High-Stability Bridge-Balancing Oscillator*

P. G. SULZER†, ASSOCIATE MEMBER, IRE

Summary—**A crystal-controlled frequency standard having a frequency stability of a few parts in 10^{10} per day is described. The oscillator employs a 1-megacycle GT-cut crystal unit in a high-sensitivity bridge-balancing frequency-correction system. The crystal serves both as the resonant element for the oscilaltor and for a bridge whose unbalance is an indication of departure of the oscillator frequency from the crystal series resonance. The unbalance, greatly amplified, is used to AFC the oscillator.**

Introduction

HIGH-STABILITY frequency standards are required for use in navigation systems, precise clocks, and for many types of scientific measurements and experiments. A frequency stability of 1×10^{-9} per day is often desirable. The oscillator to be described here is of interest because its frequency is almost entirely independent of tube, component, and supply-voltage changes. The first model, which employs relatively inexpensive, commercially available components, has a stability of a few parts in 10^{10} per day.

The resonant frequency of a quartz crystal-unit can be made to exhibit a high frequency stability. If it is connected to a suitable source of energy, such as an amplifier-limiter combination, a stable oscillator can be produced. Such an oscillator always departs from the resonant frequency of the crystal unit because of the phase shift present in the amplifier. There are two basic methods, which may be equivalent under some conditions, for decreasing the amplifier phase shift.

Oscillator Systems

One method employs a low-impedance coupling between the resonator and the amplifier.[1–4] It can be shown that the phase shift is directly proportional to the magnitude of the coupling impedance. However, with a limited transconductance, this impedance cannot be decreased below the point where gain is obtained. The second method employs inverse feedback in the amplifier to decrease the effective phase shift.[5] Recognizing the fact that phase shifts add, but gains multiply, in a multistage amplifier, it might appear that a large feedback factor could be employed to obtain excellent phase stability. Unfortunately it becomes difficult to control the sign of the feedback outside the pass band of the amplifier, particularly if three or more interstage couplings are required, and instability may be encountered. In a practical system it is difficult to obtain a phase-shift reduction of more than 50 to 1 by using inverse feedback.[6]

It should be noted here that the fractional (percentage) frequency change in a feedback oscillator is proportional to frequency, making it difficult to fully realize the capabilities of modern 1-megacycle and 5-megacycle AT-cut crystal units.[7] The use of high-frequency crystal units is desirable because they are less subject to the effects of mechanical shock and aging than are 100-kilocycle units. Furthermore they appear to be less expensive and more readily produced.

Bridge-Balancing Oscillator

The characteristics of a crystal unit can be measured to a high degree of precision in a resonance bridge. If an oscillator is adjusted to resonance with the crystal unit, frequency can be measured with a resolution of 1×10^{-10} or better.[8] Such a system constitutes a useful frequency standard, particularly if continuous frequency adjustment is made by automatic means.[9,10] Fig. 1(a), next page, is a block diagram of a suitable system. The crystal unit is connected in a low-impedance bridge so constructed that the effects of parasitic reactances are very small. The output of the null amplifier is used, in conjunction with a phase detector, to control the frequency of the oscillator exciting the bridge. In this manner the oscillator can be made to approach resonance with high precision. One limitation to the degree of precision is the noise in the system. In Appendix I it is shown that the equivalent fractional frequency deviation $\Delta f/f$ resulting from bridge thermal noise is

$$\frac{\Delta f}{f} = \frac{2}{Q}\sqrt{\frac{kT}{P_{Rt}}}, \tag{1}$$

where Q is the figure of merit of the resonator, k is Boltzmann's constant, 1.37×10^{-23} joule per degree

* Original manuscript received by the IRE, January 17, 1955; revised manuscript received, February 25, 1955.
† National Bureau of Standards, Washington 25, D. C.

[1] J. K. Clapp, "An inductance-capacitance oscillator of unusual frequency stability," Proc. I.R.E., vol. 36, pp. 356–358; March, 1948.
[2] W. A. Roberts, "An inductance-capacitance oscillator of unusual frequency stability," Proc. I.R.E., vol. 30, pp. 1261–1262; October, 1948.
[3] G. G. Gouriet, "High-stability oscillator," *Wireless Eng.*, vol. 27, pp. 105–112; April, 1950.
[4] J. K. Clapp, "Frequency-stable oscillators," Proc. I.R.E., vol. 42, pp. 1295–1300; August, 1954.
[5] L. A. Meacham, "The bridge-stabilized oscillator," Proc. I.R.E., vol. 26, pp. 1278–1294 October, 1938.
[6] P. G. Sulzer, "One-megacycle frequency standard." To be published.
[7] A. W. Warner, "High-frequency crystal units for primary standards," Proc. I.R.E., vol. 40, pp. 1030–1033; September, 1952.
[8] J. M. Shaull and J. H. Shoaf, "Precision quartz resonator frequency standards," Proc. I.R.E., vol. 42, pp. 1300–1306; August, 1954.
[9] Such an oscillator was described by Norman Lea of British Marconi during his recent visit to the U.S.A.
[10] T. A. Pendleton, "A System for Precision Frequency Control of a One-Hundred Kilocycle Oscillator by Means of a Quartz-Crystal Resonator," M.S. thesis, University of Maryland, May, 1953. (Summary of work done at the National Bureau of Standards in fall of 1952 and spring of 1953.)

Kelvin, T is the absolute temperature of the bridge, P_R is the power dissipated in the resonator, and t is the time required to make a measurement. For the oscillator to be described, $Q=10^6$, $T=320$ degrees K, $P_R=2.5\times10^{-6}$ watts, and $t\approx0.06$ sec. therefore, $(\Delta f/f)\approx3\times10^{-13}$.

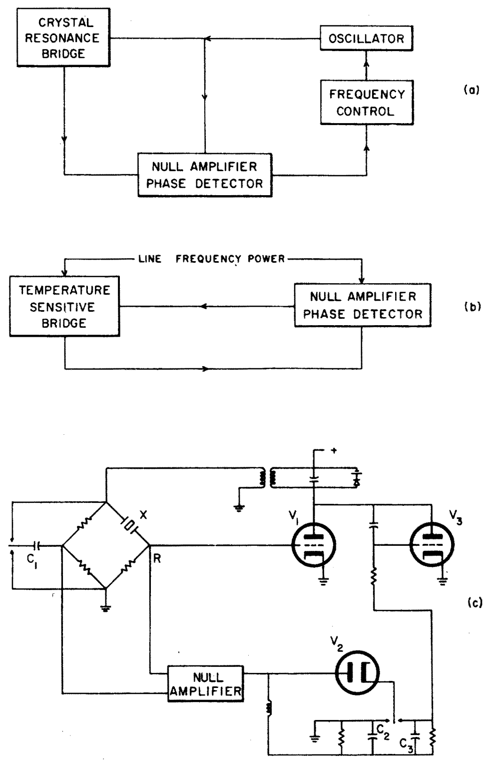

Fig. 1—(a) Block diagram of bridge-balancing oscillator, (b) block diagram of temperature-control system, (c) simplified schematic diagram showing components of bridge-balancing oscillator.

In Appendix II the feedback frequency-control system of Fig. 1(a) is analyzed, and it is found that the ratio of fractional frequency deviation with feedback $(\Delta f/f)_c$ to that without feedback, $\Delta f/f$, is given by

$$\frac{\left(\frac{\Delta f}{f}\right)_c}{\frac{\Delta f}{f}} = \frac{1}{1+\frac{1}{4}K_1QAV_1}, \tag{2}$$

where K_1 is twice the fractional frequency change produced by the frequency control per unit control voltage, Q is the figure of merit of the resonator, A is the effective voltage gain from the bridge output to the frequency-control input, and V_1 is the bridge-input voltage. For the experimental oscillator $K_1=10^{-7}$/volt, $Q=10^6$, $A=10^7$, and $V=0.01$ volt. Therefore

$$\frac{\left(\frac{\Delta f}{f}\right)_c}{\frac{\Delta f}{f}} \approx \frac{1}{2500}.$$

It is comparatively easy to produce a simple oscillator with a stability of a few parts in 10^7, and this can then be improved by a factor of 10^3 or more by a bridge-balancing frequency control, subject to the limitations of thermal noise and the stability of the crystal unit itself.

Temperature Controller

It must be pointed out that in order to realize the capabilities of such a system the crystal unit must be operated in a proper environment. Specifically, a constant, low driving power must be employed, and a constant temperature must be maintained. The crystal-unit power can be controlled by the use of the proper oscillator circuitry; however, the temperature controller deserves some additional consideration. The limitations and performance of a temperature controller are given below. Before proceeding with the analysis, however, it should be pointed out that such factors as bridge-arm instability and heat loss through leads entering the controlled chamber may make the performance of the oven poorer than an analysis assuming ideal conditions might indicate.

A simple method is shown in Fig. 1(b). A temperature-sensitive bridge is employed, and the output of a high-gain bridge-unbalance amplifier is fed back to the bridge input to provide the necessary heating power. As in the crystal bridge, one limitation to the sensitivity of this system is the thermal noise from the bridge arms themselves. In Appendix III it is shown that the thermal noise is equivalent to a temperature difference

$$\Delta T = \frac{2\sqrt{2}}{\alpha}\sqrt{\frac{kT}{P_Bt}}, \tag{3}$$

where α is the temperature coefficient of resistance of the material used in two of the bridge arms, and P_B is the signal power dissipated in the bridge. (The other quantities are defined above.) Here P_B is a constant power applied to the bridge to produce a signal for driving the amplifier, and is to be distinguished from the output of the amplifier, which is a function of signal level. In the circuit to be described below, P_B is obtained from a small alternating voltage applied to the bridge, while the major part of the bridge power, which must necessarily vary as a function of the bridge unbalance, is obtained by rectifying and filtering the amplifier output. A temperature controller has been constructed with the following constants: $\alpha=0.0045$ per degree C., $P_B=0.02$ watt, and $t=1/10$ second. Therefore $\Delta T\approx1.4\times10^{-6}$ degree C. This places a lower limit on the temperature resolution to be obtained with such a system.

In Appendix IV the feedback temperature controller is analyzed, and it is found that the ambient-temperature reduction factor S is given by

$$S = \frac{1}{A\alpha}\sqrt{\frac{K_2}{P_B(T_1-T_2)}}, \tag{4}$$

where S is the ratio of an oven-temperature change to the corresponding temperature change outside the oven, and K_2 is the power loss through the oven walls per unit temperature difference $T_1 - T_2$. If $A = 10^6$, $\alpha = 4.5 \times 10^{-3}$ per degree C., $K_2 = 0.02$ watt per degree C., $P_B = 0.02$ watt, $T_1 - T_2 = 25$ degrees C., and $S \approx (1/22{,}000)$. Thus a 22 degree C. change in ambient temperature would produce a change of but 10^{-3} degree inside the oven.

Practical Bridge-Balancing Oscillator

There are many possible choices for the components to fill the blocks of Fig. 1(a). Thus it would have been possible to use a commercial high-quality oscillator to drive the bridge, with the addition of a motor-driven frequency control, and an ordinary communications-type receiver might have been used as a null detector. However, in the interest of simplicity the circuits outlined in the incomplete schematic of Fig. 1(c) were chosen. Here V_1 is a single-stage amplifier, which is transformer-coupled to a bridge containing a one-megacycle AT-cut crystal X and a resistance R, which is made equal to the resistance of X. The other two bridge arms are equal resistors of a value somewhat higher than R. The oscillator alone tends to operate at the series-resonant frequency of X, with an over-all frequency stability of about one part in 10^7. Amplitude control is obtained by connecting a biased diode limiter across the plate circuit of V_1. The plate voltage of V_1 is approximately one volt rms. The tuned transformer decreases this to 10 millivolts at the bridge input. A crystal current of approximately 500 microamperes is obtained with a 10-ohm crystal unit.

A small capacitor C_1 is switched alternately across one or the other of the equal bridge arms by means of a chopper to obtain the sense of the unbalance. The bridge output is amplified by a factor of 10^7, rectified by V_2, and capacitors C_2 and C_3 are charged synchronously with the switching of C_1. The difference of the voltages across C_2 and C_3 is used to drive a reactance tube V_3 which controls the frequency of the oscillator over a small range. If there is no difference in frequency between the oscillator and the series-resonant frequency of X, equal outputs are obtained corresponding to the two positions of C_1, and the reactance tube is not actuated. If a frequency error does occur, a direct voltage of the proper polarity is applied to V_3 to produce a correction. The amount of correlation is given by (2), and the constants apply to this oscillator. Although the noise bandwidth of the null amplifier is approximately one kilocycle, the chopper and detector are followed by a resistance-capacitance filter with a band-width of approximately $\frac{1}{4}$ cycle. It can be shown that for small signal-to-noise ratios, the effective bandwidth reduction radio by post-detection filtering is the square root of the ratio of the pre- and post-detection filter bandwidths. Here the detector does not function as a carrier-synchronous detector, but rather as a nonsynchronous detector during each chopper half-cycle. Here the bandwidth-reduction factor is 63, with a resultant effective bandwidth of 16 cycles, and therefore the measurement time t is approximately 0.06 second.

The switching of capacitor C_1 should produce a square-wave modulation of the oscillator. The degree of modulation is very small, however, and the modulation has not been detected after multiplication to 1,000 megacycles.

One interesting feature of the oscillator is that the crystal unit is common to the bridge and the controlled oscillator, permitting some simplification of the equipment.[11] Another point worth mentioning is the use of a diode limiter rather than an automatic-gain-control system for amplitude control. The diode limiter maintains the crystal current constant to ± 1 per cent over a two-to-one plate-supply-voltage range. An automatic-gain-control system would probably require two or three tubes for equal performance. It should also be mentioned that the reactance-tube control voltage is metered, so that the departure from resonance is indicated at all times. The sensitivity of the meter is 1×10^{-11} per small division. This permits the readjustment of the oscillator if large, permanent phase shifts are encountered.

Fig. 2 shows frequency change vs plate-supply voltage for the oscillator (V_1) alone, for the oscillator plus reactance tube with the null amplifier disabled, and for the complete system. It will be noted that the reactance tube degrades the stability of the oscillator. The performance of a two-tube Meacham oscillator employing the same type of crystal unit is shown to permit comparison.

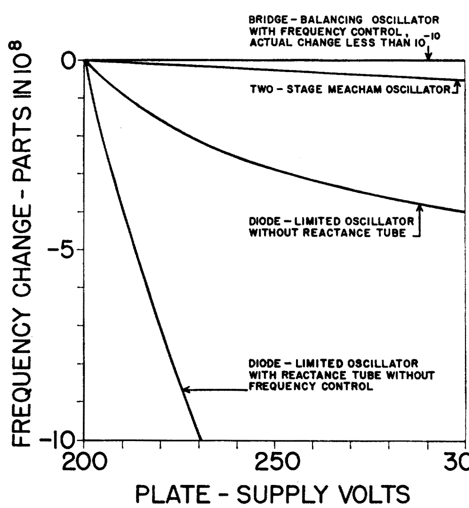

Fig. 2—Frequency change vs plate-supply voltage.

Fig. 3 (next page) is a 24-hour section of a record of beat frequency between the 1-megacycle oscillator and a reference oscillator. The outputs of the two oscillators were mixed after multiplication to a frequency of 1,000 megacycles, and the resulting beat was counted for 100-second intervals, producing a resolution of 10^{-11}. Each minor division represents a frequency change of 10^{-10}. It can be seen that the apparent stability of the oscillator, whose frequency changed $\pm 2 \times 10^{-10}$ during the

[11] N. Lea, "Quartz resonator servo," *Marconi Review*, vol. 11, pp. 65–73; 3rd quarter, 1954.

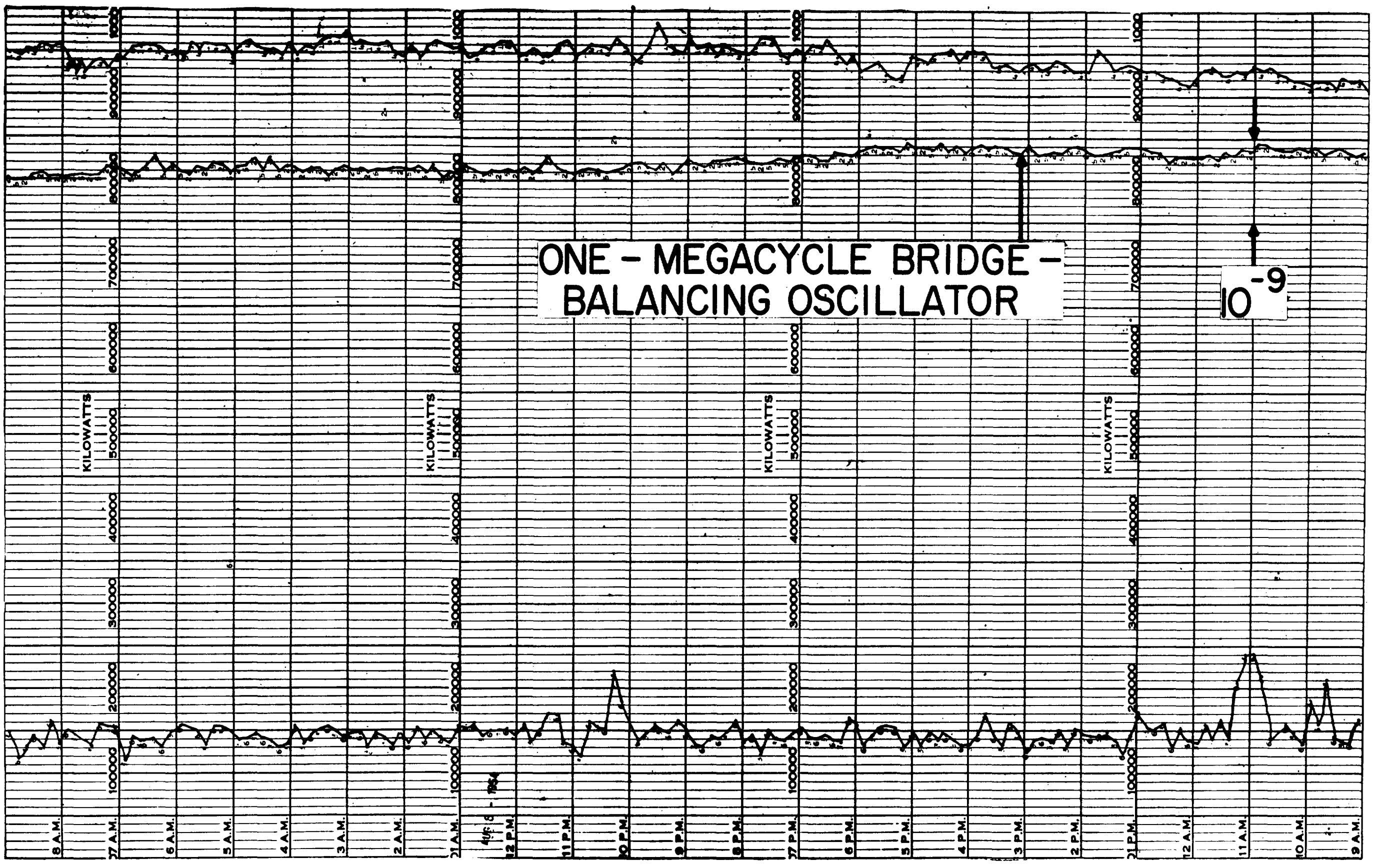

Fig. 3—24-hour frequency vs time record for the bridge-balancing oscillator.

24-hour period, compares favorably with that of the other two oscillators whose records are shown. These two oscillators, as well as the reference oscillator, are of the Meacham type, employing high-precision GT-cut crystal units. Actually the short-time stability (15 minutes) of the one-megacycle oscillator may be better than it would appear because of possible variations in the reference oscillator.

Fig. 4 is a schematic diagram showing the oscillator, reactance tube, and output (buffer) amplifier. Fig. 5 shows the null detector, which contains a three-stage tuned-radio-frequency amplifier and detector. Although a crystal filter is shown, this was not used during the tests and could well have been omitted. Such a filter would be necessary with a higher-gain amplifier. Fig. 6 is a schematic diagram of the temperature controller, which contains a conventional high-gain audio-frequency amplifier and phase detector.

The bridge arms are noninductively wound on an aluminum cylinder of $\frac{1}{4}$-inch wall thickness containing the crystal unit and its trimmer capacitor. The oven is insulated by means of a Dewar flask, whose effectiveness is demonstrated by the fact that replacing the flask with one inch of felt increased the power consumption by a factor of 10 for a given temperature difference.

Heat loss through the oven leads was minimized by the use of small-diameter wire.

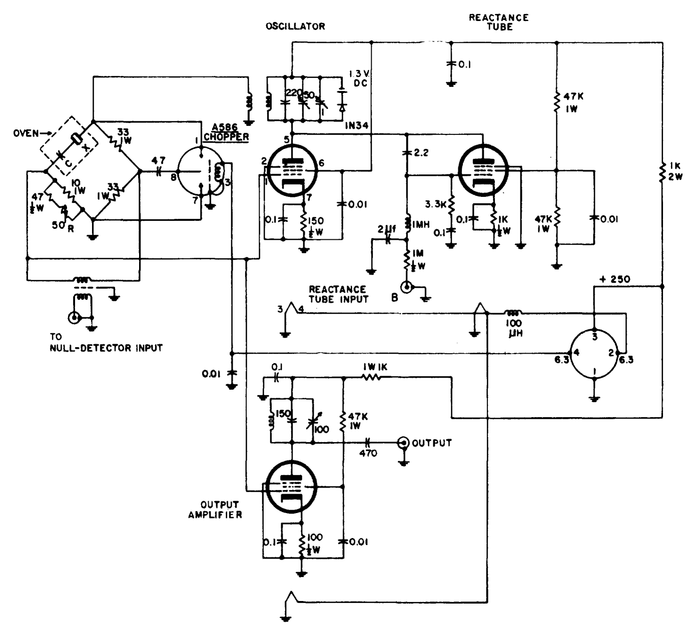

Fig. 4—Schematic diagram of the oscillator, reactance tube, and output amplifier. All tubes type 6AU6. Capacitances greater than 1 in $\mu\mu$f; others in μf unless noted. x = 1mc at cut crystal with resistance of 10 ohms ($Q = 10^6$). C = 1,500 $\mu\mu$f to adjust crystal to operating frequency.

Fig. 7, which appears on page 706, is a photograph of the experimental oscillation.

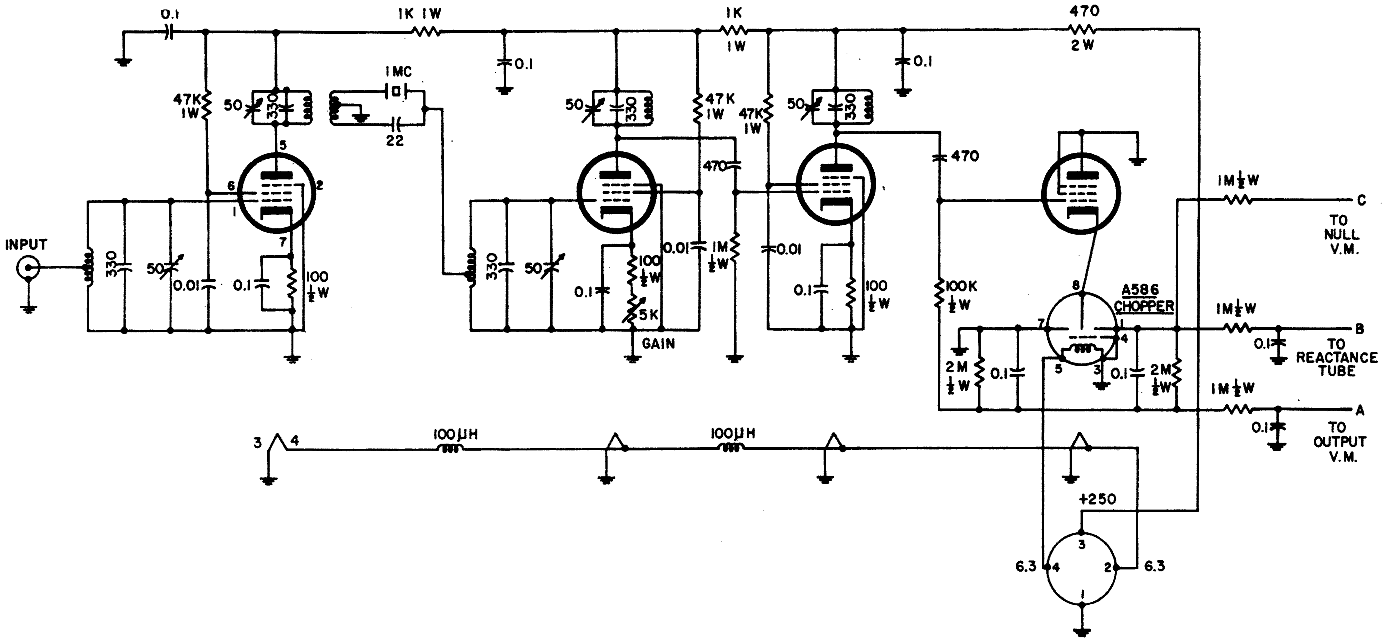

Fig. 5—Schematic diagram of the null amplifier and detector. All tubes type 6AU6. Capacitances greater than 1 in $\mu\mu$f; others in μf.

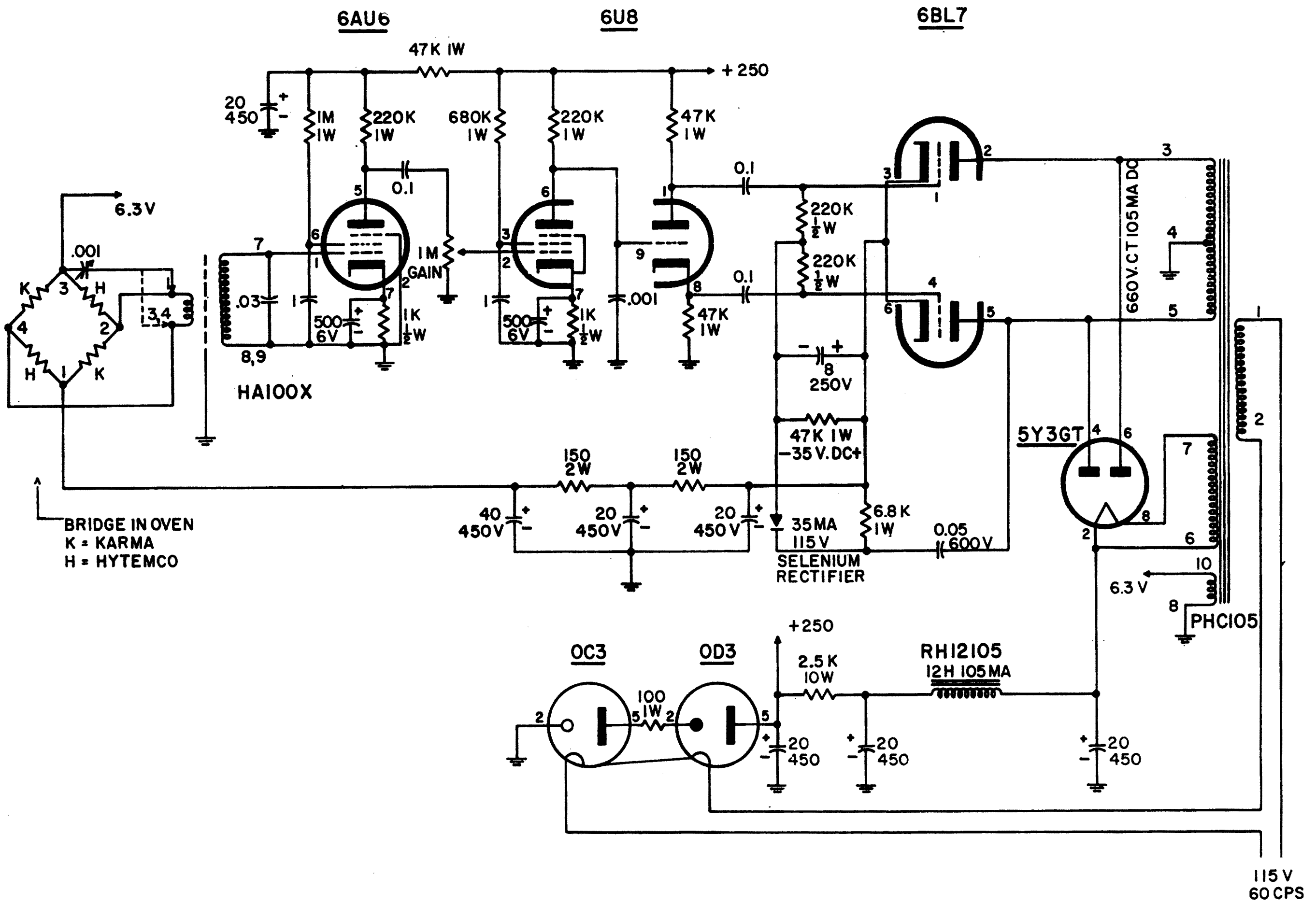

Fig. 6—Schematic diagram of the temperature controller. All bridge arms = 2,500Ω at 50 degrees C. All capacitances in μf.
All paper capacitors 600 volts working.

Appendix I

Crystal-Bridge Signal-to-Noise Ratio

The available noise power from the bridge is $P_n = kTB$, where k is Boltzmann's constant, T is the absolute temperature of the bridge, and B is the noise bandwidth of the null detector. Considering an equal-arm bridge, the equivalent bridge input voltage is $V_1 = 2\sqrt{P_R R}$, where P_R is the power dissipated in the resonator, and R is the resistance of each arm. The open-circuit bridge-output voltage resulting from a small fractional detuning $\Delta f/f$ is then $V_2 \cong Q\sqrt{P_R R}\,\Delta f/f$, and the available signal power is $P_S \cong \frac{1}{4}Q^2 P_R\,(\Delta f/f)^2$. It is assumed that the resolution of the system is limited to the condition of equal signal and noise powers, although this could be exceeded by special means. Equating the signal and noise powers, and assuming that making a measurement in t seconds requires a bandwidth $B = 1/t$ cycles,

$$\frac{\Delta f}{f} = \frac{2}{Q}\sqrt{\frac{kT}{P_R t}}\,.$$

This neglects the noise contribution of the null-detection system, which can be neglected because its noise figure can be made very low at medium frequencies.

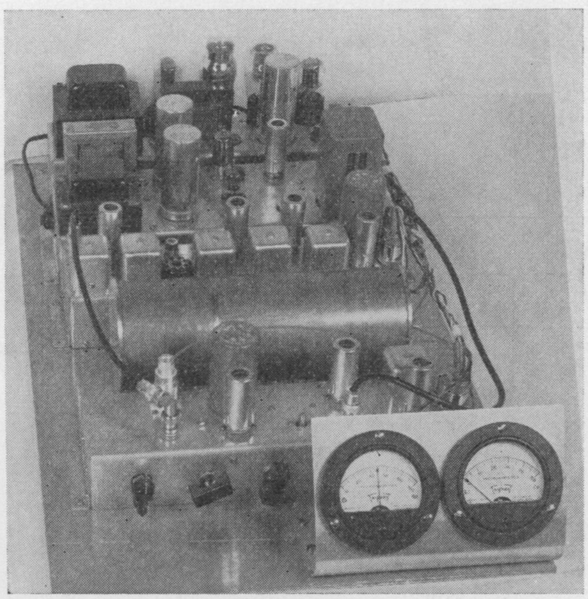

Fig. 7—Photograph of the experimental bridge-balancing oscillator.

Appendix II

Frequency-Control System

The output voltage V_2 of an equal-arm resonance bridge with input V_1 is given by

$$V_2 \cong \tfrac{1}{4}QuV_1 \tag{5}$$

where

$$u = \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega},$$

and ω_0 is the resonant frequency of the resonator.

The output voltage of V_3 of the null-detection system is then

$$V_3 \cong \tfrac{1}{4}QuV_1A, \tag{6}$$

where A is the effective voltage gain of the null detector.

A reactance tube is driven by V_3. The change produced by the reactance tube is

$$u_R = K_1V_3. \tag{7}$$

If the original (uncorrected) error is u_0, the new (corrected) error is

$$\begin{aligned} u_c &= u_0 - u_R \\ &= u_0 - K_1V_3 = u_0 - \tfrac{1}{4}K_1Qu_cV_1A. \end{aligned} \tag{8}$$

Solving for u_c/u_0,

$$\frac{u_c}{u_0} = \frac{1}{1 + \frac{1}{4}K_1QAV_1}\,. \tag{9}$$

Since

$$u \cong 2\,\frac{\omega_0 - \omega}{\omega_0} = \frac{\Delta\omega}{\omega_0} = \frac{\Delta f}{f},$$

$$\frac{\left(\frac{\Delta f}{f}\right)_c}{\frac{\Delta f}{f}} = \frac{1}{1 + \frac{1}{4}K_1QAV_1}$$

Appendix III

Temperature-Control Signal-to-Noise Ratio

The available noise power from the bridge is $P_n = kTB$, where B is the noise bandwidth of the system following the bridge, and the other quantities are defined above. If the signal power supplied to the bridge is P_B, the equivalent input voltage V_1 is $V_1 = \sqrt{P_B R}$, when R is the resistance of each arm at the balance temperature. The open-circuit output voltage V_2 is then $V_2 = \frac{1}{2}\alpha\Delta T V_1 = \frac{1}{2}\alpha\Delta T\sqrt{P_B R}$, where α is the temperature coefficient of resistance of the material used in two of the bridge arms, and ΔT is the difference between the balance and operating temperatures. Thus the available signal power P_S is $P_S = \frac{1}{8}\alpha^2 P_B(\Delta T)^2$. Equating P_n and P_S, and letting $B = 1/t$, where t is time required for a measurement,

$$\Delta T = \frac{2\sqrt{2}}{\alpha}\sqrt{\frac{kT}{P_B t}}\,.$$

Appendix IV

Temperature-Control System

T_0 = bridge-balance temperature
T_1 = bridge temperature (very nearly the oven temperature)
T_2 = ambient temperature
Pb = constant signal power supplied to bridge
P_H = variable bridge-heating power
R = resistance of each bridge arm at T_0.

V_2 = bridge-output voltage (alternating)
V_3 = detector-output voltage (direct)
$A = V_3/V_2$
α = temperature-coefficient of resistance of two opposite bridge arms
K_2 = coefficient of power transfer through oven walls
S = ambient-temperature reduction factor.

The temperature-sensing bridge is supplied with a small power P_B at the line frequency. The bridge output voltage is amplified and rectified in a synchronous detector. The detector output, which is a direct voltage proportional to bridge unbalance, is filtered and applied to the bridge to supply the necessary heat, P_H. The principal advantage of rectification is the avoidance of oscillation at the signal frequency. An additional benefit is the fact that, with synchronous detection, the effective bandwidth of the system is twice that of the filter following the detector, rather than that of the audio amplifier preceding the detector. It is easier to obtain a narrow bandwidth with a low-pass filter following the detector than with a band-pass filter at the power line frequency. Actually the effective bandwidth of the system may be somewhat less than this because of the thermal inertia of the oven itself, which could produce some additional filtering.

It is easily shown that the bridge output voltage is

$$V_2 = \frac{\sqrt{P_B R}\,\alpha}{2}(T_0 - T_1) \qquad (10)$$

$$V_2 = AV_2 \qquad (11)$$

$$\text{power in} = \text{power out} \qquad (12)$$

$$P_H = K_2(T_1 - T_2) \qquad (13)$$

$$V_3 = \sqrt{P_H R} = \sqrt{RK_2(T_1 - T_2)} \qquad (14)$$

Combining (10) and (14).

$$T_0 - T_1 = \frac{2}{A\alpha}\sqrt{\frac{K_2}{P_B}(T_1 - T_2)}. \qquad (15)$$

Differentiating (15), the ambient-temperature reduction factor S is obtained,

$$S = \frac{d(T_0 - T_1)}{d(T_1 - T_2)} = \frac{1}{A\alpha}\sqrt{\frac{K_2}{P_B(T_1 - T_2)}}. \qquad (16)$$

An Inflection-Point Emission Test*

E. G. HOPKINS† AND K. K. SHRIVASTAVA†

Summary—**The paper describes a method by which the inflection-point-emission of an oxide-cathode tube may be indicated as a meter reading after subjecting the tube to a single triangular current pulse. The testing current rises from zero at a constant rate and the anode voltage is differentiated twice with respect to time. The time which elapses between the commencement of the test and the appearance of the first positive voltage from the second differentiator is indicated on a meter calibrated directly in inflection-point emission. The testing current is arrested at the inflection point. An experimental equipment is described which tests receiving-tubes for inflection-point emission using a rate of current rise of 4 amperes per millisecond. The duration of the test is of the order of one millisecond and the emission readings tend towards pulse emission values rather than dc emission values. Experiments are described which demonstrate the potential usefulness of the test for quality control in tube manufacture and for taking readings of emission while subjecting a tube to particular operating conditions.**

Introduction

THE CURRENT-VOLTAGE characteristic of an oxide-cathode tube executes a gradual increase in slope throughout the space-charge-limited region followed by a gradual decrease during transition to the temperature limited region. "Emission" can be defined only as the current corresponding to some arbitrarily chosen point on this characteristic such as "breakway point," "inflection point," or "flection point." The location of such points has hitherto required the interpretation of a simultaneous display of both current and voltage,[1,2] over a large portion of the tube characteristic. This is too lengthy a procedure for the commercial testing of receiving tubes or for rapid observations of emission during tube operation.

The system of emission testing described in this paper enables inflection-point emission to be read directly on a meter by subjecting the tube under test to a single current pulse. It was developed to enable the emission of a self-heating tube[3] to be measured with the minimum of interruption to the normal electron bombardment of its cathodes and is applicable to the investigation of oxide-cathode behavior under operating conditions and

* Original manuscript received by the IRE, November 8, 1954; revised manuscript February 11, 1955.
† School of Electrical Engrg., N.S.W. University of Technology, Broadway, Sydney, Australia.

[1] Standards of the IRE, Electron Tubes, Part I, "Methods of Testing," PROC. I.R.E., vol. 38 pp. 922–924; August, 1950.
[2] L. A. Marzetta, "High-power pulser aids cathode studies," *Electronics*, vol. 27, pp. 178–180; March, 1954.
[3] E. G. Hopkins, "Self-heating thermionic tubes," *Proc. A.I.E.E.*, Part III, p. 77; March, 1954.