Characterization of frequency noise on a broadband infrared frequency comb using optical heterodyne techniques

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Abstract: We measure the frequency noise across a Cr:forsterite infrared frequency comb through the optical heterodyne beat of different comb teeth against stable continuous wave (CW) lasers. This sensitive measurement shows strong correlations of the frequency noise between spectral components of the comb, relative to a fixed optical frequency near the 1.3 micron carrier of the Cr:forsterite laser. The correlated frequency fluctuations are shown to arise from amplitude noise on the pump laser. We also report a preliminary comparison of excess noise that occurs during supercontinuum generation in both highly nonlinear fiber and an extruded glass microstructured fiber.

OCIS codes: (120.3940) Metrology; (320.7090) Ultrafast lasers; (320.7140) Ultrafast processes in fibers;

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1. Introduction

Supercontinuum generation with low power mode-locked femtosecond lasers and highly nonlinear optical fibers has provided optical frequency combs that are used in a variety of high precision frequency metrology and spectroscopy applications [1, 2]. With the goal of understanding the possible limitations in these applications, several recent papers have addressed the measurement, physical origin, and reduction of the noise on broadband frequency combs [3, 4, 5, 6]. In this paper, we analyze the frequency noise on an infrared frequency comb generated with a Cr:forsterite femtosecond laser. The most straightforward manner to characterize the frequency noise is to measure the fluctuations in the arrival times of the optical pulses (e.g. the frequency noise on the pulse repetition rate). However, as illustrated here one can gain a factor of $\sim 10^6$ in sensitivity by instead measuring the frequency noise of the optical comb components. We employ this approach with heterodyne measurements between specific comb teeth and CW lasers at different wavelengths across several hundred nanometers in the 1-2 micron range. For our Cr:forsterite laser, the frequency fluctuations are caused primarily by the amplitude noise on the pump laser, which drives a breathing motion of the comb about a central fixed point near 1.3 microns. With the sensitive optical heterodyne technique, we explore the strong correlations (anti-correlations) between the frequency noise on the same (different) sides of spectrum relative to this fixed point [4, 7, 8, 9]. Furthermore, we see evidence that there is additional frequency noise arising in the process of supercontinuum generation and that it depends on the specific nonlinear fiber.

2. The Cr:forsterite optical frequency comb and the elastic tape model

The frequency comb is based on a 433 MHz Cr:forsterite laser that generates 1.2 nJ, 35 fs pulses centered at 1.26 μm [10]. The output pulses are injected into two different nonlinear fibers. The first is a $\sim$2-m long piece of dispersion-flattened highly nonlinear optical fiber (HNLF) that produces an octave-spanning supercontinuum from 1.0 μm to 2.2 μm. In this case, the carrier envelope offset (CEO) frequency f_0 is observed using self-referencing f to $2f$ technique. [11, 12] This HNLF has a nonlinear coefficient of $\gamma \sim 10.6$ W^{-1}km^{-1}, a dispersion of 1.74 ps/(nm·km), and a dispersion slope of 0.009 ps/(nm^2·km) at 1550 nm [13]. The second fiber is a 3 cm long piece of SF-6 extruded glass photonic crystal fiber (PCF) which has zero group velocity dispersion at 1.3 μm [14] The supercontinuum generated from each of these fibers is shown in Fig. 1

The Cr:forsterite laser is optically pumped by a 10-W Yb:glass fiber laser. This fiber laser

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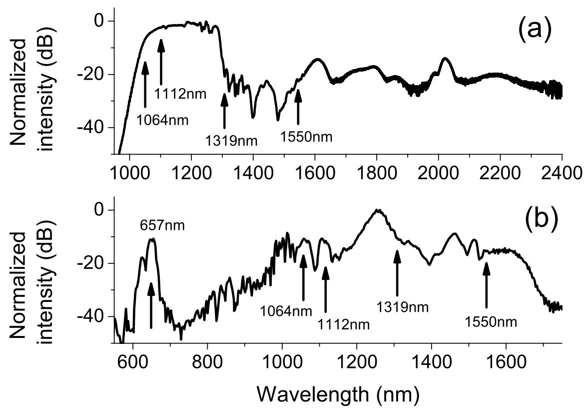

Fig. 1. The supercontinuum spectra generated with femtosecond Cr:forsterite laser pulses after (a) 2-m HNLF and (b) 3-cm PCF. The arrows indicate the wavelengths of available stable CW lasers for heterodyne measurements.

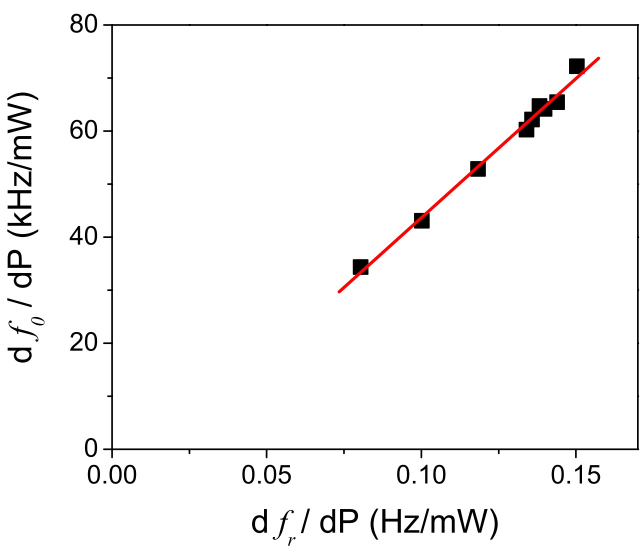

Fig. 2. The changes of f_0 and f_r with pump power modulation, measured at different modulation depths. The solid line is a linear fit to the data.

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has large amplitude fluctuations (see Fig. 3(b) of Ref. [15]) that are expected to drive frequency fluctuations in the output frequency comb. As a framework to discuss the influence of these pump power fluctuations on the frequency comb, we follow the formalism outlined in the recent papers by McFerran and Washburn *et al.* [5, 6, 16]. On long timescales, the frequency comb underlying the super continuum spectrum follows the well-established equation

$$f_n = n f_r - f_0, \tag{1}$$

with the repetition rate f_r, the carrier envelope offset frequency f_0, and integer mode index n. Variations in the pump power, P, will cause fluctuations in the frequency of the n_{th} comb line,

$$\frac{d f_n}{dP} = n \frac{d f_r}{dP} - \frac{d f_0}{dP} \tag{2}$$

To measure the low-frequency response of the comb parameters to pump power fluctuations, we weakly modulate the pump laser power with a 1-Hz square wave applied to the RF power driving an acousto-optic modulator (AOM) in the pump beam path, and we monitored the resulting modulation of f_0 and f_r using frequency counters. This measurement was repeated at several settings of the pump power modulation depth, thus generating a series of values of $d f_0/dP$ and $d f_r/dP$, which are plotted versus each other in Fig. 2 as described by Washburn *et al.* [16]. The fixed point for these fluctuations is given by setting Eq (2) to zero such that $n_{fix} = (d f_0/dP)/(d f_r/dP)$. From the data of Fig. 2, we find $n_{fix} \sim 5.3 \times 10^5$. The corresponding optical frequency is near the center of the laser spectrum at $f_{fix} \sim 2.3 \times 10^{14}$ Hz ($\sim 1.3\mu$m). With this definition, pump-driven fluctuations in the frequency comb are described by

$$\frac{d f_n}{dP} \approx (f_n - f_{fix}) \frac{d f_r}{dP}. \tag{3}$$

This equation shows that we expect the frequency fluctuations of the comb to be anti-correlated about f_{fix}.

3. Noise measurements via optical heterodyne with frequency comb elements

To investigate the frequency noise in the supercontinuum generation in different regions of the optical spectrum, we heterodyned comb modes from the supercontinuum with stable CW lasers at 657 nm, 1064 nm, 1112 nm, 1319 nm, and 1550 nm. All of these CW lasers have average linewidths (on 0.1s timescale) that are below 10 kHz, so their noise contribution to the following measurements is negligible. Using an RF spectrum analyzer, we measured the approximate average linewidths (on 0.1 s time scale) for the beats at 1064 nm, 1112 nm, 1319 nm, 1550 nm, and f_0 to be 0.8 MHz, 0.5 MHz, $<$ 100 kHz (resolution-limited), 0.8 MHz, and 4.5 MHz, respectively. This linewidth broadening of the optical beat notes is consistent with the model described above and originates from amplitude noise on the Yb fiber pump laser.

Correlations of the frequency noise at different wavelengths were observed in the time and frequency domains. Two photodiodes simultaneously detect the heterodyne beats ($f^{\lambda 1}$ and $f^{\lambda 2}$) between two CW lasers and different wavelength regions of the continuum. Assuming the frequency noise of the CW lasers is negligible, the resulting beats are mixed electronically and the two outputs of the mixer ($f^{\lambda 1} + f^{\lambda 2}$ and $|f^{\lambda 1} - f^{\lambda 2}|$) yield information about the correlations of the frequency noise in the different wavelength regions (see Fig. 3).

If the frequency noise is uncorrelated, the two mixing products should both have the same width given by the convolution of the linewidths of the two components. However, if correlations in the noise exist, the two mixer outputs will show different linewidths. When the beat notes from 1064 nm and 1550 nm are mixed, as shown in Fig. 4(a), $f^{1064} + f^{1550}$ yields a

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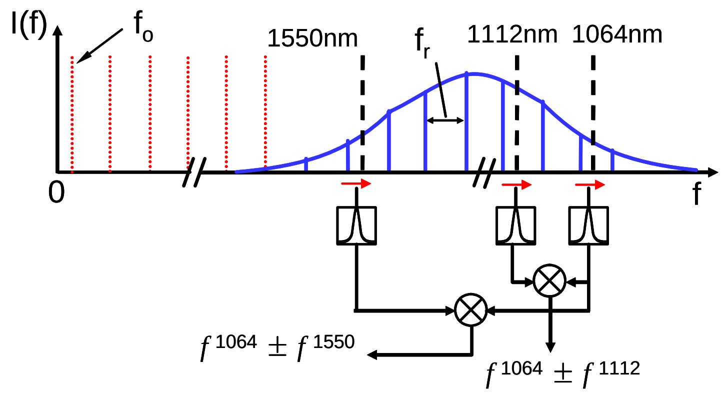

Fig. 3. Scheme of frequency mixing between the optical heterodyne RF beat notes at two different wavelengths from 1064nm, 1112nm, and 1550nm.

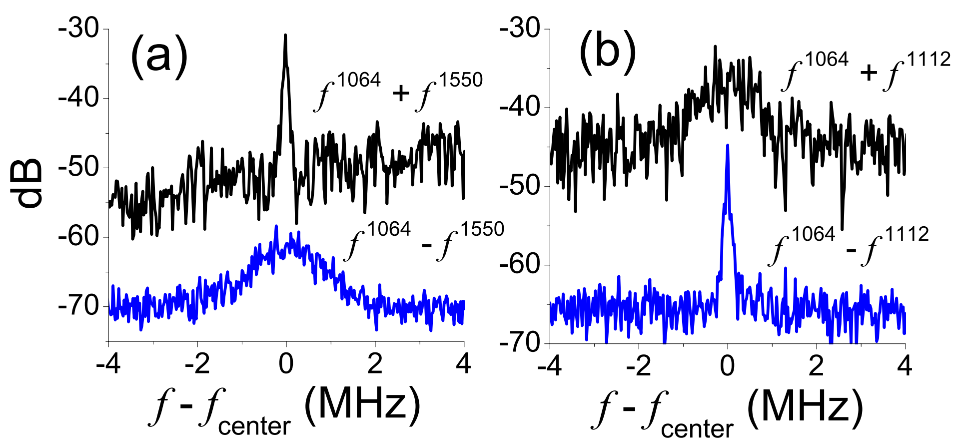

Fig. 4. The RF spectra of the two mixing products of optical beats between modes from the supercontinuum and stable CW lasers. See text for details. (a) 1064 nm, 1550 nm and (b) 1064 nm, 1112 nm. The resolution bandwidth was 10 kHz. The various RF beats are offset from their nominal center frequencies f_{center} and also vertically offset for clarity and ease of comparison.

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narrow, resolution-limited linewidth, while the beat $f^{1064} - f^{1550}$ has a linewidth of 1.9 MHz. These two resulting beats are significantly narrower and broader, respectively, than the convolution of two 0.8 MHz linewidths. On the other hand, when the beat notes at 1064 nm and 1112 nm are mixed, $f^{1064} + f^{1112}$ is 1.3 MHz wide and $f^{1064} - f^{1112}$ is a narrow resolution-limited feature (Fig. 4(b)). Note that for these data, we took care to ensure that the CW frequencies were both located to the same side of the respective comb modes (i.e. both CW lasers had higher frequencies than the respective comb modes against which they were heterodyned as shown in Fig. 3). Using similar techniques, we also confirmed that two adjacent comb modes shared strong frequency correlations.

These results suggest a real-time anti-correlation between the comb elements at 1064 nm and 1550 nm, while clear correlations exist between the comb elements at 1064 nm and 1112 nm. As the fixed point frequency is around 1.3 μm, the repetition rate changes due to the amplitude noise of the pump laser causes a breathing-like expansion of the frequency comb about this center wavelength, with the motion correlated on the same side and anti-correlated between the different sides.

To further diagnose the correlations between the different spectral regions of the supercontinuum, we measured the amplitude and phase of the frequency response of the optical beat notes at 1064 nm, 1319 nm, and 1550 nm to variations in the pump power. The heterodyned beat notes were filtered, amplified, and sent to a delay-line frequency discriminator that has a linear voltage output with respect to frequency (bandwidth $>$ 10 MHz). The pump power is modulated via the AOM while a digital FFT spectrum analyzer recorded the synchronous variations in the output of the frequency discriminator. Through this method, we determined the effective transfer function between the pump power and the frequency of particular frequency comb elements. In other words, we measured the full frequency response of the $\frac{df_n}{dP}$ term in Eq. (2). For each of the optical beats, the amplitude and associated phase of the frequency response are shown in Fig. 5(a) and 5(b) under the condition of the same signal-to-noise (S/N) ratios, signal levels, carrier frequencies, and calibration of the frequency discriminator. The frequency noise of the stable CW lasers and possible thermal effects in the Cr:forsterite laser result in the rise of the frequency responses below Fourier frequencies of 10 kHz in Fig. 5(a). However, from the data at Fourier frequencies higher than 10 kHz one sees that the amplitude of the frequency response to input pump variations is largest for the 1550 nm and 1064 nm beats, while it is significantly smaller near the fixed point at 1319 nm. The phase difference between 1064 nm and 1550 nm is $\sim 180^\circ$ as plotted in Fig. 5(c), and this confirms the anti-correlation between them. As an additional example, when the pump power is modulated at 200 kHz using the AOM, the frequency shifts of the beat notes at 1064 nm and 1550 nm are out of phase in the time domain, as shown in Fig. 5(d). We further note that when we apply the 1 Hz-modulated df_r/dP measured with a frequency counter to Eq. (3), we get a response magnitude consistent with Fig. 5(a) when the contribution from the noise of the CW lasers is ignored. Finally, we note that the response of the frequency comb to pump power variations shows no evidence of relaxation oscillations. In other words, the Cr:forsterite laser response to pump fluctuations is overdamped, as with femtosecond fiber laser frequency combs. [4, 5, 6, 16]

Using the same heterodyne beats and the frequency discriminator system, we also measured and compared the amplitude of the frequency noise directly on the Cr:forsterite laser output with that on the supercontinuum generation after two different fibers described above (see Fig. 1). In this case, we simply relied on the intrinsic amplitude noise of the pump laser to drive the frequency noise in the comb. As this amplitude noise of the pump laser is the largest around Fourier frequencies of 100 kHz [15], we recorded the frequency noise on the comb at this same offset frequency. The results are plotted in Fig. 6. For each measurement, the delay-line

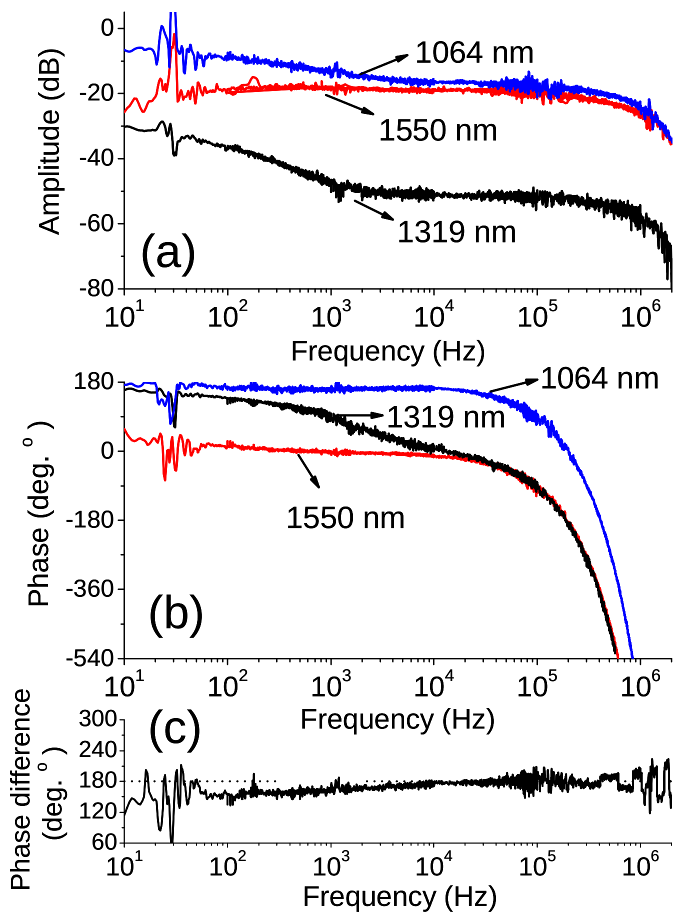

Fig. 5. For the optical beat notes heterodyned with stable CW lasers at 1064 nm, 1319 nm, and 1550 nm, a delay-line discriminator was used to measure the (a) amplitude and (b) phase of the frequency comb response to modulation of the pump laser power. That is, we measure and plot the complex transfer coefficient between amplitude modulation on the pump and frequency modulation on specific comb lines. (c) The difference of the measured phase of beats at 1064 nm and 1550 nm. (d) The frequency shift of the beats at 1064 nm and 1550 nm in the time domain while the pump power was modulated at 200 kHz.

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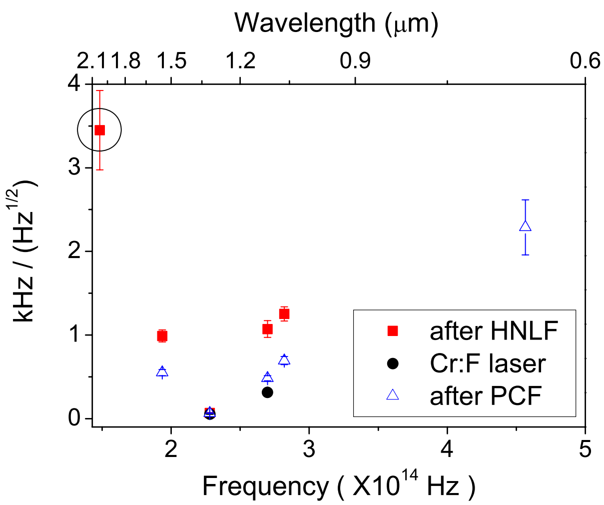

Fig. 6. The frequency noise (at a Fourier frequency of 100 kHz) was measured at different wavelengths of the comb at both the Cr:forsterite laser output and after supercontinuum generation in HNLF and PCF. The error bars on each point are representative of the uncertainty in the measurements and calibration of the delay-line frequency discriminator

frequency discriminator was carefully calibrated so that meaningful comparison can be made between the various data points. The spectrum directly from the Cr:forsterite laser was broad enough that beats with the 1319 and 1112 nm light could be measured without the nonlinear fiber. Moreover, the comparison of the 1112 nm point to the measured noise obtained from the outputs of the two fibers at the same wavelength shows evidence for the addition of frequency noise in the process of supercontinuum generation through both fibers. Apparently, fluctuations in the comb elements coming directly from the laser are amplified in the nonlinear fibers, with the longer HNLF fiber exhibiting more excess noise than the much shorter PCF. The data point of HNLF fiber at 2060 nm (circled in plot) is derived from the noise on f_0 around 100 kHz as follows. We took the measured noise on f_0 and subtracted the independently measured noise at 1064 nm on the assumption of exact anti-correlation. We then divided this value in half to account for the increase due to the second harmonic generation of 2060 nm. From a general point of view, the data of Fig. 6 show an approximately linear growth of the noise away from the fixed point at 1.3 microns. Qualitatively, this is in agreement with theoretical models [4, 5, 6]. However, it is interesting to note that the noise rises more rapidly on the low frequency side of the spectrum–a feature not present in current theories. Additionally, these data show that extra frequency noise (above the noise of the laser itself) is added in the process of supercontinuum generation in the nonlinear fibers, and that the amount of excess noise depends on the specifics of the nonlinear fiber

4. Conclusion

We used heterodyne optical beat notes with stable CW lasers at different wavelengths to measure the frequency noise on the optical frequency comb generated with a femtosecond Cr:forsterite laser. This technique had an enhanced sensitivity of $\sim 10^6$ over techniques that measure noise on the repetition rate. Strong anti-correlations were observed in the pump power driven frequency noise on the comb components on different sides of spectrum relative to the fixed point frequency near 1.3 microns. Finally, the amplitude of the frequency noise was amplified in the continuum generation process with the amount of amplification depending on the type of nonlinear fiber used.

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