. 1. Phase and group delays of a signal propagated over a 1000-km path as a function of frequency.

Recent observations[7] at Maui, Hawaii, of the 20-kHz WWVL transmission from Fort Collins, Colo. ($d=5200$ km), indicate that the standard deviation in he time of arrival of the signal δt (about the mean time) s about 0.1 μs for an integration time of 15 minutes. These observations refer to noon at the midpoint of the path. Experimental evidence[5] indicates that δt varies as $d^{1/4}$ so that we will assume that at 20 000 km $\Delta t_o \sim 0.14$ μs. Thus, from (2), assuming that $\rho=0$ and $\gamma=1$, we obtain

$$\sigma(B) \sim \sqrt{2}\ 2.8\ \mu\text{s}.$$

If we assume that a similar result would be obtained for a frequency pair near 10 kHz, then the standard deviation in the difference measurement Δt_B is

$$\sigma(\Delta t_B) \sim 5.6\ \mu\text{s}.$$

This assumes there is no correlation in the time of arrival fluctuations on the two beat notes. Thus, we would expect the result of our measurement to be

$$\Delta t_B \sim 100\ \mu\text{s} \pm 5.6\ \mu\text{s}$$

which, in terms of distance, is

$$r \sim 20\,000\ \text{km} \pm 1120\ \text{km}.$$

We would now like to improve our distance measurement by picking one carrier frequency from each of the two pairs and comparing, say, the positive-going zero-rossing of any two of the cycles which left the transmitting antenna in phase. The beat notes are used to identify which cycles at 10 and 20 kHz left the antenna in phase. However, there are two places where ambiguity may arise in the two steps outlined above. First, the selected pair of cycles may have slipped in phase by more than one cycle over r. Second, the beat note travels with group velocity V_g, whereas the two carriers producing the beat note will travel with their respective phase velocities which in a dispersive medium will both differ from V_g. So, the beat note will not necessarily point to the correct cycle of the selected carrier.

We will consider the beat note–carrier signal ambiguity first. For the example discussed here, the beat note and carrier signal will have slipped over the 20 000-km path, from Fig. 1, by about 240 μs for the signals near 10 kHz and about 60 μs for the signals near 20 kHz. Thus, the beat note will not unambiguously point to the correct carrier cycle at either frequency since the period for 20 kHz is 50 μs and for 10 kHz is 100 μs. However, from our gross estimate of the distance, we can calculate that the slippage should be about 240 ± 13.2 μs at 10 kHz and 60 ± 3.3 μs at 20 kHz, allowing us to select the correct cycles at both frequencies. Similarly, the slippage between the selected cycles of the two carriers over the 20 000 km-path will, from Fig. 1, have slipped about 280 μs, which, again, is ambiguous since it exceeds the period of either one of the carrier frequencies near 10 and 20 kHz. Again, however, the coarse distance measurements predict, using Fig. 1, that the slippage will be about 280 ± 16.5 μs, which indicates how we should correct our actual measurement, by adding or subtracting cycles to obtain an unambiguous result.

Having corrected our measurements for possible ambiguities, the final distance measurement will depend only upon our ability to measure the difference in time of arrival Δt_o of the correct cycles of the two selected carriers near 10 and 20 kHz and upon our knowledge of their respective phase velocities. As mentioned earlier, the standard deviation in the time of arrival of a 20-kHz signal over a 5200-km path was about 0.1 μs, for a 15-minute integration time, which becomes about 0.14 μs for a 20 000-km path. Assuming a similar result would be obtained at 10 kHz and that the phase fluctuations on the two carriers are independent, the standard deviation of Δt_ϕ would be

$$0.14\sqrt{2}\ \mu\text{s}$$

which corresponds to a distance measurement error of about 13 km for our particular example. From the timing point of view, a 13-km range error would produce a clock setting error of about 43 μs.

It should be emphasized that the preceding discussion is presented only to illustrate the method, to demonstrate possible points of ambiguity in the measurements, and to give order of magnitude estimates of the measurements which might be encountered in actual practice. Other ionospheric models will give different values for the phase and group velocities. In fact, it probably will not be possible to design a working system until more experimental information is available about propagation characteristics at VLF. However, at the present time there is considerable interest in VLF for navigational and time dissemination systems, and much experimental effort is being directed to this area.[2],[8]–[10]

V. Conclusions

Because of dispersion in the earth–ionosphere waveguide, VLF signals which simultaneously leave the same location will not arrive simultaneously at some distant point. This difference in arrival time may be used to determine the distance between transmitting and receiving locations if the signal velocities are known. A particular system is discussed which consists of four CW signals grouped into two pairs. Although various ambiguities may arise in the measurements, it is shown how these ambiguities may be resolved by first making a coarse range determination using the difference in arrival time of the beat notes associated with each pair of frequencies. Based upon available experimental information, an order of magnitude estimate indicates that the system could be used to synchronize clocks to about 40 μs. However, the ultimate utility and accuracy of the system must await further experimental results which are now being obtained in basic studies of VLF navigational and time dissemination systems.

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Phase-Sensitive Detector Nonlinearity at the Signal Detection in the Presence of Noise

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***Abstract*—The essential nonlinearity characteristics of a phase-sensitive detector are studied theoretically, assuming that the input signal is a sine wave in the presence of additive narrow-band Gaussian noise. Three interesting cases of the nonlinearity of the phase-sensitive detector characteristics are analyzed by means of the expression which has been derived for the output signal-to-input noise ratio as a function of the input signal-to-noise ratio, the reference wave-to-noise ratio, and the phase angle between the input signal and reference wave. In the first case, the detector nonlinearity N_A as a function of the input signal-to-noise ratio is determined. In the second and third cases, the detector nonlinearities N_B and N_C as a function of the phase angle between the input signal and the reference wave are obtained. The results are presented as closed-form analytical expressions. Several interesting cases are plotted as a function of the significant parameters. Besides being important in themselves, the results are of a general interest because they may be used to estimate essential nonlinearities in some other more complicated cases.**

I. INTRODUCTION

PHASE-SENSITIVE detection often finds application in various fields, such as experimental physics instrumentation,[1]–[3] electrical measurements,[4],[5] and automatic control systems.[6] More recently, its use increased in other areas of experimental work, where either extremely narrow band is desired or where the derivative of the signal is required.[7] Although literature dealing with the phase-sensitive detection is extensive,[8]–[10] to the knowledge of the author there are no theoretical considerations about the phase-sensitive detector nonlinearity at the signal detection in the presence of additive narrow-band Gaussian noise. Though for some purposes linearity of the detector is not essential, there are other cases of its applications in modern instrumentation where the linearity is of primary importance. For use of phase-sensitive detection systems in the nuclear magnetic resonance and electron-spin resonance spectrometers, the effect of detector nonlinearity must be evaluated since the shape of the recorded resonance can be altered.[11]–[13] For phase-sensitive detector circuits used in the amplitude or phase measurement systems, the effect of detector nonlinearity directly determines both the measurement accuracy and the system sensitivity on changing of various parameters which are involved in the inherent behavior of the detector.

The purpose of this paper is to obtain new and useful results of the essential nonlinearity characteristics of a phase-sensitive detector whose input signal is a sine wave which has been corrupted by the additive narrow-band Gaussian noise. Under the detector's essential nonlinearities, the nonlinearities are understood resulting from the inherent behavior of the phase-sensitive detector of the amplitude or phase detection of the input signal in the presence of noise; but they do not involve nonlinearities resulting from the characteristic nonlinearity of the diode elements used. Three impor-

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