APPARENT DIURNAL EFFECTS IN THE GLOBAL POSITIONING SYSTEM

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ABSTRACT

Since the Global Positioning System (GPS) has been used for common view time and frequency transfer between remote locations various systematic effects have been observed. These effects have been discussed on various occasions appearing as biases between different daily measurements as well as obstructing closure in around-the-world time transfer. In addition we may attempt to look at GPS satellites from several locations around the world, after linking the ground station clocks using GPS. The results are that there are apparent diurnal variations in many of the SV clocks. We study these systematic effects here, the biases in common view time transfer, the lack of closure in around-the-world time transfer, and the diurnal variations in the SV clocks. We conclude that the diurnal effects are primarily due to errors in the transmitted satellite ephemeris and ionospheric model.

INTRODUCTION

Since the Global Positioning System (GPS) has been used for common view time and frequency transfer between remote locations various systematic effects have been observed. These effects have been discussed on various occasions and appear as biases among different once-per-sidereal-day measurements as well as obstructing closure in around-the-world time transfer[1]. In addition we may attempt to look at GPS satellites from several locations around the world, after linking the ground station clocks using GPS. In this paper we look at GPS from the international time standards laboratories, referencing all stations to UTC(NBS), thus creating a global network of UTC(NBS). The results are that there are apparent diurnal variations in many of the space vehicle (SV) clocks and in GPS system time. The SV's are denoted by two different numbers: the pseudo-random code number (PRN), corresponding to the code the SV transmits, and the Navstar number for the sequential order in which the SV was launched. We look at space vehicle PRN numbers (Navstar's) 3 (11), 6 (3), 11 (8), 12 (10), and 13 (9), and the GPS system clock from MJD 46987 to 47046, July 11 - September 8, 1987. These effects, the biases in common view time transfer, the lack of closure in around-the-world time transfer, and diurnal variations in the GPS clocks, are not completely understood. We know that the first two must be either satellite ephemeris errors, propagation modelling errors, multi-path variations, or poor ground station coordinates. The diurnal variations could be any of these also, plus possibly a real effect in the SV clocks. We show that there are systematic diurnal variations in the ephemeris and ionospheric correction as transmitted by each of the SV's in the GPS system. We see also in most cases that diurnal variations in the individual clock corrections, the transmitted value of GPS time minus SV clock time, if they exist, are below the noise level of the clock correction itself.

GLOBAL UTC(NBS) AS A REFERENCE FOR GPS

Looking at GPS satellites from several locations is a two step process. We must first link the reference clocks at the remote locations by computing their offsets from a common reference clock. In our case this clock was UTC(NBS). Thus we produce a Global UTC(NBS) from which to view the GPS satellites. We then subtract this estimate from the measurements made against satellites at the respective locations.

We link remote clocks using a Kalman smoother on common view time differences weighted according to measurement noises computed by a multi-station separation of variance technique[2]. The "clocks" involved are the reference standards at the GPS receivers at each of the National Bureau of Standards (NBS), Boulder, CO; the Jet Propulsion Laboratory (JPL) Deep Space Network station at Goldstone, CA; the Applied Physics Laboratory (APL), Laurel, MD; the U.S. Naval Observatory (USNO), Washington D.C.; the Paris Observatory (OP), Paris, France; the Physikalisch Technische Bundesanstalt (PTB) in Braunschweig, Fed. Rep. of Germany; the Tokyo Astronomical Observatory (TAO) and the Radio Research Lab (RRL) both in Tokyo, Japan; and the WWVH radio station of NBS in Kauai, HA. These stations and their estimated time differences from UTC(NBS) form the Global UTC(NBS) network for this research.

Measurements are made regularly at each of these locations of GPS satellite clocks against the local clock averaged for 13 minutes, using the SV's transmitted ephemeris and ionospheric model. In addition a transmitted SV clock correction is applied to this measurement to obtain GPS system time against the local clock. These measurements are repeated every sidereal day since the SV's are in 12 hour sidereal orbits, thus maintaining essentially the same geometry of each SV measurement. We apply our estimates of local reference minus UTC(NBS) to each of these data sets to obtain measurements of individual SV's against Global UTC(NBS) throughout their orbits, as well as measurements of GPS minus UTC(NBS) over periods much less than one day. With a few exceptions, each measurement is made by at least three locations at once. The difference of these measurements against the local clock are the input to the Kalman smoother which estimates Global UTC(NBS) at each site. Also, after subtracting the local offset to obtain SV or GPS minus Global UTC(NBS), which we denote [SV - Global UTC(NBS)] or [GPS - Global UTC(NBS)] respectively, we average across the simultaneous measurements to obtain better estimates of the clocks, both SV and GPS. Finally, we run a Kalman smoother on the individual [SV - Global UTC(NBS)] or [GPS - Global UTC(NBS)] weighted according to the separation-of-variance data. The model allows clocks to have a certain amount of white and random walk frequency modulation (FM), with white phase modulated measurement noise. Residuals from the smoothed [SV - Global UTC(NBS)] are assumed to be propagation plus ephemeris modelling errors, since the receiver noise is sub- nanosecond [3]. When we smooth [GPS - Global UTC(NBS)] we use as input the smoothed SV values plus the transmitted clock correction. Thus, the residuals here are clock correction errors.

The most striking feature in the results is a diurnal variation which appears in the data for every SV against Global UTC(NBS), as well as in the smoothed [GPS - Global UTC(NBS)] data. This diurnal variations must be due to either: 1) a true diurnal variation in the SV clock, 2) a diurnal variation in the transmitted data: the SV clock correction to GPS system time, the ephemeris, or the ionospheric model as transmitted by the SV, 3) a local effect at the receiver: multi-path or coordinate errors, or 4) a bias in the estimated value Global UTC(NBS). The local effects we believe to be minimal as will be discussed later. The other effects all reflect a system diurnal variation in the GPS. Even 4), a Global UTC(NBS) bias, does so in that the GPS is used to estimate this value. We show, however, that the systematic diurnal variation we report here is not caused by any such bias. We find that the diurnal variation is a systematic error either in SV ephemerides or the ionospheric model or both.

Possibly the GPS has a systematic diurnal modulation in that the control segment is linked to the satellites by a measurement and control system with diurnal variations built in: the system of the earth rotating under the 12 hour sidereal SV orbits. Any error in GPS monitor station coordinates or clock estimates would feed into the system with a diurnal signature.

ANALYSIS TECHNIQUE

The primary tools for our analysis are the Fast Fourier Transform (FFT) and the Allan or two point variance. As summarized elsewhere[4] these two are related. A power law dependence in the frequency domain corresponds to a power law dependence of the Allan variance on integration time. This is applicable for clocks with white, flicker or random walk fluctuations in either phase or frequency. We expect clock noise to exhibit f^{β} behavior in the time spectrum, where β = -2, -3, or -4, for white FM flicker FM or random walk FM respectively. A sinusoidal modulation with frequency f_m appears as a sharp spike in the frequency domain. The Allan variance, $\sigma_y(\tau)$, can also reveal this phenomenon in that there is a dependence on integration time, τ, quite different from a power law:

$$\sigma_y(\tau) = \frac{x_{pp}}{\tau}\sin^2(\pi f_m \tau) \tag{1}$$

Thus if we see a variance with a peak value at 0.5 day, and this value is inconsistent with the slope associated with a power law, we conclude there is a systematic fluctuation with a period of 1.0 day.

RESULTS

The first data we look at are GPS system time minus Global UTC(NBS). This is the most generic form of data in that all possible sources of systematic variations are present. GPS system time is used as it is represented in the transmitted offset from the physical clocks on board the satellites. Included in this number is the range correction derived from the transmitted ephemeris and ionospheric models, and the transmitted value of GPS system time versus the SV clock. Global UTC(NBS) is also an estimated offset applied to various clocks as described above. When we look at these comparisons over the period in question, Figure 1, we seem to see some daily variation. If we look at the Fourier spectrum, $S_y(f)$, in Figure 2, we indeed see modulation at 1 cycle per day. The output of our linearized GPS smoother, Figure 3, gives a clear visual appearance of a systematic variation after removing the white PM. In the variance we see a sudden drop in value from $\tau =$ 0.5 sidereal day to 1 day, Figure 4. Using Equation 1 above we find that the peak-to-peak variations average 9.4 ns, which closely agrees with the estimate from our $S_y(f)$ plot. These data indicate that something is wrong somewhere without giving us a particularly good idea where. The possibilities are: 1) a true diurnal variation in the SV clock, 2) a diurnal variation in the transmitted data: the SV clock correction to GPS system time, the ephemeris, or the ionospheric model as transmitted by the SV, 3) a local effect at the receiver: multi-path or coordinate errors, or 4) a bias in the estimated value Global UTC(NBS). Biases in Global UTC(NBS) could in turn be due to either local or system effects.

A. Local Effects

Coordinates have been checked in the past and we are fairly certain coordinate errors cannot be the cause of this diurnal variation. From other experiments, multi-path errors should be in the range of 3-5 ns[3]. Thus we are left with system errors.

B. Biases across SV's

Let us first consider that GPS minus Global UTC(NBS) values might be biased differently by different SV's. When we scan across [GPS - Global UTC(NBS)] from individual SV's we still see a significant diurnal variation. Figure 5 illustrates SV 13, which is representative. The diurnal variation appears slightly lower than in the combined data. Thus we see the systematic effect in question is not due to combining data across satellites, though there may be a slight penalty.

C. Biases in Global UTC(NBS)

Let us consider in depth the possibility of biases in Global UTC(NBS). Since the satellites are in 12 hour sidereal orbits and the earth rotates underneath them once per sidereal day with respect to the orbit positions, the geometry of a measurement of a satellite vehicle (SV) against a reference station is repeated once per sidereal day. If there is any bias in the measurement of reference minus Global UTC(NBS) this must appear as a diurnal variation, actually a once-per-sidereal day variation, in the measurement of that SV against Global UTC(NBS). Biases between different satellite paths have been reported in the past in performing common view time transfer[1]. In the absence of a more accurate method of time transfer we have no way of knowing the unbiased value of reference minus Global UTC(NBS).

In this work we have attempted to minimize these effects in two ways. First we average across SV's combining with weights which optimize time stability, as mentioned in [1]. Second, each measurement of an SV is made by at least three locations simultaneously. Thus when we combine measurements of the SV against Global UTC(NBS) across the various locations, biases in the various reference minus Global UTC(NBS) estimates will be averaged.

To simplify the question of biases in Global UTC(NBS) we look at SV minus reference data for single locations. Looking at Figures 6-9 we see both in SV 11 measured directly against NBS, and in SV 13 measured against PTB there is clearly the presence of a systematic daily variation. Indeed it seems to be a larger effect than we obtain from looking with our Global UTC(NBS) network. This suggests the presence of a systematic error in the transmitted ephemeris and ionospheric correction, since these effects would tend to cancel in our Global UTC(NBS) measurement system.

D. Ephemeris and Ionospheric Models

If we look at FFT's of unsmoothed [SV - Global UTC(NBS)] we see the presence of diurnal modulation in all of them. Figure 10 for PRN 13 is representative. So we conclude there must be a diurnal variation in either the SV clocks or in the transmitted ephemeris or ionospheric models. We smooth the [SV - Global UTC(NBS)] data allowing variations in the data only of the size the various clocks were capable of. Table I below gives the parameters of white and random walk FM.

TABLE I:
Parameters used in the Kalman Smoother

SV ID #	White FM (ns/day)	Random Walk FM (ns/day)
3	8.00	2.00
6	5.00	10.00
11	8.00	2.00
12	8.00	2.00
13	8.00	4.00

We do not model the ephemeris or ionosphere in the Kalman smoother, hence errors in these models can pass through if they are smaller than the clock noise. When we look at the $S_y(f)$ plot of Kalman smoothed [SV 13 - Global UTC(NBS)], Figure 11, we see that the diurnal variation is significantly reduced, almost to the noise level. This is fairly typical of the other SV's. For PRN 11 (Navstar 8), though the diurnal variation is reduced, it is still fairly large (Figure 12). We conclude nevertheless that most of this diurnal effect is in the ephemeris and ionospheric models as transmitted. This conclusion is largely based on the lack of a mechanism to drive a diurnal variation in the SV clocks without driving a 1 cycle per 0.5 sidereal day modulation. We assume the diurnal variation that remains after smoothing is residual ephemeris and propagation error, since we do not model these directly in the Kalman smoother.

E. SV Clock Corrections

There remains the question of whether a systematic diurnal variation is introduced in GPS system time in the transmitted SV clock correction in addition to ephemeris and ionospheric values. We find the diurnal variation in [GPS - Global UTC(NBS)] as transmitted by each SV separately. There are three possibilities: either the transmitted clock correction removes some of the diurnal variation, or it passes it from the [SV - Global UTC(NBS)] to the [GPS - Global UTC(NBS)] without changing its magnitude, or it adds additional diurnal effects. We may look at these effects for PRN 13 by viewing Figures 10, 5 and 13, the FFT of [SV 11 - Global UTC(NBS)] data, [GPS from SV 11 - Global UTC(NBS)] data, and the transmitted clock correction for PRN 13. The diurnal variation is largest in the SV data, smaller in the SV clock correction, and smallest in the GPS data. This seems to imply that there is a diurnal variation in the transmitted clock correction which somewhat counteracts the diurnal effect in the ephemeris and ionospheric models as reflected in the SV data.

F. Around-the-World Closure

Finally, we may transfer time around the world in a closed loop with the stations in our network to test the consistency of time transfer by GPS. If we do this using two independent closed paths, errors in common between the two paths must be in the GPS, that is independent of the stations used. We do around-the-world closure via measurements of: (PTB-NBS)+(TAO-PTB)+(NBS-TAO) and (OP-USNO)+(RRL-OP)+(USNO-RRL). See Figures 14-15, respectively. What we see in these Figures is a consistency of slope in the lack of closure between the two paths, and agreement on a systematic bias less than zero. The mean of the first path (Figure 14) is -18.9 ns. The mean of the second path (Figure 15) is - 9.3 ns.

CONCLUSIONS

We have found significant evidence indicating a systematic diurnal variation in the ephemeris and the propagation terms as transmitted from GPS satellites. The proper way to sort out between these would be with a two frequency receiver. Results at USNO indicate the presence of a similar diurnal variation even with a two frequency P-code receiver. The significance of using the P-code is that the chip rate of the pseudo-noise code is 10 times higher, thus dividing by 10 the effect of multi-path variations.

We show finally that a GPS clock can be characterized for periods less than one day by removing the diurnal variation. We assume the diurnal variation is a systematic error, not a feature of the clock. We remove it by taking only those relevant FFT values, inversely transforming them, and subtracting them in the time domain. The resultant [GPS - Global UTC(NBS)] data appears in Figure 16, with its Allan variance in Figure 17.

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1. An NBS Calibration Procedure for Providing Time and Frequency at a Remote Site by Weighting and Smoothing of GPS Common View Data. M. A. Weiss, D. W. Allan. IEEE Trans. I& M-36, No.2, June 1987.
2. Using Multiple Reference Stations to Separate the Variances of Noise Components in the Global Positioning System. M.A. Weiss, D.W. Allan. Conference on Precision Electromagnetic Measurements, June 23-27, 1986, National Bureau of Standards, Gaithersburg, MD.
3. D. Davis, Time and Frequency Division, 524, National Bureau of Standards, 325 Broadway, Boulder, CO, 80303, private communication.
4. Time and Frequency (Time-Domain) Characterization, Estimation, and Prediction of Precision Clocks and Oscillators. D.W. Allan. IEEE Trans. UFFC, November 1987.
5. A Modified "Allan Variance" with Increased Oscillator Characterization Ability, D.W. Allan and J.A.Barnes, Proc. 35th Annual Symp. on Frequency Control (1981).

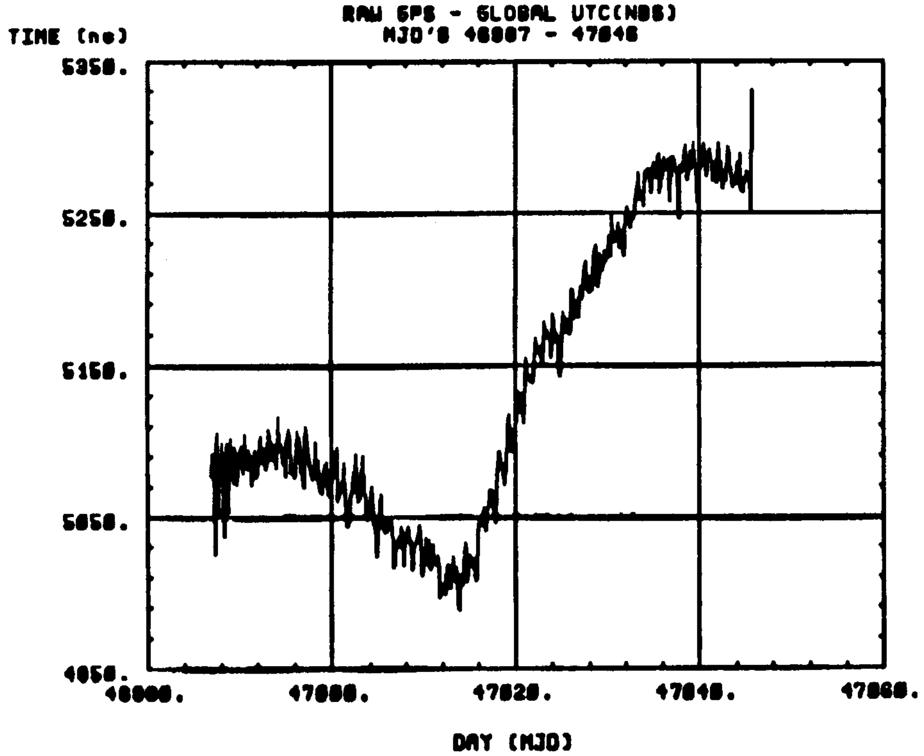

Figure 1: GPS system time minus Global UTC(NBS) data shows some daily variations.

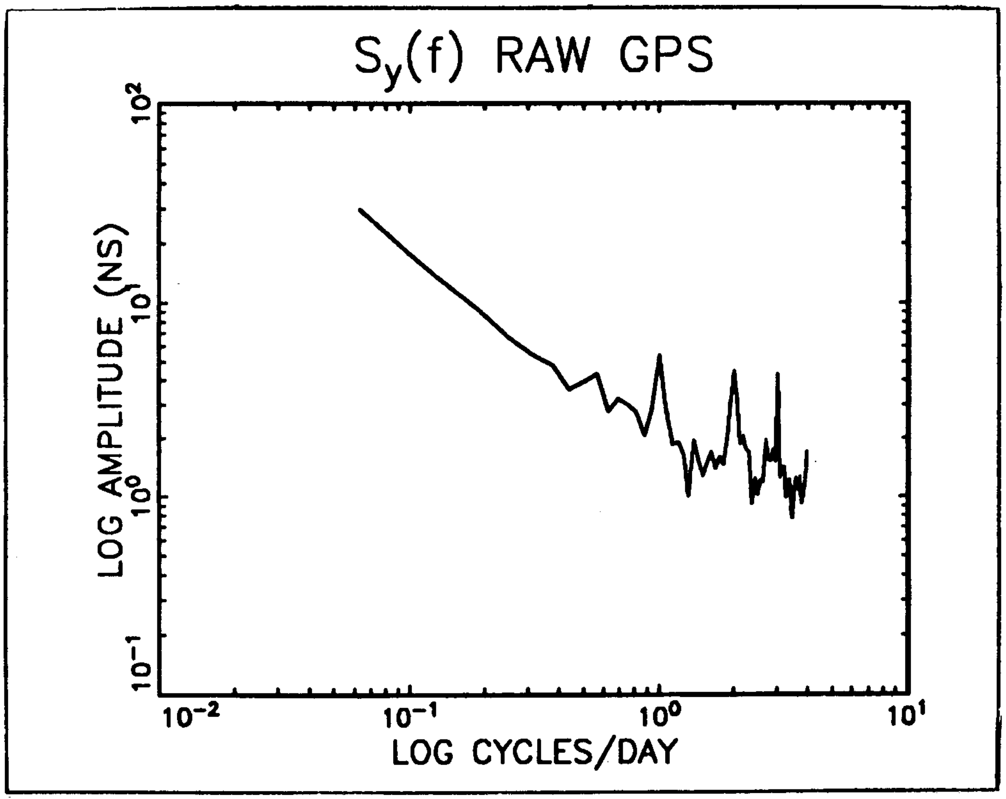

Figure 2: The Fourier spectrum of GPS system time minus Global UTC(NBS data reveals modulation at 1 cycle per day.

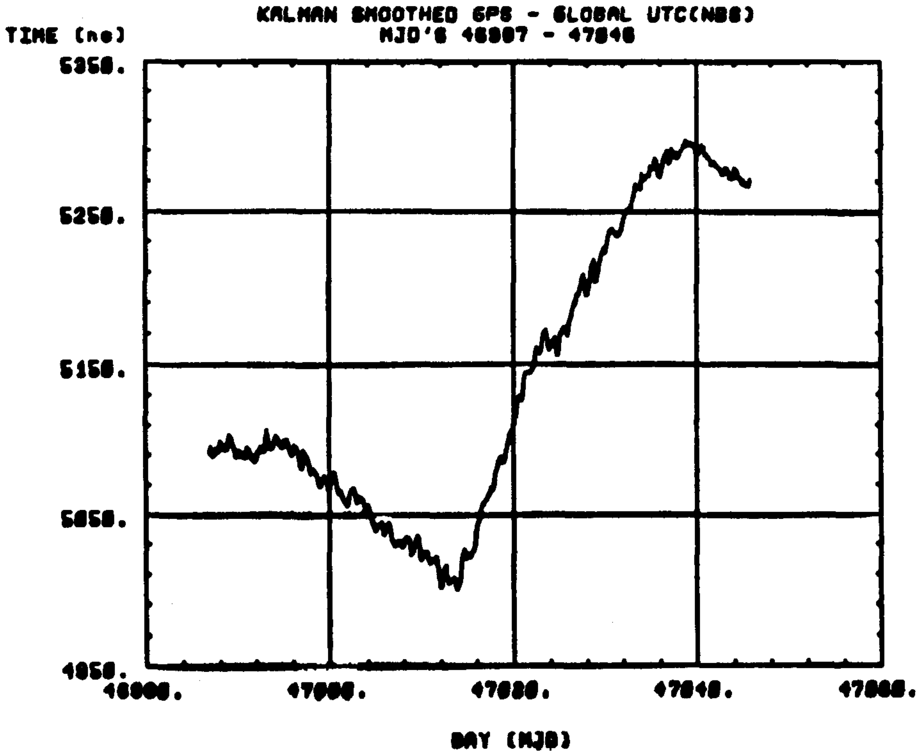

Figure 3: The Kalman smoothed output of GPS system time minus Global UTC (NBS) data has the white phase modulation removed and gives a clear visual appearance of a diurnal variation.

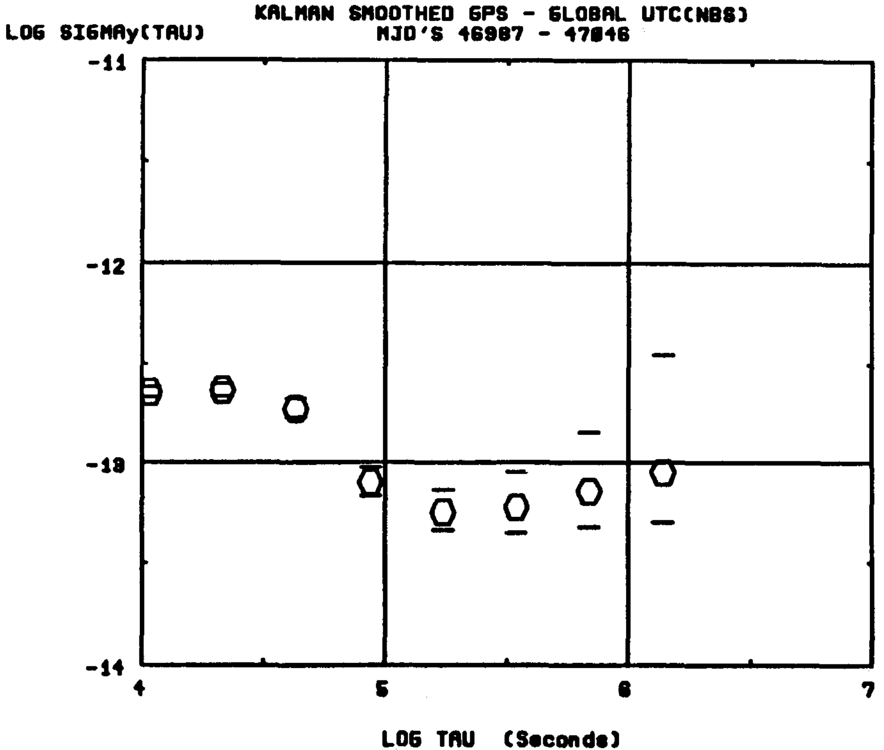

Figure 4: The two-point variance of the data in figure 3 shows a sudden drop in value from tau equals 0.5 sidereal day to 1 day. This reflects a diurnal modulation of 9.4 ns peak-to-peak.

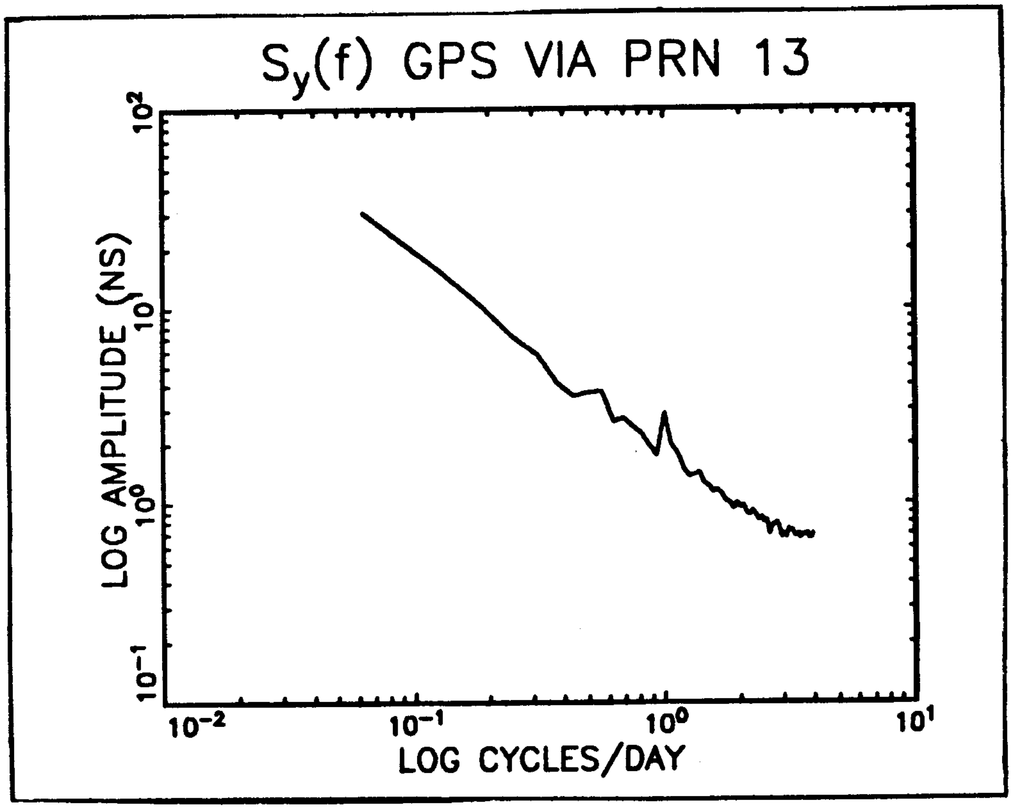

Figure 5: The Fourier transform of GPS minus Global UTC(NBS) from PRN 13 only has a significant diurnal variation. This is representative of the data via any individual SV.

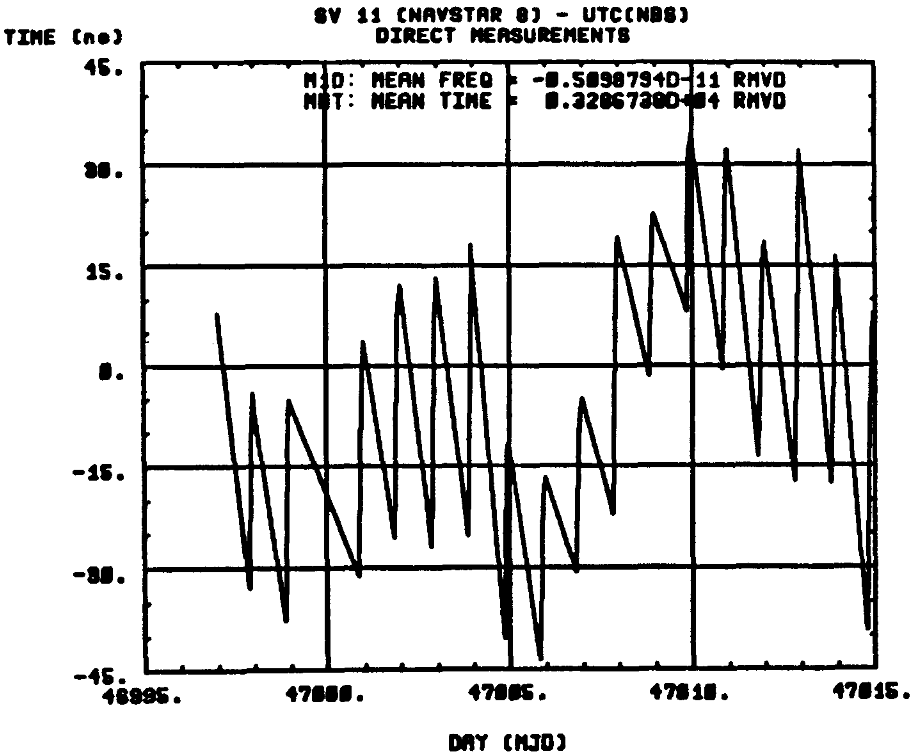

Figure 6: Measurements of PRN 11 (Navstar 8) against NBS only show a systematic variation.

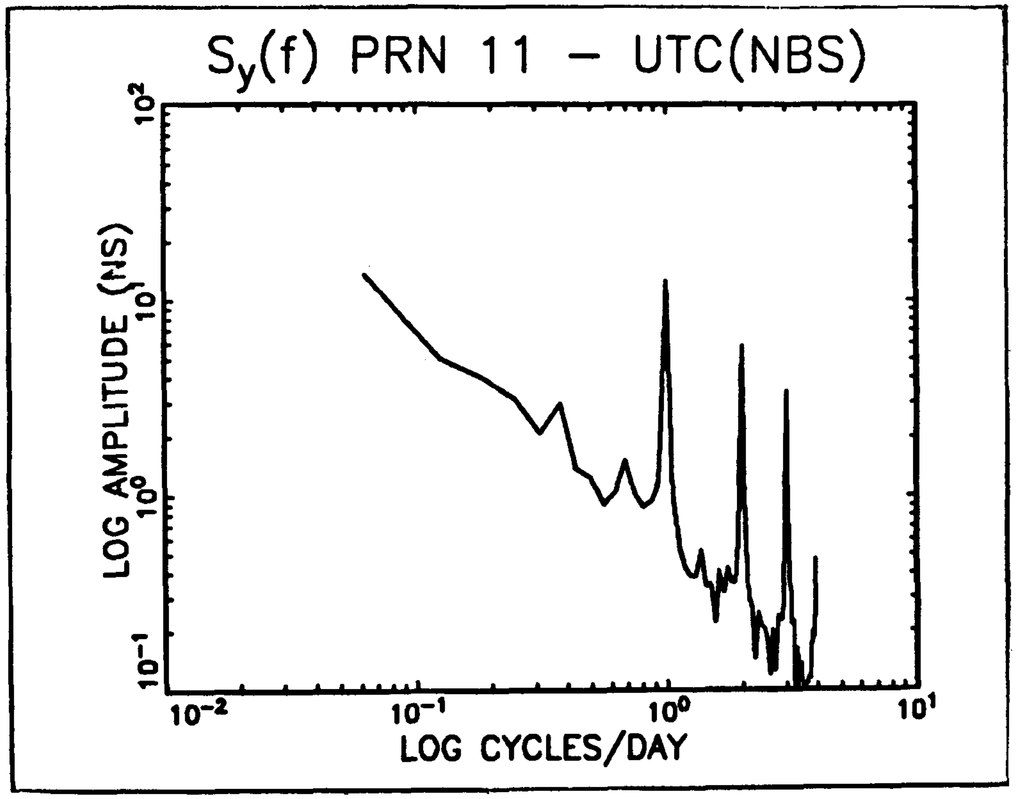

Figure 7: The Fourier transform of data from figure 6 showing a diurnal effect, as well as higher harmonics.

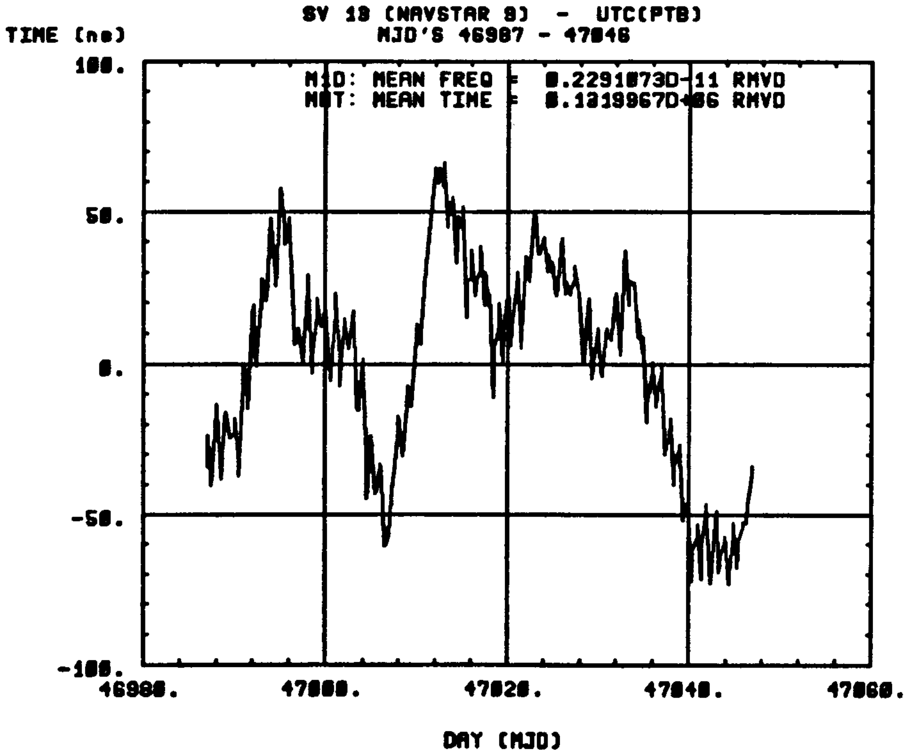

Figure 8: Measurements of PRN 13 (Navstar 9) against PTB only show a systematic variation.

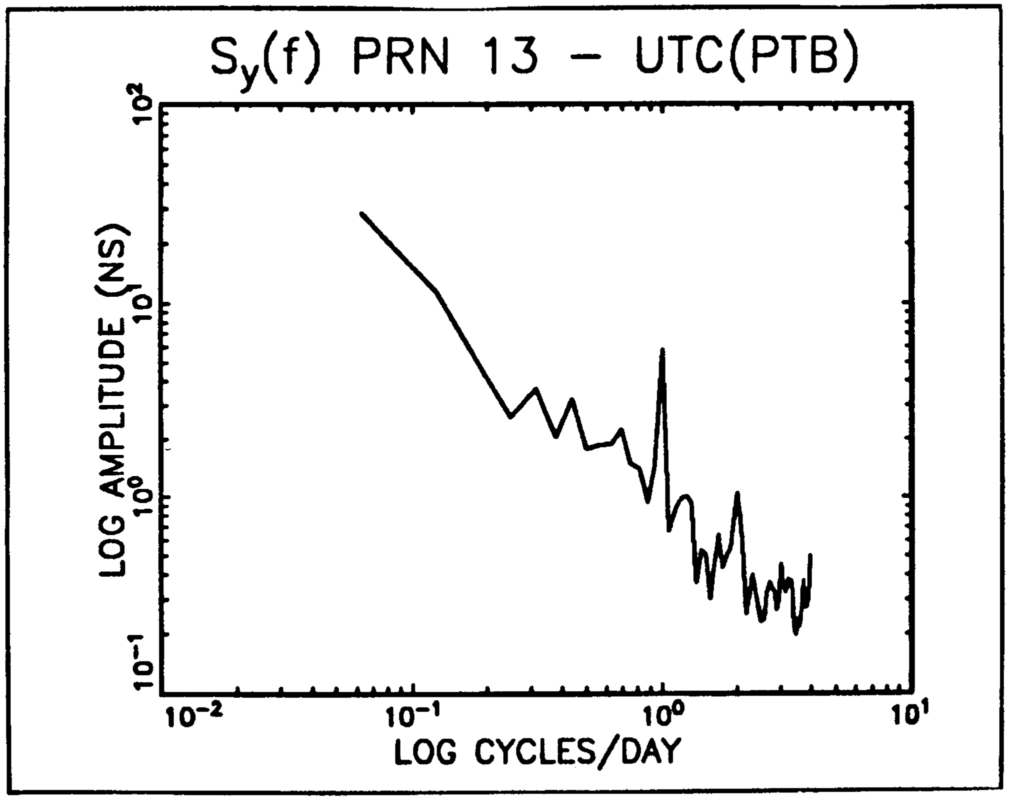

Figure 9: The Fourier transform of data from figure 8 showing a diurnal effect.

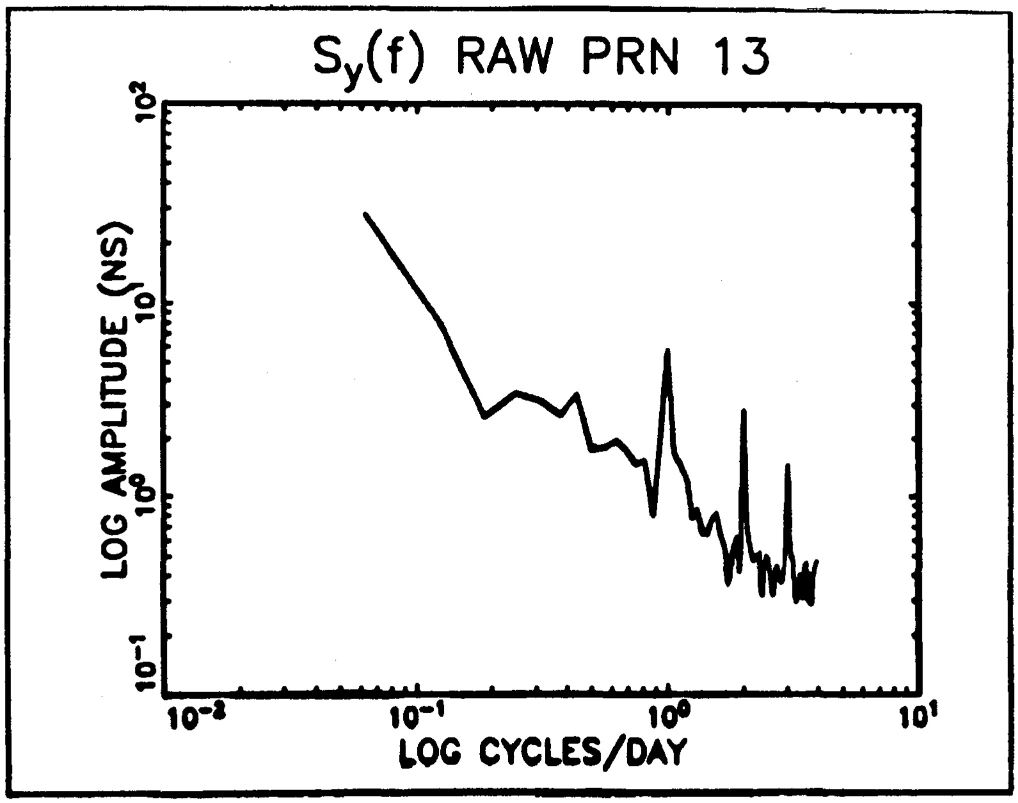

Figure 10: The Fourier transform of PRN 13 minus Global UTC(NBS) data without Kalman smoothing. A diurnal variation is present here.

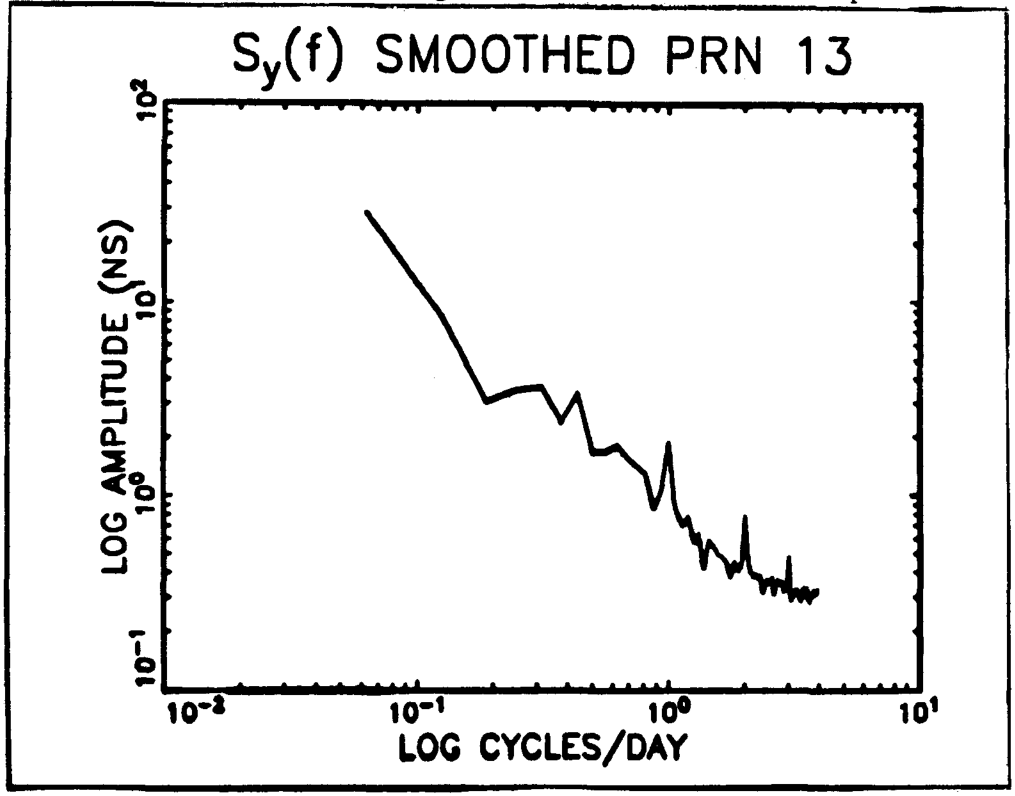

Figure 11: The Fourier transform of PRN 13 (Navstar 9) minus Global UTC(NBS) data after Kalman smoothing. A diurnal variation remains, though it is very much reduced.

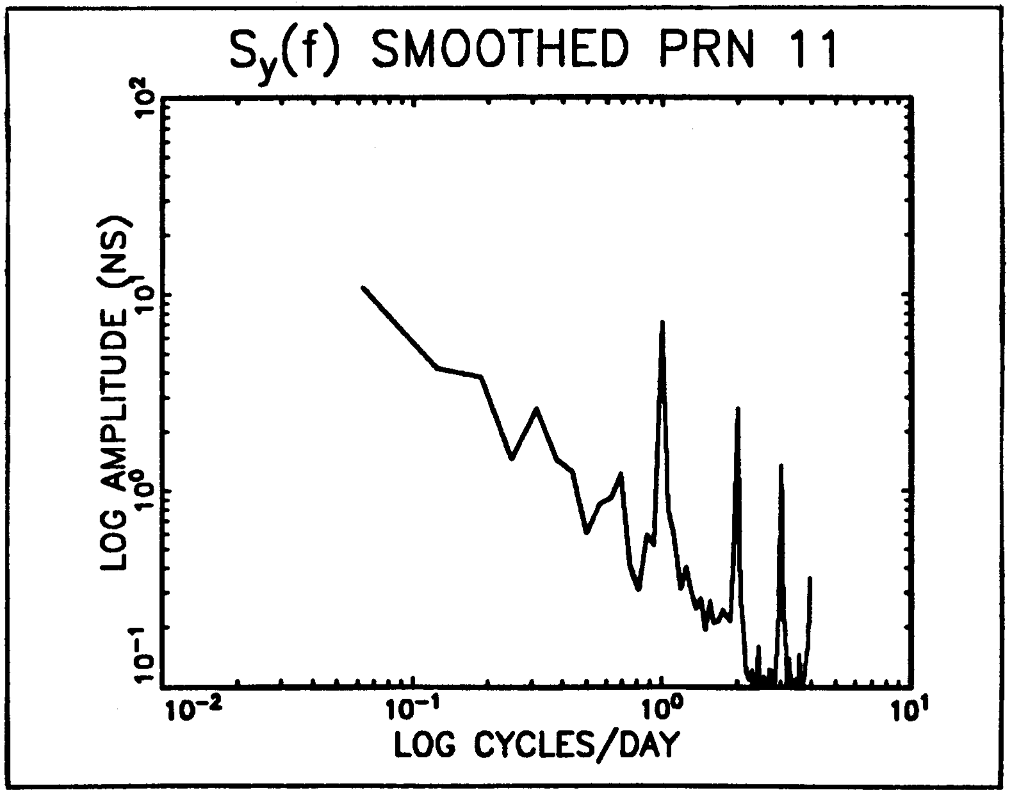

Figure 12: The Fourier transform of PRN 11 (Navstar 8) minus Global UTC(NBS) data even after Kalman smoothing shows a large diurnal variation.

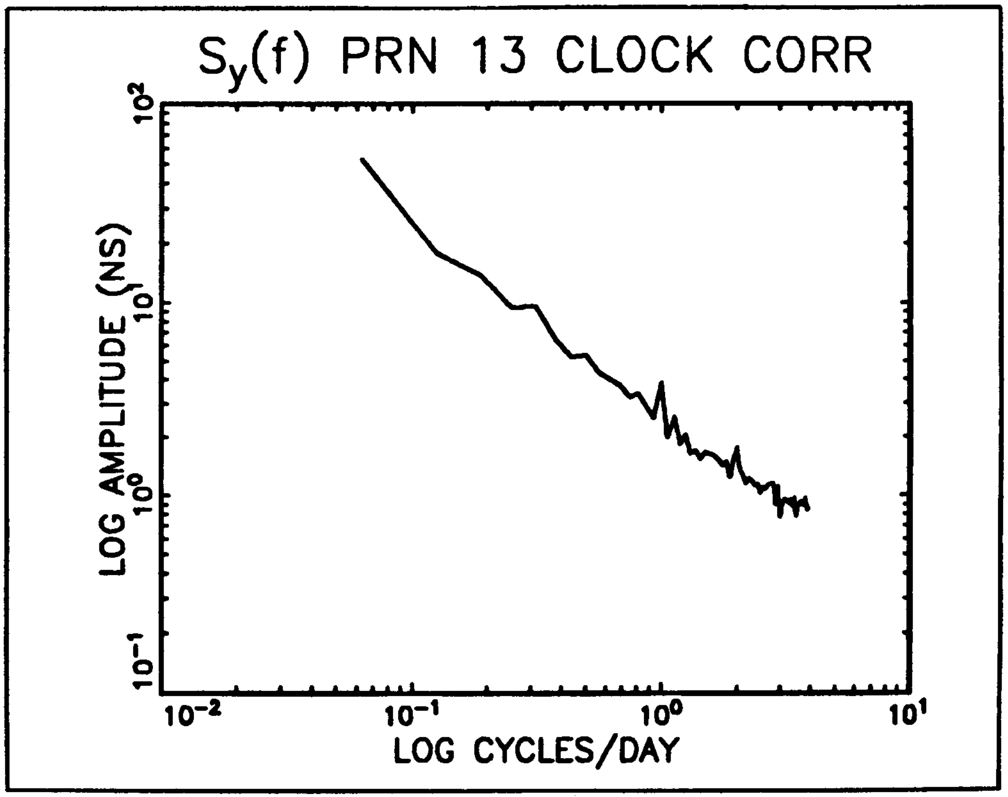

Figure 13: The Fourier transform of the PRN 13 clock correction from the GPS system time as transmitted shows a small diurnal variation.

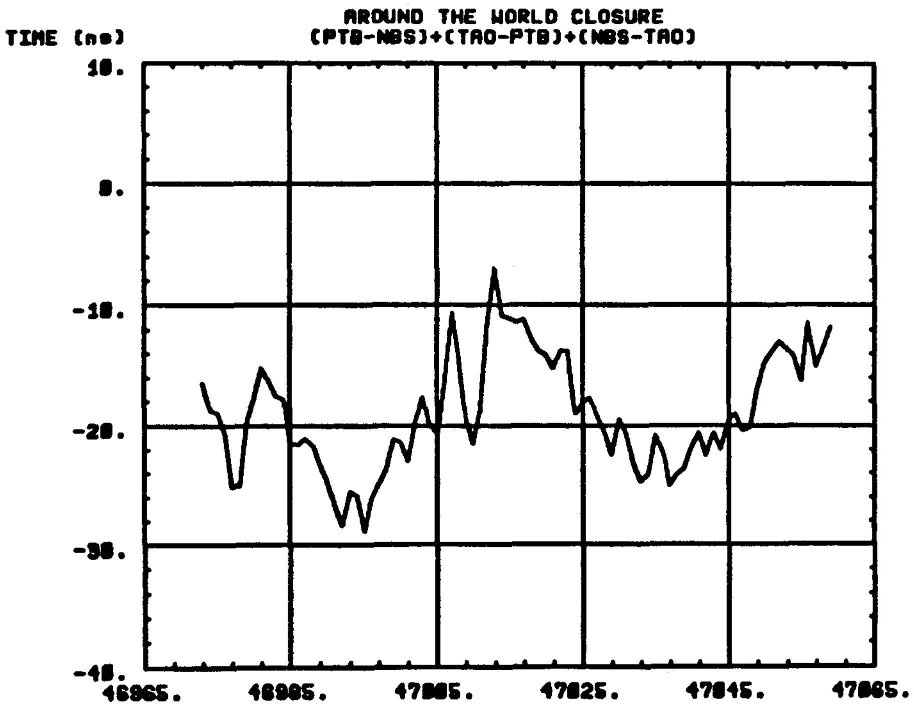

Figure 14: Transferring time around the world from NBS, Boulder, CO to PTB, Braunschweig, FRG, to TAO, Tokyo, Japan, and then back to NBS should yield 0. The mean of -18.9 ns we believe reflects a systematic error in the GPS.

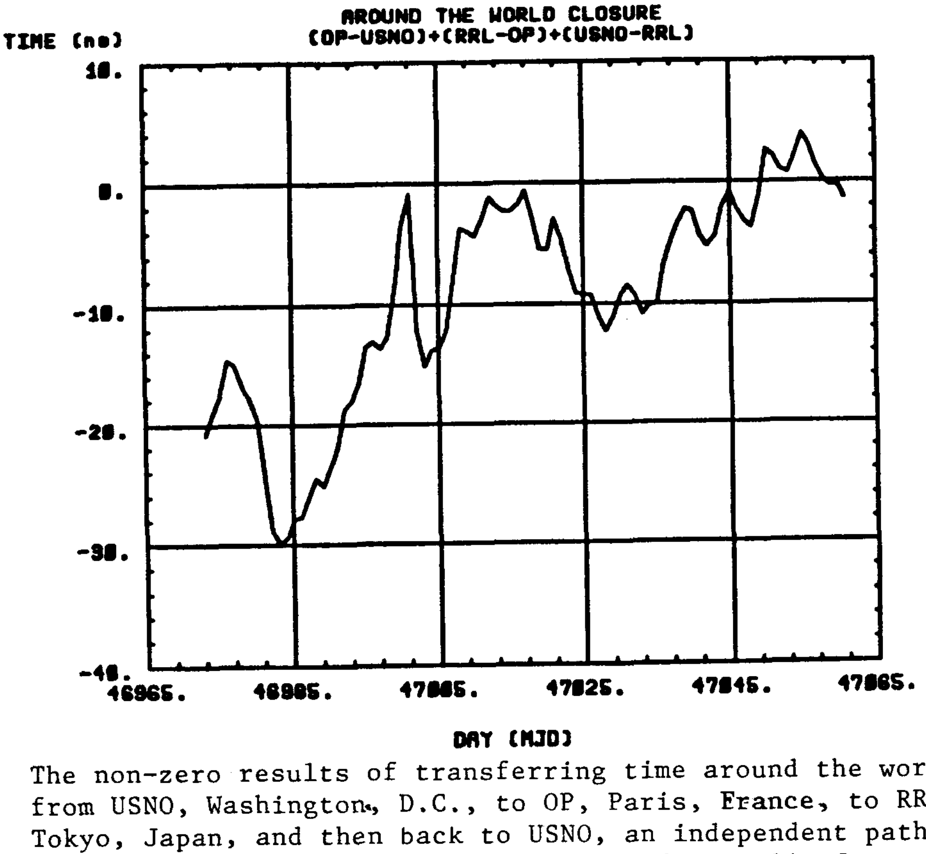

Figure 15: The non-zero results of transferring time around the world from USNO, Washington, D.C., to OP, Paris, France, to RRL, Tokyo, Japan, and then back to USNO, an independent path from that in figure 14, support evidence in figure 14 of a systematic error in the GPS. The mean is -9.3 ns.

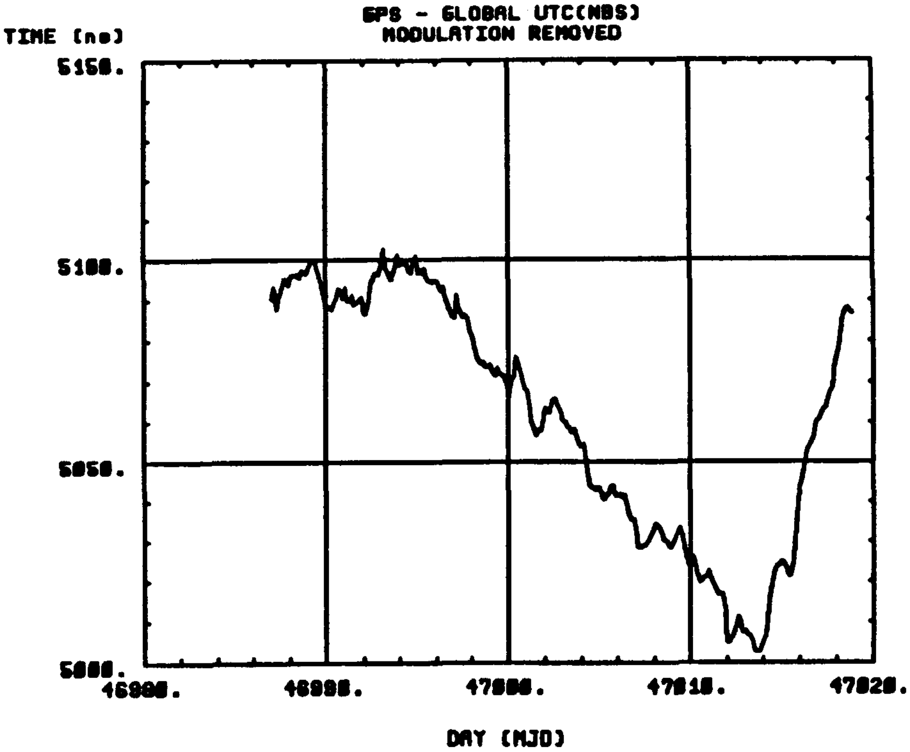

Figure 16: Removing the diurnal part of the Fourier transform from the GPS minus Global UTC(NBS) data gives a better estimate of the GPS master clock.

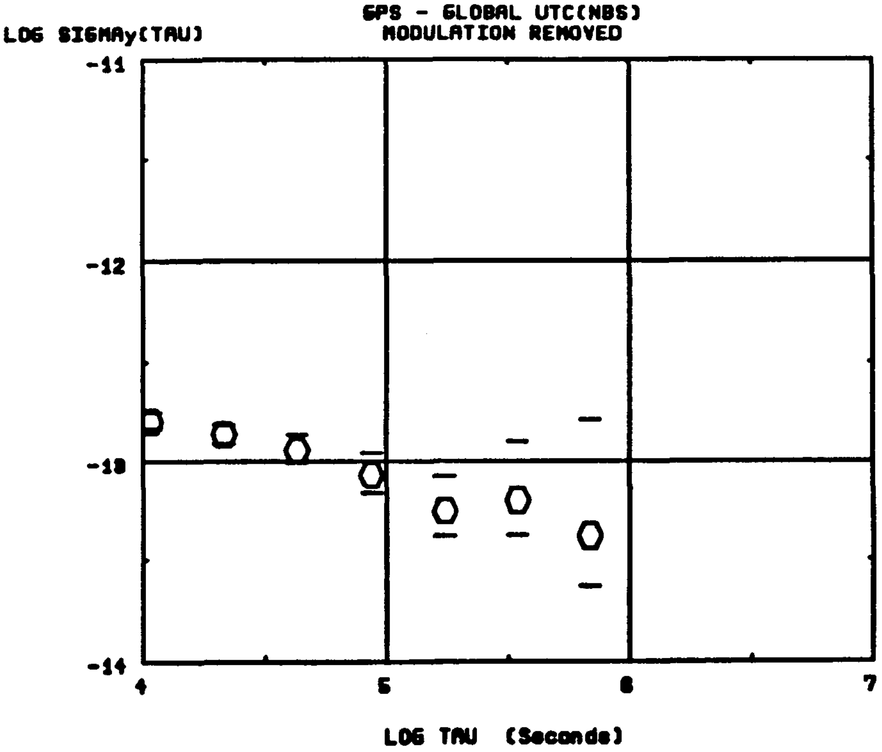

Figure 17: The Allan or two-point variance of the data in figure 16 gives an estimate of the performance of the GPS master clock at integration times less than one day.

QUESTIONS AND ANSWERS

Prof. Don Percival, University of Washington: The first question is "Why in the world are you using the sigma–tau to estimate the amplitude of the variation when you are doing a spectral analysis? Can't you just get that from the spectrum directly?"

Dr. Weiss: Yes, you can and they tend to agree. I did want to point out that you can use the sigma–tau analysis to estimate the amplitude of a modulation. The sigma–tau is good for eliminating the long term effects, the low frequency terms. The sigma–tau is a better estimator of these low frequency effects that are sometimes wiped out in a Fourier transform.

Prof. Percival: OK. Were we seeing a random walk component in the spectral analysis, or was I misinterpreting the scale of the spectrum that you plotted?

Dr. Weiss: Yes, the clock behavior was present there. For some of the clocks there was a random walk behavior.

Prof. Percival: Is there possibly a linear trend in this? When you are doing a spectral analysis there is a mapping of a linear component into a random walk component.

Dr. Weiss: No, I had removed the linear component.

Ken Uglow, Uglow Electronics: I have a strong feeling that if you delayed one of those curves, the correlation would be a bit stronger. If the around–the–world one were set to the left.

Dr. Weiss: You think that there is a lag between them? Is that what you are suggesting?

Mr. Uglow: Yes.

Dr. Weiss: One of the things that can cause a bias like that is if there are coordinate errors at the different locations. In particular, Lewandowski pointed out that he found a coordinate error at PTB, which is in one of these closures.

Mr. Uglow: I would move the upper curve to the right.

Dr. Weiss: In fact, you are right. I see that the horizontal axes are not the same, that is my mistake.

Prof. Brad Parkinson, Stanford University: It seems to me that there are some things that you would want to eliminate before you draw any strong conclusions: the coordinate errors of position, which I think that you could clearly do; also the errors induced by the ephemeris. The question is " Are you using a precise ephemeris and do you plan to go into a geodetic coordinate system actually using differential geodetic positions?"

Dr. Weiss: It would be good to use a precise ephemeris. In our data set that is difficult to implement. From our point of view, this is what GPS looks like for the international community where we are using broadcast ephemeris and C/A code. You are right, it would be nice to use precise ephemeris, but we don't have time to implement that. The positioning errors are something that I would like to get more into. The precise ephemeris is not guaranteed to be perfect either.