ESTABLISHMENT OF THE SIM TIME SCALE

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Abstract: The SIM time and frequency metrology working group has developed a comparison network for the Americas, with the goals of improving metrology in the SIM region and to allow as many countries as possible to participate in the network. As of May 2008, ten National Metrology Institutes (NMIs) were participating, and six additional NMIs are expected to join the network by the end of 2009. This paper describes how measurements from the SIM network will be used to generate a time scale named SIM-time to be used as the time reference for the SIM region.

1. INTRODUCTION

The Sistema Interamericano de Metrología (SIM) consists of the National Metrology Institutes (NMI's) located in the 34 states that are members of the Organization of American States (OAS). Currently, about one half of the countries that are members of the OAS operate and maintain a time and frequency metrology laboratory, and some of the remaining countries plan to establish a laboratory in the future. With the goal of developing an efficient comparison mechanism throughout the Americas for time and frequency metrology, the SIM time and frequency metrology working group has built a regional comparison network that uses the GPS common view technique. The SIM time and frequency comparison network is helping to improve the development of metrology in SIM region by promoting the participation of as many countries as possible, and allowing each country to see their comparison results in real time. The time and frequency (TF) comparison network of SIM began operation in 2005 with the participation of three national laboratories: National Research Council (NRC) of Canada, National Institute of Standards and Technology (NIST) of the United States of America and the Centro Nacional de Metrología (CENAM) of Mexico [1]. As of May 2008, ten countries of the region were taking part in the comparison network, and 16 SIM NMIs are expected to participate by the end of 2009. A report with the most recent results of the SIM TF comparison network was presented in [2].

To make efficient use of the comparison results produced by the SIM TF network and to reinforce the continuing development of metrology in the SIM region, a SIM time scale, called SIM-time, will be generated. The SIM-time scale will provide a time reference for the SIM region that is as stable and accurate as possible. The comparison results of each of the NMIs time scales participating in the SIM network with respect to the SIM time scale will be available in real-time through the network itself.

This paper presents the algorithm that will be employed in the generation of the SIM-time scale, and also presents preliminary results obtained by the Centro Nacional de Metrología, CENAM, where the time scale algorithm is implemented on four Cs clocks and one active hydrogen maser. Much of the basic analysis presented in Section 2 is presented in detail elsewhere [3, 4, 5], but is summarized here for completeness.

2. ALGORITHM OF THE SIM-TIME SCALE

If a set of N atomic clocks is available, then an Atomic Time scale *TA* can be defined by the equation

$$TA(t) = \frac{1}{N}\sum_{i=1}^{N} h_i(t) \quad , \qquad (1)$$

where $h_i(t)$ is the reading of the clock *i* at the time *t*. It is important to notice that in Eq. (1) every participating clock has the same weight. If different weights are assigned to each clock, then the time scale *TA* is defined as:

$$TA(t) = \frac{1}{N}\sum_{i=1}^{N} \omega_i h_i(t) \quad , \qquad (2)$$

where ω_i is the weight of the clock *i* and the condition of normalized weights is given by

$$\sum_{i=1}^{N}\omega_i = 1. \tag{3}$$

It is possible to define each weight ω_i as constant in time. However, the best scenario is given when a dynamic weighting approach is implemented, that increases or decreases the weight given to a clock according to its frequency stability characteristics, which can change over time for different reasons. In this case the weights can be defined as inversely proportional to the frequency stability of the clock under consideration, where the frequency stability is measured in terms of the Allan deviation. In this scheme we can define the weights as

$$\omega_i(t) \propto \frac{1}{\sigma_i(\tau)} \ , \tag{4}$$

where $\sigma_i(\tau)$ is the Allan deviation of the clock i for an integration time τ. The proportional constant is defined by the normalization condition on weights (3). In this way, the *TA* time scale definition in terms of the dynamic weighting takes the following form:

$$TA(t) = \frac{\sum_{i=1}^{N}\omega_i h_i(t)}{\sum_{i=1}^{N}\omega_i} = \frac{\sum_{i=1}^{N}\frac{1}{\sigma_i(\tau)}h_i(t)}{\sum_{i=1}^{N}\frac{1}{\sigma_i(\tau)}} \tag{5}$$

Unfortunately, Eq. (5) cannot be implemented experimentally as written, because the readings from clocks $h_i(t)$ are not observables. Experimentally it is possible to know only the time difference between pairs of clocks.

We can rewrite Eq. (5) in terms of the time differences between participating clocks, by subtracting from each side of the equation the reading of clock *k* at the time *t* as follows:

$$TA(t) - h_k(t) = \frac{\sum_{i=1}^{N}\omega_i\left[h_i(t) - h_k(t)\right]}{\sum_{i=1}^{N}\omega_i} \ . \tag{6}$$

If we define $x_k(t)$ as the time difference between clock *k* and the time scale *TA* at the time *t*, and define x_{ki} as the difference between clock *k* and clock *i* and we take in consideration the normalization condition on the weights (3), then equation (6) takes the form

$$x_k(t) = \sum_{i=1}^{N}\omega_i x_{ki}(t). \tag{7}$$

This last equation defines the time scale *TA* in terms of the time differences between participating clocks. It is important to notice that the weighted average of $x_k(t)$ is zero, because $x_{kj}(t) = -x_{jk}(t)$; that is,

$$\begin{aligned}\sum_{k=1}^{N}\omega_k x_k(t) = \sum_{k=1}^{N}\omega_k\sum_{i=1}^{N}\omega_i x_{ki}(t) = \\ \sum_{k,i=1}^{N}\omega_k\omega_i x_{ki}(t) = 0.\end{aligned} \tag{8}$$

Once the time differences $x_{ki}(t)$ are known, it is then possible to compute the time scale *TA* by using Eq. (7). However, it is important to notice that this computes the time scale using a post-processing scheme. In cases where it is necessary to generate a time scale *TA* in real time, we must introduce a scheme of prediction for the time differences among clocks and the time scale *TA*.

One of the simplest models to predict the time difference of a clock with respect to a reference (we assume the reference is more stable and accurate than the clock under consideration) is

$$\hat{x}_k(t+\tau) = x_k(t) + \left[y_k(t) + \frac{D_k\tau}{2}\right]\tau + \ldots \ , \tag{9}$$

where $\hat{x}_k(t+\tau)$ is the prediction of the time difference of clock *k* respect to the reference for the future time $t+\tau$. $x_k(t)$ is the (known) time difference between the clock *k* and the reference, and $y_k(t)$ is the fractional frequency difference at the time *t*. Finally, D_k is a constant accounting for changes of $y_k(t)$ during the interval.

Equation (9) can be seen as expansion of the x_k in terms of a Taylor series around the value $x_k(t)$ for a time interval of τ. Once the time difference $x_k(t)$ is known through Eq. (7), it is possible to predict the time scale TA for the time $t+\tau$ using Eq. (9). Here it is important to note that the weighted average of the predictions $\hat{x}_k(t+\tau)$ is not necessarily zero; that is,

$$\sum_{k=1}^{N} \omega_k \hat{x}_k(t+\tau) \neq 0. \tag{10}$$

Once the (future) time $t+\tau$ is reached, it is possible to know the time differences between participating clocks, and then it is possible to compute the value of the time scale *TA* for that time $t+\tau$. Of course, the predicted value of the time scale *TA* computed at time *t* for the time $t+\tau$ will not necessarily be equal to the computation of *TA* at time $t+\tau$. Under this scheme, the time scale prediction for $t+\tau$ can be corrected by the time difference measurements by using

$$x_k(t+\tau) = \sum_{j=1}^{N} \omega_j \left[\hat{x}_j(t+\tau) - x_{jk}(t+\tau)\right] \ . \tag{11}$$

The prediction $\hat{y}_i(t+\tau)$ of the fractional frequency deviation of clock i for time $t+\tau$ is made according to

$$\hat{y}_k(t+\tau) = \frac{\hat{x}_k(t+\tau) - x_k(t)}{\tau}. \tag{12}$$

Once the (future) time $t+\tau$ is reached, the correction for the frequency prediction can be made through the exponential filtering defined by

$$y_i(t+\tau) = \frac{1}{1+m_i}\left[\hat{y}_i(t+\tau) + m_i(\tau) y_i(t)\right], \tag{13}$$

where m_i is given by the equation

$$m_i(\tau) = \frac{1}{2}\left[\sqrt{\frac{1}{3} + \frac{4}{3}\frac{\tau_{\min,i}^2}{\tau^2}} - 1\right] \quad ,(14)$$

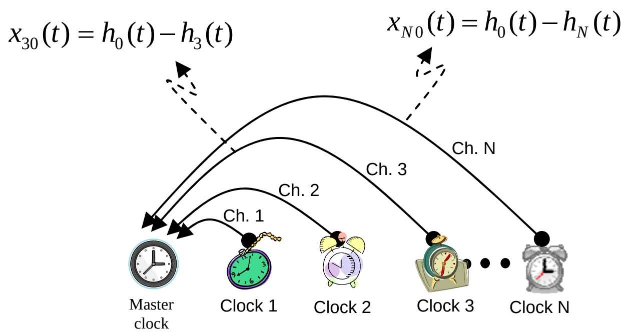
and $\tau_{\min,i}$ is the integration time at which the noise floor of the clock i is reached.

3. RESULTS

The algorithm presented in the previous section has been implemented at CENAM on four Cs clocks and one active hydrogen maser. The time scale *TA* is computed every hour with the dynamic weighting algorithm described in [4]. The system that measures the time differences between clocks, also referred to as the phase comparator, utilizes the dual mixer frequency technique and has a resolution of 20 ps. The phase comparator has 32 input channels, which allows it to compare that same number of clocks. It performs one time difference measurement every second for each of the 32 channels in use. One Cs clock (labeled as Cs I) is selected as the master clock, and all of the other clocks are compared to the master.

In this section we show the results obtained at CENAM during the implementation of the time scale algorithm. The results were obtained from six weeks of continuous time scale generation (from April 5 to May 16, 2008). The following results are considered as preliminary because it is necessary to measure the performance of the time scale algorithm for a longer period to obtain more confidence in the results.

Figure 1 presents a schematic of the time difference measurement system (the phase comparator). The clocks are compared through their 5 MHz output signals. The phase comparator measures the time differences between clocks modulo one signal period (200 ns). To obtain the total phase difference it is necessary to post-process the phase comparator results.



$$x_{ij}(t) = h_j(t) - h_i(t) = [h_0(t) - h_i(t)] - [h_0(t) - h_j(t)]$$

Figure 1. Schematic of the time difference measurement system used at CENAM, also referred to as a phase comparator. The system performs one measurement every second.

To transform the time scale *TA* from a virtual time scale (with no physical signal defining the time scale) to a real time scale (with a physical signal defining the time scale) we use a micro phase stepper (MPS). The MPS steers the 5 MHz frequency signal of the active hydrogen maser so that it follows the virtual *TA* time scale.

The output of the MPS is used to measure the time difference between the time scale and the corrected frequency of the hydrogen maser. The MPS is controlled by an automatic servo loop so that the time difference between the TA and the MPS output remains constant (Figure 2).

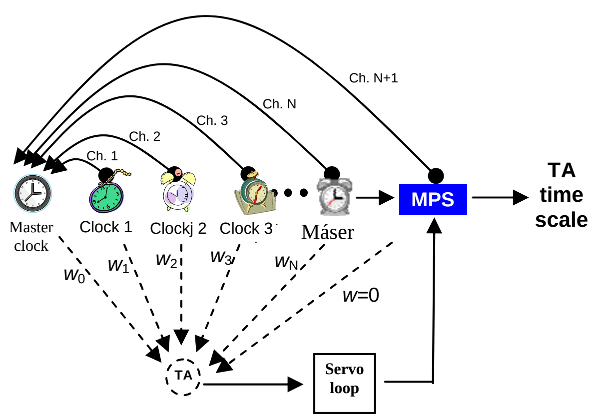


Figure 2. Schematic showing how the output of the micro phase stepper is held in agreement with the time scale TA.

Figure 3 shows the measurement results from the phase comparator during six weeks, from April 4th to May 16th, 2008. The vertical axis corresponds to the time difference of participating clocks with respect to Cs I, the master clock. Discontinuities observed at 0 ns and 200 ns are due to the fact that the phase comparator performs time difference measurements modulo one period of the 5 MHz signals (200 ns). To correct for these discontinuities, it is necessary to post-process the phase comparator measurements. It is also important to notice the frequency change of clock J on April 12th, 2008, marked in the figure within a circle.

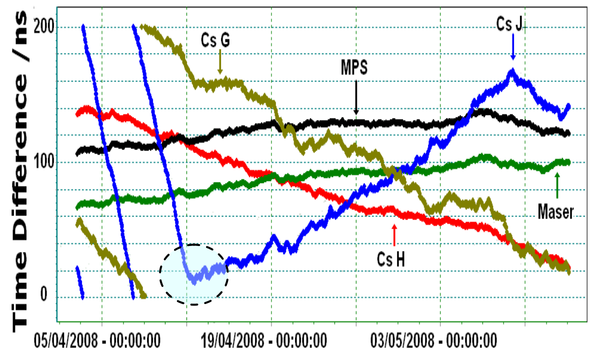


Figure 3. Time difference measurement of participating clocks with respect to the master clock (Cs I). The black line corresponds to the MPS output.

Figure 4 shows the time differences of the participating clocks with respect to the time scale TA. It is interesting to note how the frequency change of clock J on April 12th, 2008 affected the time scale *TA* performance.

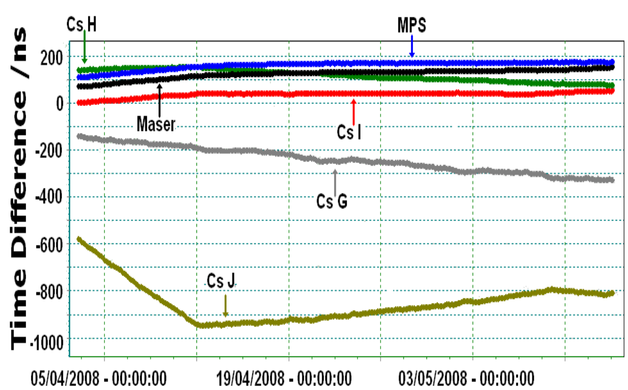


Figure 4. Time difference of participating clocks with respect to the time scale TA.

Figures 5 and 6 show the frequency stability of the comparisons shown earlier in Figures 2 and 3.

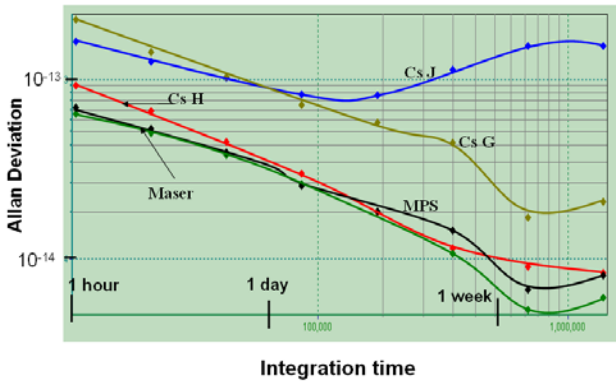


Figure 5. Frequency stability of the time difference of participating clocks with respect to the master clock (Cs I). The black line corresponds to the frequency stability of the MPS output.

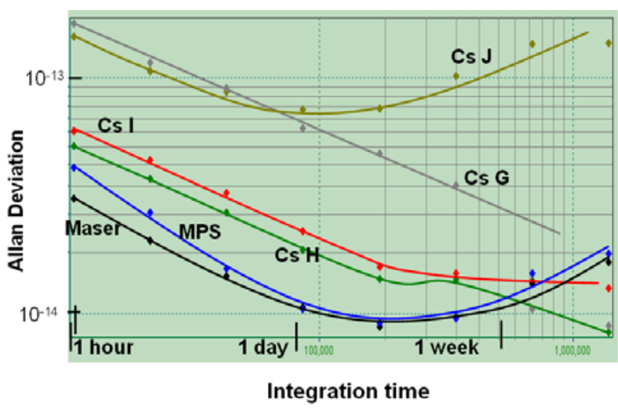


Figure 6. Frequency stability of participating clocks with respect to the time scale TA.

4. DISCUSSION

Here we first present a discussion of the obtained results when the Cs clock is used as the master clock. Then, we will discuss results when the time scale *TA* is used as the master clock.

4.1 Measurement results when Cs I is used as the master clock (Figure 5)

Due to the high frequency stability of the active hydrogen maser in the short term, the relative frequency stability between Cs I and the maser is a measure of the frequency stability of the master clock Cs I, which is about 8×10^{14} at 1 hour. If the Cs I clock is compared to another reference more stable than the maser, we will expect no significant change in this number, because it is already limited fundamentally by the Cs I clock itself. A similar argument can be applied to the results for the stability measurement of clock H, because clocks Cs I and Cs H have the same manufacturer's specifications for stability and perform similarly when they are measured in the laboratory. According to the manufacturer's specifications, Cs I and Cs H should have a frequency stability around 7×10^{-14} for an average time of 1 hour, which is in close agreement with our results shown in Figure 5.

Both Cs J and Cs G are less stable than Cs I and Cs H. This was expected, because both of them are low performance clocks. It is also interesting to note that the frequency correction on clock Cs J has an effect on the frequency stability results in the long term.

For integration time around one day or longer, note that the frequency stability of Cs H approaches the stability of the maser and MPS. It is actually because the frequency stability of the maser approaches the performance of a Cs clock. Finally, the frequency stabilities of clocks Cs J and Cs G are interesting for the purpose of evaluating the time scale performance. The frequency stability of these two clocks agrees with the manufacturer's specification for low performance Cs clocks. Because of their relatively large instabilities, if these two clocks are compared with a reference more stable than the Cs I clock, like the time scale *TA*, we will expect no significant changes in the frequency stability results.

4.2 Measurement results when the time scale TA is used as the master clock (Figure 6)

As we discussed in the previous section, the frequency stability results for Cs J and Cs G are similar to the results when those clocks were compared with respect to Cs I.

The frequency stability for Cs I clock is slightly better than when it was compared with the maser. For an integration time of 1 hour the stability of Cs I when it was compared to the maser was about 8×10^{-14}, as opposed to about 7×10^{-14} when compared to the time scale. This slight difference indicates that the stability numbers are limited by the (absolute) frequency stability of Cs I, and that the time scale is more stable than the maser itself. A similar discussion also applies to the Cs H clock.

The difference in stability between the maser and the MPS is due to the corrections of the maser frequency made by the MPS. The MPS corrects the maser frequency once per hour if necessary, causing a small perturbation on the maser frequency stability, as we can see in Figure 6. These results suggest that corrections to the maser frequency output can be made less often than once per hour, perhaps once per day, and that the magnitude of the corrections can be made smaller. These possibilities will be carefully analyzed during the coming months.

The results of the *TA* time scale performance are satisfactory. The *TA* time scale appears to be more stable than the clocks participating on its generation. However, the stability of the time scale can probably be improved by optimizing the method used to apply frequency corrections to the maser output.

During the implementation stage of the SIM time scale with real data from the SIM TF comparison system will be necessary to take into account some important aspects, such as noise from the comparison system. To face that inconvenient we estimate the necessity to use of a previous filtering scheme on the measurement data before applying the algorithm here discussed. We estimate that about 30 Cs clocks and about 10 Hydrogen masers will contribute to the SIM time scale. Many of the individual clock signals are combined into a single time scale that is measured by the SIM network. For example, there are four Cs clocks and six Hydrogen Masers involved in the generation of UTC(NIST). Because the SIM network compares UTC(NIST) and not the individual clocks behind it, the weight of time scales like UTC(NIST) will be larger than the weight of signals currently supported by a single clock like UTC(ICE) in Costa Rica.

5. CONCLUSIONS

The Centro Nacional de Metrologia, CENAM, has developed and implemented an algorithm similar to the NIST AT1 algorithm [4] with the goal of providing a time scale for the SIM region, called SIM-time. The SIM-time scale will be generated using the algorithm discussed here, utilizing the time difference measurements provided by the SIM time and frequency network that is already in operation. The preliminary results obtained by implementing the SIM-time algorithm at CENAM on four Cs clocks and one active hydrogen maser are satisfactory. Special attention has been taken to implement the algorithm with the robustness required when generating a high performance time scale. Software is currently being added to the SIM time and frequency network that will feed the time scale algorithm with the necessary time difference data. We expect to generate the SIM-time scale by the end of 2008, and to make the results available in near real-time through the SIM time and frequency network.

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