A Note on a Selective RC Bridge*

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Summary—**A simple resistance-capacitance bridge is described The circuit is capable of providing higher selectivity than the Wien bridge, although it employs the same number of circuit elements.**

RESISTANCE-CAPACITANCE bridges have been used for the measurement of frequency,[1] resistance, and capacitance,[2] and have also been employed as selective elements in feedback amplifiers.[3] The Wien bridge and the parallel-T are commonly used in such applications. It is the purpose of this note to describe a simple RC bridge capable of providing a moderate increase in selectivity without requiring an increase in the number of circuit elements. The circuit is an RC form of a generalized six-arm bridge described by Anderson.[4]

Before describing the subject bridge, let us consider the Wien bridge of Fig. 1(a) as a basis for comparison.

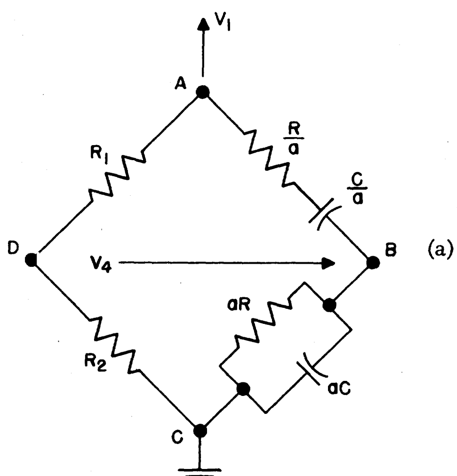

Fig. 1(a)—The Wien bridge.

In this circuit, provision has been made for modifying the values of resistance and capacitance in arms $A-B$ and $B-C$ to obtain the maximum selectivity. Applying Kirchoff's laws, it is found that

$$\beta = \frac{-ju}{b(b+ju)}, \tag{1}$$

and therefore

$$|\beta| = \frac{u}{b\sqrt{b^2+u^2}}, \tag{2}$$

where

$$\beta = \frac{V_4}{V_1},$$

$$u = \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega},$$

$$\omega_0 = \frac{1}{RC},$$

$$b = 1 + a^2 + \frac{1}{a^2},$$

and $R_1 = (b-1)R_2$ (the balance condition).

The voltage V_1 and V_4 are defined by the figure, as is the parameter a; ω is the angular frequency.

It is convenient to work in terms of the frequency variable u. Defining the selectivity S (at balance) in terms of this quantity,

$$S = \left.\frac{d|\beta|}{du}\right|_{u=0}. \tag{3}$$

For the Wien bridge under consideration,

$$S = \frac{1}{b^2} = \frac{1}{\left(1 + a^2 + \frac{1}{a^2}\right)^2}. \tag{4}$$

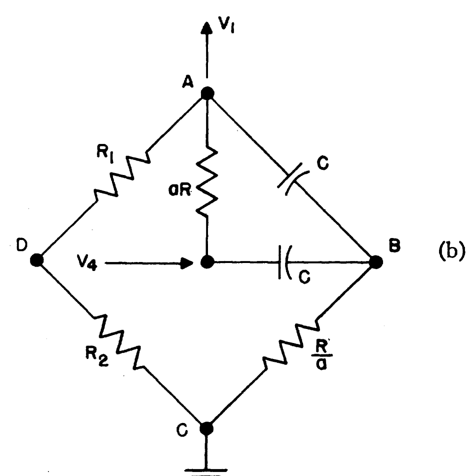

Fig. 1(b)—An RC bridge providing higher selectivity than the Wien bridge.

Examination of (4) shows that the maximum selectivity, numerically equal to 1/9, occurs for $a=1$, that

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[1] F. E. Terman, "Radio Engineer's Handbook," McGraw-Hill Book Co., Inc., New York, N. Y., p. 957; 1943.

[2] J. G. Ferguson and B. W. Bartlett, "The measurement of capacitance in terms of resistance and frequency," *Bell Sys. Tech. Jour.*, vol. 7, p. 420; July, 1928.

[3] G. E. Valley and H. Wallman, "Vacuum Tube Amplifiers," McGraw-Hill Book Co., Inc., New York, N. Y., pp. 384–408; 1948.

[4] B. Hague, "Alternating Current Bridge Methods," Pitman Publishing Corp., New York, N. Y., pp. 54–56; 1938.

is, with equal resistances and capacitances in arms A-B and B-C.

Consider, now, the circuit of Fig. 1(b), which contains the same number of components as Fig. 1(a). Here,

$$\beta = \frac{jua}{\left(\frac{2}{a} + a\right)^2 + ju\left(\frac{2}{a} + a\right)}, \quad (5)$$

and therefore

$$|\beta| = \frac{ua}{\left(\frac{2}{a} + a\right)\sqrt{\left(\frac{2}{a} + a\right)^2 + u^2}}, \quad (6)$$

where a is defined by the figure. The null condition requires

$$R_1 = \frac{a^2}{2} R_2.$$

It is interesting to note that for $a=1$, (6) is identical with (2), indicating that, with equal resistances and capacitances in the right-hand arms, the bridges of Figs. 1(a) and 1(b) have identical properties, including selectivity as defined. The selectivity of the bridge of Fig. 1(b) is given by

$$S = \frac{a^3}{(2 + a^2)^2}. \quad (7)$$

As expected from the above, $S=1/9$ when $a=1$. However, S increases with a to a maximum at $a=\sqrt{6}$, where $S_{max}=3\sqrt{6}/32$, or approximately 0.23. Consequently, the maximum selectivity is more than double that of the Wien bridge, permitting more accurate measurements of frequency, resistance or capacitance.

The bridge under discussion employs equal capacitances as frequency-determining elements. The alternate form shown in Fig. 1(c) employs equal resistances, which is convenient when variable-frequency operation is to be obtained with a dual variable resistor.

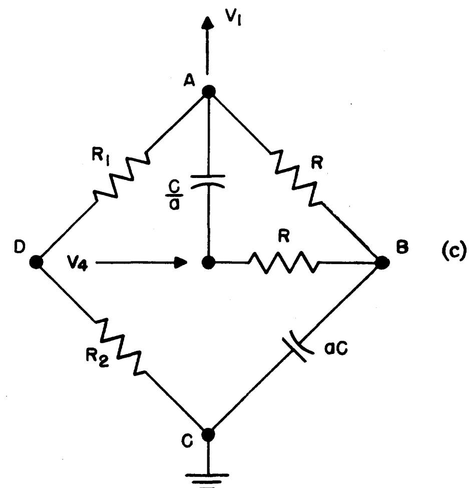

Fig. 1(c)—Another RC bridge providing higher selectivity than the Wien bridge.

The bridges described have been employed by the writer as frequency-determining elements in audio-frequency oscillators. It is thought that they should also be useful in some other applications requiring measurement of resistance, capacitance, or frequency.

Waves on Inhomogeneous Cylindrical Structures*

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Summary—An analysis is given of some of the basic properties of exponential modes on passive cylindrical structures in which the material constants of the medium vary over the cross section. The bounding surface is assumed to be opaque, in the form of an electric or a magnetic wall; it is therefore always nondissipative. Major consideration is given to structures in which the internal medium is also nondissipative.

Each mode is usually a $TE-TM$ mixture. Some of the conventional orthogonality conditions no longer remain valid. In certain circumstances, however, the instantaneous and vector power that flow along the system are still additive among the various modes. Stored and dissipated energies per unit length generally are not additive. The propagation constant for modes on a nondissipative structure cannot be complex. The relation between the direction of the time-average Poynting vector at any point of the cross section, and that of the phase and group velocities, is no longer necessarily conventional, and the space angle between the transverse electric and magnetic fields may vary over the cross section. The field distribution of each mode varies with frequency in a manner which is clarified by physical interpretation.

I. Introduction

THE MOST COMMON waveguide structures consist of a perfectly conducting metal tube, uniformly filled with a dissipationless dielectric material. Under these physical conditions, the analysis of the steady-state electromagnetic fields in the guide can be carried out completely in terms of the set of

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