where $e[n] = \phi[n] - \hat{\phi}[n]$. From Eq. (6), an update equation for the filter coefficients is constructed:

$$h_{i,j}^{n+1} = h_{i,j}^{n} - \mu \frac{\partial e[n]^2}{\partial h_{i,j}} = h_{i,j}^{n} + 2\mu e[n] a_i(n-j). \quad (7)$$

The coefficient μ, referred to as the step size, determines the convergence rate and stability of the LMS algorithm. This algorithm performs a gradient search for the set of coefficients $h_{i,j}$ that minimize the phase prediction error. At each time step the filter coefficients are updated via Eq. (7), and the current filter coefficients are used to calculate the next phase prediction $\hat{\phi}$. This prediction is compared to the measured phase ϕ to determine $e[n]$ for for the next filter coefficient update. With the appropriate choice of μ, the set of $h_{i,j}$ quickly converges (∼10 s) to the values obtained using the Wiener filter.

To perform real-time noise cancellation the filter updating is turned off ($\mu = 0$), and the now static set of filter coefficients is used in conjunction with the accelerometer measurements to predict the laser phase excursions. The laser phase is corrected by subtracting the predicted phase excursion via the AOM in Fig. 1. Figure 4 shows the performance of real-time noise cancellation. Unlike the case of post-processed filtering, the real-time filtering shows some residual phase noise in the 8 Hz-20 Hz spectral region. The residual noise can be attributed to a combination of excess accelerometer noise that was present in the real-time measurements and the fact that the FPGA update rate had substantial timing jitter. Nevertheless, this initial implementation of real-time active vibration cancellation reduces the vibration sensitivity by 25 dB for the dominant ambient vibrations.

In future investigations it will be useful to carefully filter accelerometer signals and control the timing jitter of the FPGA output signals improve the real-time noise cancellation. Also, a study of the time-invariance of the cavity response will be required to determine how often the filter coefficients should be updated to maintain optimal noise cancellation.

5. Conclusion

We have measured the acceleration sensitivity of an optical cavity frequency reference with a Wiener filter algorithm that requires only ambient noise as the driving source. Furthermore, we have shown that the accelerometer measurements can be used for real-time cancellation of vibration-induced noise by up to 25 dB. Together with more robust and environment-insensitive cavity designs, the measurement and cancellation techniques presented here may allow cavity stabilized lasers to act as ultra-stable frequency references for non-laboratory applications.

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