

## RESEARCH PAPER RP925

Part of Journal of Research of the National Bureau of Standards, Volume 17,  
October 1936

# HARMONIC METHOD OF INTERCOMPARING THE OSCILLATORS OF THE NATIONAL STANDARD OF RADIO FREQUENCY

By Evan G. Lapham

## ABSTRACT

A method of precisely measuring the frequency difference between two frequency standards which have nominally the same fundamental frequency is outlined. The application of this method to the continuous intercomparison of the piezo oscillators in the primary frequency standard maintained by the National Bureau of Standards is described.

The customary method of measuring the frequency difference between two piezo oscillators utilizes the beat frequency between the two standards at the fundamental frequency or one of the harmonics. The method described utilizes the frequency produced by heterodyning two consecutive harmonics to obtain any desired precision of frequency comparison without necessitating the operation of any circuits at exceedingly high frequencies.

## CONTENTS

	Page
I. Introduction.....	491
II. Method.....	492
III. Circuit arrangement.....	493
IV. Results.....	496

## I. INTRODUCTION

The primary frequency standard maintained by the National Bureau of Standards is described in a previous paper.<sup>1</sup> As described in that paper, the output frequencies of the individual piezo oscillators, which together constitute the primary frequency standard, are continuously measured in terms of the frequency of one of the oscillators as a reference standard. This measurement has hitherto been made by heterodyning the output of each of the standards with the output of the reference standard, and recording graphically the frequency difference. The range of the recorder was from 0 to 0.4 cycle per second. Since the measurement was made at the fundamental frequency, 100 kilocycles per second, 0.4 cycle per second corresponded to a change in the frequency of the standard of 4 parts per million. There are 100 divisions on the recorder paper, so that, other conditions being constant, the changes of the frequency difference could be read to  $\pm 2$  parts per 100 million.

The precision of this method of intercomparison was adequate at the time the standard was originally installed. The frequency variations were of such a magnitude that better precision was unnecessary.

<sup>1</sup> Hall, Heaton, and Lapham. *The National Primary Standard of Radio Frequency*, J. Research NBS 14, 85 (1935) RP759.

In the course of time, however, the variations of frequency were reduced considerably, with the result that the record obtained with this arrangement was practically a straight line. It showed the gradual changes in the frequency differences over a period of a few days, but did not resolve the random variations of small magnitude which take place during shorter time intervals, such as an hour or a few hours.

The simplest method of increasing the precision of the recorder was to measure the beat frequency at a harmonic of the fundamental frequency. If the reference standard has a frequency  $F$ , and a standard to be compared with it has a frequency  $F \pm f$ , the frequency difference is  $f$  cycles per second. Multiplying both standards by a factor  $n$ , the frequency difference is given by

$$n(F \pm f) - nF = nf.$$

The precision of the measurement is multiplied by a factor equal to the harmonic number if the accuracy of the recording equipment remains the same. This condition can be realized if the frequencies of the different standards are sufficiently close to one another that the range of beat frequency to be recorded is unchanged.

A consideration in the choice of methods was the desirability of making maximum use of the existing equipment. This included 100-kilocycle mixing circuits which produced the beat frequency to operate the recorder. In order not to discard those circuits it was necessary to use a method which produces the beat-frequency difference for the harmonic frequencies by mixing appropriate 100-kilocycle outputs.

## II. METHOD

The method utilizes the frequency difference between two successive harmonics of the fundamental frequency. If the frequency of any standard is multiplied by a factor  $n$ , and the frequency of the reference standard is multiplied by a factor  $(n-1)$ , the frequency difference is given by

$$n(F \pm f) - (n-1)F = nF \pm nf - nF + F = F \pm nf.$$

This intermediate frequency differs from the fundamental frequency of the reference oscillator by an amount equal to  $n$  times the frequency difference between the two standards at their fundamental frequency or  $nf$ . If this process is repeated, multiplying the above intermediate frequency by the factor  $n$ , and subtracting from it the fundamental frequency of the reference standard, multiplied by the factor  $(n-1)$ , a new intermediate frequency is obtained, and is equal to

$$n(F \pm nf) - (n-1)F = nF \pm n^2f - nF + F = F \pm n^2f.$$

The intermediate frequency now differs from the fundamental frequency of the reference standard by a factor  $n^2f$ . This process of multiplication can be repeated until the frequency difference is of any desired magnitude.

The above arrangement as applied to the intercomparison of the oscillators of the primary standard is shown schematically for a single unit in figure 1. The frequency of the reference standard is multiplied by 9 by means of a harmonic amplifier. If the fundamental frequency of the reference standard is 99,999.98 cycles per second, the output of the frequency multiplier will have a frequency of 899,999.82 ( $9 \times 99,999.98$ ) cycles per second. In a similar manner,

the frequency of the standard to be compared with the reference standard is multiplied by 10, which gives an output of 1,000,000.00 ( $10 \times 100,000.00$ ) cycles per second if this standard has a fundamental frequency of exactly 100,000.00 cycles per second. Combining these two outputs in a detector-circuit arrangement a difference frequency is produced which is nominally 100,000 cycles per second and in the illustrative example is exactly 100,000.18 cycles per second. This 100-kilocycle output is amplified and coupled, along with the output of the reference standard, into the mixing circuits provided in the old arrangement. The difference frequency so produced is recorded just as in the old method. The beat frequency would be 0.20 cycle per second ( $100,000.18 - 99,999.98$ ), which is exactly 10 times the beat frequency at the fundamental. The result is that the recorder indicates the frequency changes on the 10th harmonic, 1,000 kc/s. If a higher order of precision is desired, this process can be repeated, using the 100-kilocycle output of the detector amplifier as the input to the succeeding multiplier.

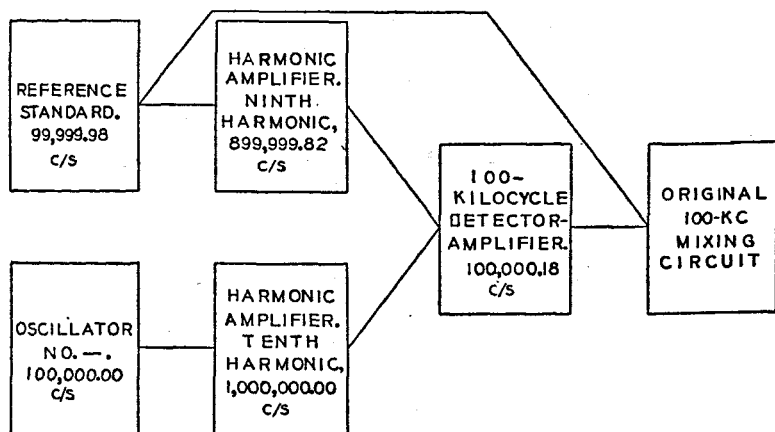


FIGURE 1.—Schematic diagram of method of comparing a standard with reference standard.

The equipment for the intercomparison of the entire group of standards consists of a frequency multiplier for the reference standard, to change its frequency from 100 to 900 kc/s a frequency multiplier for each of the standards to be recorded to change their frequency from 100 to 1,000 kc/s, a detector-amplifier for each standard, and the present intercomparison equipment.

### III. CIRCUIT ARRANGEMENT

The harmonic-amplifier circuit arrangement is shown in figure 2. Two types of harmonic amplifiers are used as frequency multipliers, the one providing a 9-to-1 and the other a 10-to-1 multiplication. In the frequency multipliers the plate circuits of the multiplier stages are tuned to the desired harmonic. The tuned circuits must be low-loss circuits in order to reduce the troublesome effect of amplitude modulation at the input frequency. The multiplier stages are followed by an output amplifier which provides the necessary power

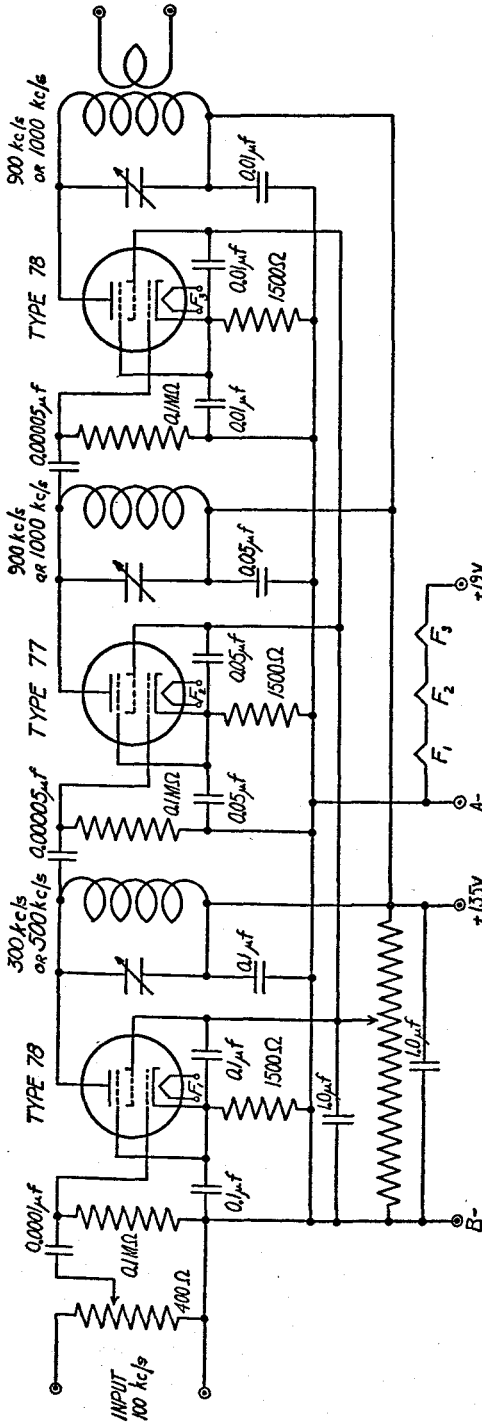


FIGURE 2.—Circuit arrangement of harmonic amplifiers.

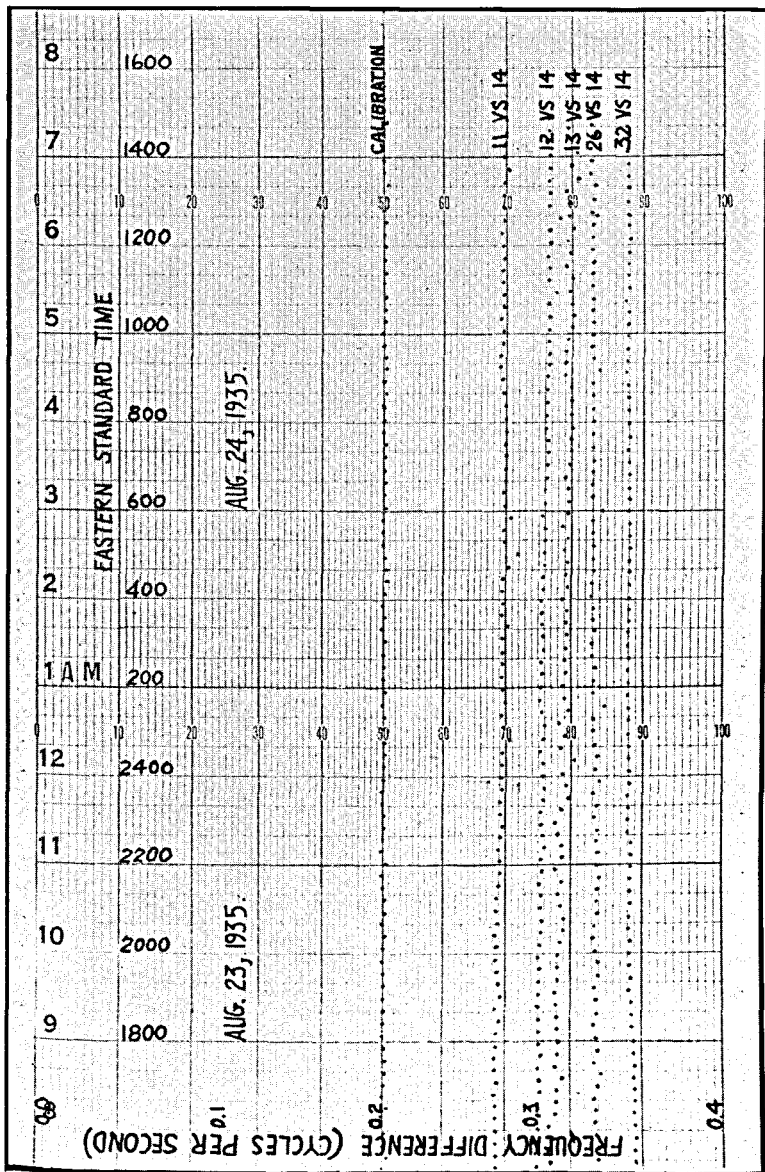


FIGURE 4.—Sample record of beat frequency at fundamental frequency.

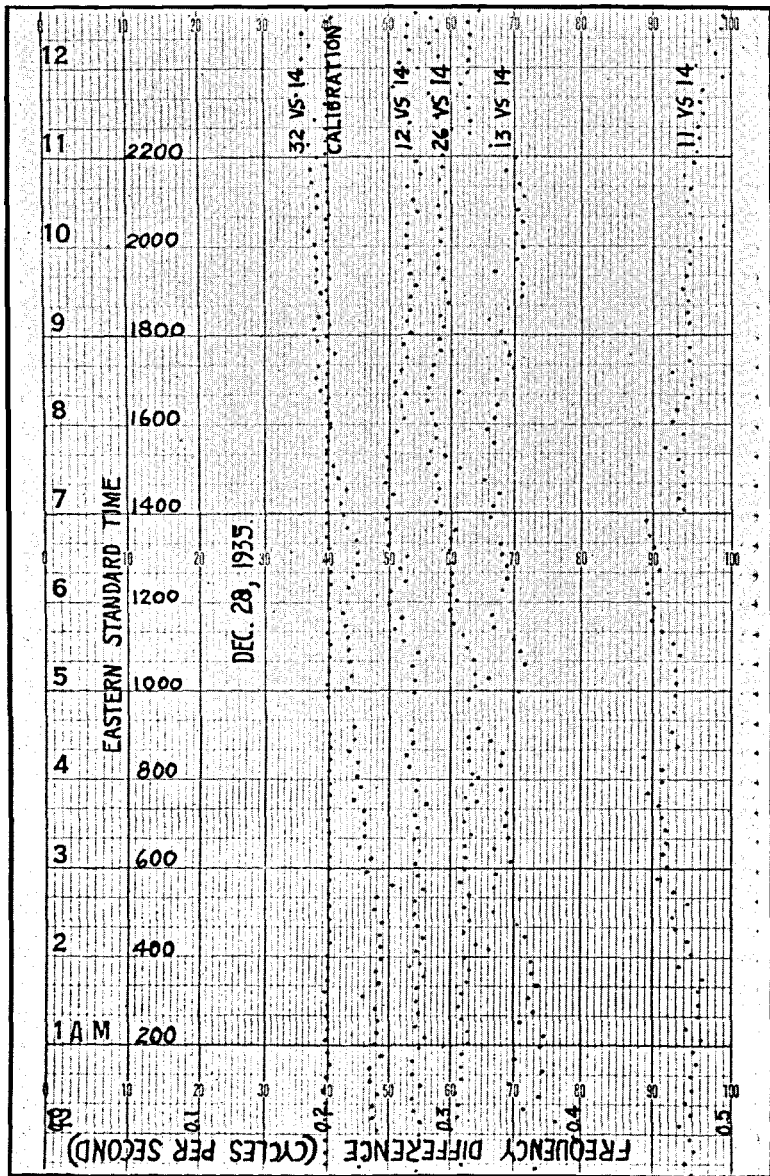


FIGURE 5.—Sample record of beat frequency at tenth harmonic.

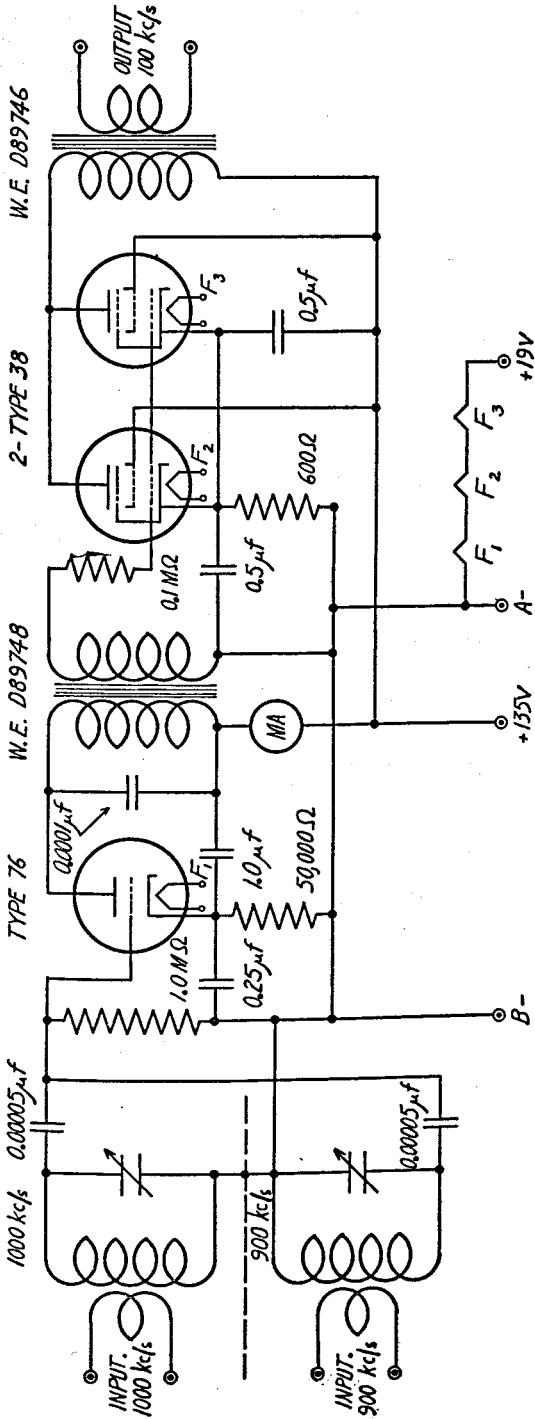


FIGURE 3.—Circuit arrangement of 100-kilocycle detector-amplifier.

output and also serves to further reduce the amplitude of undesired components of the input voltage. A low-impedance link circuit, consisting of a single turn closely coupled to the amplifier plate coil, is used to couple the output of the harmonic amplifier to the 100-kilocycle detector and amplifier.

The circuit diagram of the 100-kilocycle detector and amplifier is shown in figure 3. The outputs of the 900 and 1,000-kilocycle harmonic amplifiers are link-coupled to 900 and 1,000-kilocycle tuned circuits. These tuned circuits are capacitively coupled to a detector biased for plate detection. A meter which reads detector-plate current is provided as an aid in adjusting the circuits. The detector is followed by a 100-kilocycle amplifier using two type 38 tubes in parallel. At a little sacrifice in voltage two separate outputs can be obtained by connecting these tubes separately. Space is provided for the additional transformer if the need arises.

#### IV. RESULTS

The records of the frequencies of the different standards show the effectiveness of this arrangement in increasing the accuracy of the frequency intercomparisons. A sample record obtained with the recorder operating on the fundamental frequency is shown in figure 4. A similar record when the frequency difference is measured on the 10th harmonic is shown in figure 5. In figure 4, one small division on the record corresponds to a frequency change of 4 parts per hundred million, while in figure 5 it corresponds to a change of 0.5 part per hundred million.

---

A method of increasing the precision of intercomparison of two frequency standards, which is identical in principle to the one described, was suggested by my colleague, W. D. George, prior to the development of this equipment.

WASHINGTON, July 10, 1936.