

The Realization of the SI Second and Generation of UTC(NIST) at the Time and Frequency Division of the National Institute of Standards and Technology (NIST)⁺

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Abstract

The Time and Frequency Division of the National Institute of Standards and Technology (NIST) maintains the laser-cooled cesium primary fountain frequency standard NIST-F1. NIST-F1 is used to accurately realize the SI (International System) second, defined as 9,192,631,770 periods of the radiation of the ground state hyperfine transition in cesium. Since its completion in 1998, NIST-F1 has undergone 20 formal accuracy evaluations, which have been submitted to the Bureau International des Poids et Mesures (BIPM). NIST-F1 has been continuously improved over the years and presently has a fractional frequency inaccuracy of $\Delta f/f_0 \approx 5 \times 10^{-16}$. The low atomic temperatures (less than 1 μK) attainable with laser-cooling allow for Ramsey (interaction) times of ~ 1 s, a factor of $\sim 10^2$ longer than that achieved in thermal beam frequency standards. Since the frequency stability as well as the magnitudes of many systematic biases scale with the Ramsey time, atomic fountains easily outperform thermal beam devices. Presently, the accuracy of NIST-F1 is limited by the blackbody radiation shift. NIST-F2, the second-generation atomic fountain standard under construction at NIST, will operate at 77 K, thus reducing the blackbody shift to a negligible level. Also located on the NIST premises is a continuously operating timescale, comprised of five hydrogen masers and several commercial cesium beam atomic standards. The clocks are continuously inter-compared and their rates calibrated against the SI second whenever NIST-F1 is evaluated. A modified, weighted averaging algorithm is used to generate Universal Coordinated Time at NIST, or UTC(NIST). Frequency measurements from NIST-F1 as well as the rate of UTC(NIST) are compared with clocks and timescales located at other national standards laboratories using time-transfer techniques such as common-view GPS and two-way satellite time-transfer.

Keywords: CES, GEN, TIME

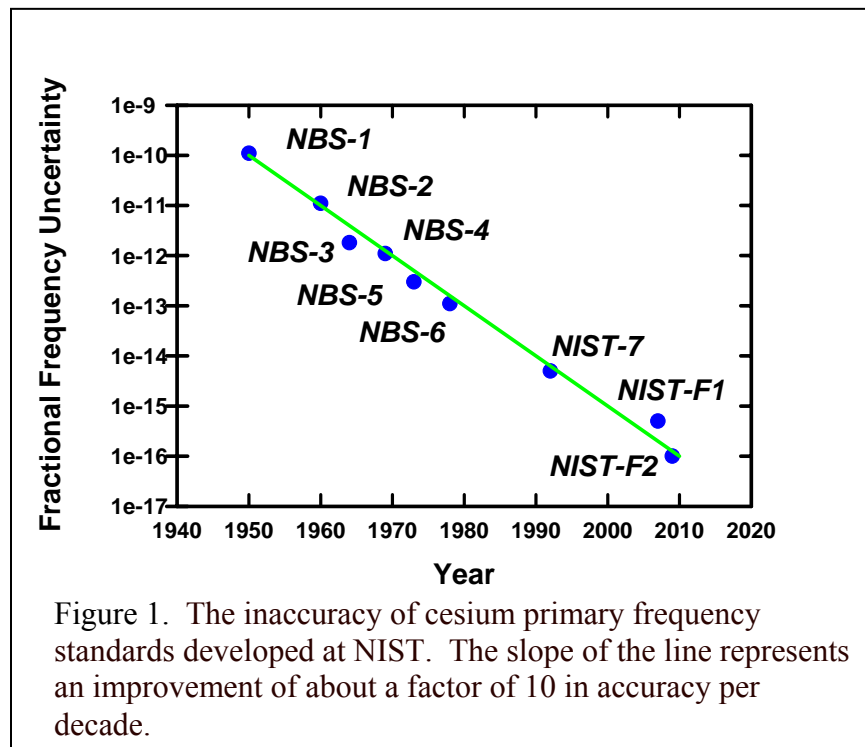
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1. Introduction

The second is one of seven base units of the international system (SI) and is used to define several of the other base units. The meter is in fact defined with respect to the second along with the speed of light, and the volt is maintained by the Josephson effect in terms of a constant times a frequency (and the frequency is, of course, defined in terms of the second). Because frequency can be measured easily with very low uncertainty, many physical quantities are measured with transducers that convert the measurement into a frequency measurement, as with the Josephson Effect voltage standard. The development and operation of high-quality frequency standards is therefore a priority at many national metrology laboratories around the world.

Since 1967, the SI second has been defined as the duration of 9,192,631,770 periods of the radiation corresponding to the ground-state hyperfine splitting of cesium-133 (^{133}Cs), where the atom is taken to be unperturbed, at rest, on the reference geoid of the Earth [1, 2]. This definition is essentially impossible to realize in a laboratory, as it requires that the atom be in a zero magnetic field; further, the environment seen by the atom would have to be at a temperature of absolute zero and the atom would be required to have no residual velocity. Primary frequency standards, such as those operated at national metrological laboratories, attempt to measure the effect of residual magnetic fields: cesium atoms not at absolute zero, etc., on the output frequency of the standard. Measurements of these biases are used to correct the output frequency to achieve an accurate realization of the SI second.

In the late 1940's and early 1950's work began in several national laboratories, notably the National Physical Laboratory (NPL) in England and the National Bureau of Standards (NBS; now the National Institute of Standards and Technology, NIST) in the United States to build frequency references based on atomic transitions [3, 4]. Prior to the introduction of atomic clocks, quartz oscillators calibrated to the orbital motion of the Earth about the Sun were the best



frequency standards available, with fractional frequency uncertainties of about 2×10^{-8} (throughout this paper the quoted frequency uncertainty is given as $\delta f / f$, where δf is generally the one-sigma uncertainty of the measurement of the accuracy of the clock frequency, f). The first atomic standards quickly surpassed the quartz standards of the day and since then have continued to improve at the rate of about a factor of 10 every ten years. Figure 1

illustrates this trend for the primary frequency standards at NBS/NIST, and this curve is representative of the development in primary frequency standards over time worldwide. The point corresponding to NIST-F2 is the anticipated performance of the standard which is presently under development at NIST.

The primary standards named NBS-1 through NBS-6 as well as NIST-7 were thermal cesium beam devices and operated by directing a beam of Cs atoms through a microwave cavity to interrogate the 9.192631770 GHz clock transition. The relatively high speed of the thermal atoms (≈ 100 m/s) fundamentally limited the ultimate accuracy of these devices. NIST-7 was the last thermal beam frequency standard developed at NIST.

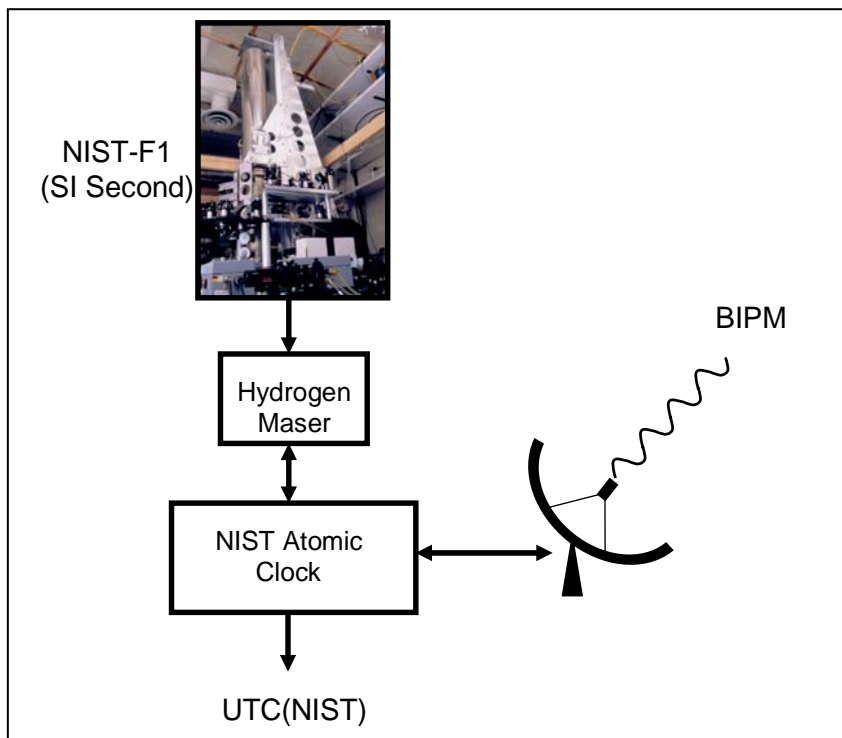


Figure 2 – A block diagram illustrating how the SI second as realized by NIST-F1 is used to calibrate secondary atomic clocks, which in turn are used to generate UTC(NIST). Additionally, measurements are transferred to the BIPM and other national standards laboratories using satellite time and frequency transfer techniques.

NIST-F1 was the first cesium standard constructed at NIST to use the technique of laser-cooling. Laser-cooling allows a sample of $\approx 10^8$ Cs atoms with a volume of approximately 1 cm^3 to be cooled to a temperature of $\approx 0.5 \text{ }\mu\text{K}$ in a few hundred milliseconds. Cs atoms at this low temperature have a thermal velocity on the order of 1 cm/s and can therefore be observed, or measured, for periods much longer than possible in a thermal beam standard. This technology allowed for laser-cooled atomic frequency standards to quickly surpass the highest accuracies

achieved in the thermal beam standards. In addition to the primary standard NIST-F1, two other systems are needed to realize and disseminate the SI second at NIST: the NIST time scale and satellite-based time and frequency transfer systems. The NIST time scale is composed of a collection of commercial hydrogen masers and commercial cesium beam standards that operate continuously and are intercompared to generate an average time by use of a modified weighted mean algorithm. When NIST-F1 is operated, a hydrogen maser in this clock ensemble is used as a reference. Although the masers are not inherently accurate, they are very stable, and once calibrated against NIST-F1 they serve as a flywheel to maintain accuracy when NIST-F1 is not

operational. This ensemble of calibrated commercial atomic clocks is used to generate UTC(NIST). Comparisons with other standards laboratories and calibration contributions to the BIPM are made by satellite-based time and frequency transfer techniques. Fig. 2 shows the calibration pathway from NIST-F1 to other users.

Since its completion in 1998, NIST-F1 has undergone 20 accuracy evaluations (calibrations), which have been submitted to the BIPM. Typically, an evaluation of NIST-F1 takes 20 to 40 days of near continuous operation during which period a hydrogen maser at NIST is calibrated with respect to the SI second. The length of the calibration procedure is typically set by the noise of the satellite time transfer process used to relay the measurements to the BIPM. During this interval, many frequency measurements are made to reduce the statistical uncertainty and to calibrate systematic bias.

2. Atomic Clocks: Background

Primary atomic frequency standards work by frequency-locking an external oscillator to an atomic transition. Generally speaking, the narrower the resonance, the more stable and accurate the clock can be made. The width of the resonance, Δf_a , is set by the period, τ_a , the atoms spend

interacting with the external oscillator. The fractional width of the resonance is $\frac{\Delta f_a}{f_o} \propto \frac{1}{\tau_a f_o}$. It

can be seen that the optimum performance is obtained by making the interaction period, τ_a , as long as possible and the frequency of the atomic resonance, f_o , as high as possible. These simple considerations led to cesium being adopted for the international definition of the second; cesium is a relatively heavy atom (133 amu) and as a result its mean thermal velocity is fairly small (~130 m/s at room temperature). This relatively low velocity allows the cesium to stay in the interaction region for longer periods than for example hydrogen, which has a mean thermal velocity of about 1600 m/s at room temperature. Cesium also has one of the highest hyperfine frequencies of any atom, at 9.2 GHz, compared to 6.8 GHz for rubidium and 1.4 GHz for hydrogen.

Most of the basic concepts of modern atomic frequency standards were developed by Rabi and his co-workers at Harvard in the 1930's and 40's [5]. In the early work of Rabi the atomic transition (resonance) was interrogated with one long (microwave) interrogation pulse. This provided the needed long interaction time, but led, for various reasons, to the output frequency of the standard being subject to Doppler shifts, and being critically dependent on the amplitude of the microwaves as well as the uniformity of a small DC magnetic field in the interaction region (for historical reasons, this field is known as the C-field). Ramsey's method of separated oscillatory fields provided a critical improvement that has been adopted by all modern primary frequency standards. In Ramsey's method, the microwave excitation occurs during two relatively short pulses at the beginning and end of the interaction zone. This two pulse process (now universally known as Ramsey interrogation) reduces the sensitivity to microwave power fluctuations and magnetic field inhomogeneity (by factors of 10 to 100 or more) and essentially eliminates the Doppler effect [6].

Cesium is a complicated atom with $F = 3$ and $F = 4$ ground states. Associated with the F quantum number is another quantum number, m_F , that is limited to $2F + 1$ integer values between $-F$ and $+F$. There are therefore $7 + 9 = 16$ possible ground states for the cesium atoms in the atom beam (labeled $|F, m_F\rangle$), and all of these states are essentially equally populated in the atomic beam from the oven. Unfortunately, only the $m_F = 0$ state is useful in a primary frequency standard, because the other states are sensitive in first order to magnetic fields (Zeeman effect), while the transition between the $|4,0\rangle$ and $|3,0\rangle$ states used by the clock is insensitive to magnetic fields (to first order).

3. Cesium Beam Frequency Standards

Figure 3 shows a schematic diagram of a conventional magnetically state-selected cesium beam standard. The design can be directly traced back to Rabi and Ramsey's seminal work, and essentially all commercial cesium atomic clocks available today (2007) operate using this general design, as did all primary frequency standards until quite recently.

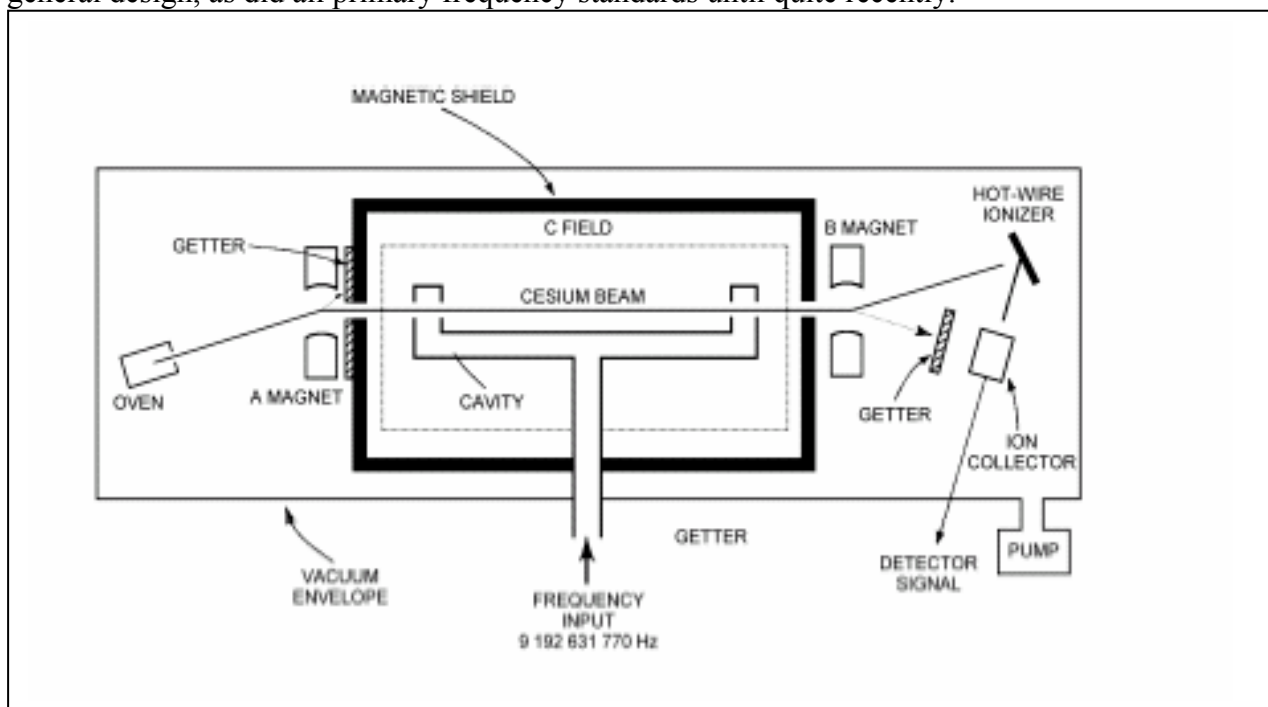


Figure 3. Diagram of a cesium-beam frequency standard using magnetic state-selection and detection. The form of Ramsey interrogation involves a U-shaped microwave cavity (the Ramsey cavity) where the microwave interrogation fields are spatially separated.

On the left of Fig. 3 is a cesium oven from which a beam of cesium atoms emerges when the oven is heated. This beam of cesium atoms is collimated and directed through a Stern-Gerlach magnet (the A-magnet in Fig. 3) that deflects and focuses those cesium atoms in the beam that are in the desired state through a hole in the magnetic shield, so the atoms are now traveling from left to right in the figure. Atoms that are not in the desired state are defocused and absorbed by

the getter shown in the figure. The net result of this is that the A-magnet selects only the $|4,0\rangle$ state and “throws away” most (15/16) of the flux of cesium atoms from the oven. The remaining atomic beam next enters the first microwave interaction region (cavity) and continues through the vacuum chamber until it reaches the second interaction region. The atoms then leave the magnetically shielded region at the right edge in the figure and pass through another Stern-Gerlach magnet (B-magnet). Atoms that have changed state, from $|4,0\rangle$ to $|3,0\rangle$, as a result of the microwave interaction, are directed to the hot-wire ionizer, and the resulting ions are collected in the ion-collector. Atoms that have not changed their atomic state are directed to the getter. The current induced in the hot-wire ionizer and ion collector is proportional to the flux of atoms hitting the hot wire. Maximizing this current, all other things being equal, maximizes the number of atoms making the transition and thus assures that the frequency of the microwaves is on resonance.

There are several challenging aspects of this design. Loss of the majority of the atoms compromises the potential signal to noise (S/N) ratio. The Stern-Gerlach magnets have large magnetic fields associated with them that can cause frequency shifts. Finally, the current at the hot wire and ion collector is quite small, further complicating the S/N situation. Nonetheless this design provided the world with all of its primary frequency standards up until about 1990, and several national metrological laboratories continue to operate such clocks with great success. The CS-1 and CS-2 clocks at the Physikalisch-Technische Bundesanstalt (PTB) in Germany use this basic design and are currently contributing to international atomic time. CS-1 has the lowest stated frequency inaccuracy of any clock of this type ever built, with a claimed $\delta f / f \approx 5 \times 10^{-15}$ [7].

4. Cesium Fountain Frequency Standards

The original concept of a cesium fountain was introduced in the 1950's by Jerrold Zacharias [6]. Zacharias' idea was simply to build a cesium beam clock vertically with one Ramsey interaction zone. Slow atoms from the cesium oven would traverse the microwave interaction zone while traveling upward, stop and reverse their velocity under the influence of gravity, and traverse the same microwave interaction zone while traveling downward. The two interactions with the microwaves reproduced Ramsey's two-pulse interaction scheme, and a ballistic flight traveling only a meter upwards would give interaction times approaching one second instead of the 10 ms typical of thermal beam clocks up to that time. Unfortunately, Zacharias' idea was premature; for obscure technical reasons involving collisions between cesium atoms in a thermal beam (ideas that Zacharias was, eventually, the first to understand!) no signal was ever seen in Zacharias' clock.

Zacharias' idea was resurrected in the late 1980's by Steven Chu and coworkers at Stanford, who made the world's first working fountain (Chu's group built fountains that used either Na (first) and (later) Cs) using a technique known as laser-cooling, which was made possible by advances in laser technology [8]. Researchers at the LPTF (now BNM-SYRTE) in Paris later built the first primary frequency standard based on Zacharias' fountain concept [9]. Many other researchers in metrology laboratories around the world have built (or are building) laser-cooled cesium fountain clocks. Currently fountain clocks at NIST, PTB (Germany), BNM-SYRTE,

Istituto Nazionale di Ricerca Metrologica (formerly IEN, now INRM) in Turin, Italy, the National Physical Laboratory (NPL) in England and NMIJ (Japan) contribute to international atomic time using cesium fountain clocks.

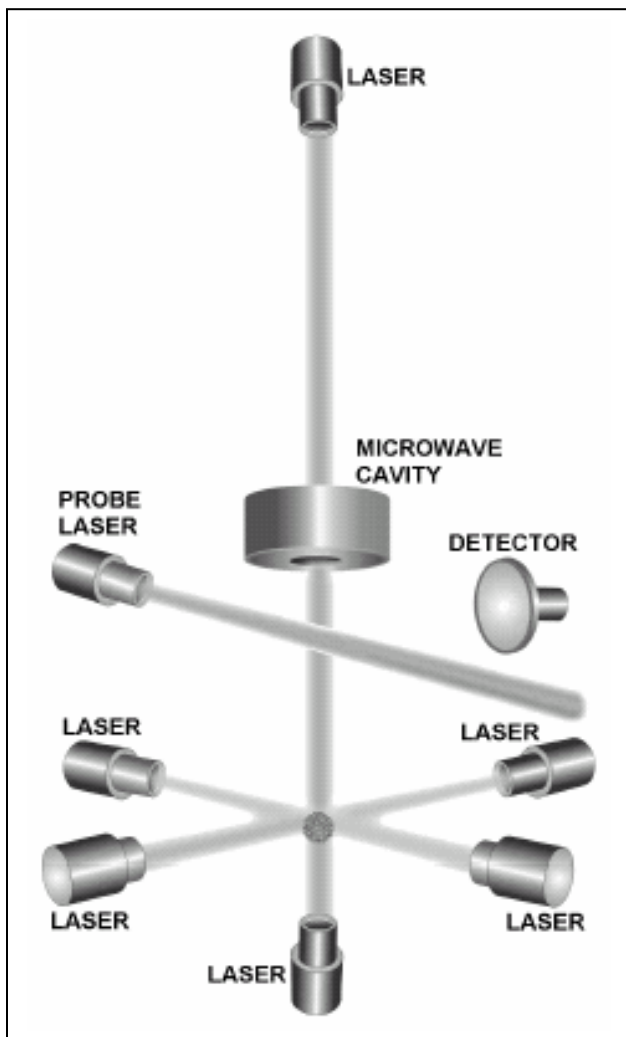


Figure 4. Schematic diagram of a cesium fountain. Atoms are collected and cooled to $\approx 1 \mu\text{K}$ at the intersection of six orthogonal laser beams and are tossed vertically with a slight frequency detuning of the vertical lasers. The atoms follow a ballistic trajectory vertically through the microwave cavity (both on the way up and down). The population in each of the two ground-state hyperfine levels is measured by the probe laser after Ramsey interrogation.

Laser-cooling of atoms, the key to making a fountain clock work successfully, is only briefly explained here—there are numerous good references on the subject [10]. A schematic of a cesium-fountain clock is shown in Fig. 4. Atoms from a background gas of room-temperature cesium vapor are cooled at the intersection of the six laser beams to temperatures around $1 \mu\text{K}$. Laser-cooling can be explained as essentially a refrigerator; a “sink” at very low entropy (the laser beam) interacts with a sample with much higher entropy (the atomic sample). Entropy is transferred from the atoms to the light field, via optical interactions between the atom and the light field. The entropy of the light field is raised (the atom scatters many photons out of the laser beam with random direction and phase), while the entropy of the atomic sample is lowered substantially. In the case of the scheme used in cesium fountains, known as optical molasses, the room temperature cesium atoms can be cooled to a temperature of a few microkelvins (or below) in a few hundred milliseconds. At these temperatures the thermal velocity of the cesium atom is around 10 mm/s , as opposed to the 100 m/s velocity at room-temperature. This has the result that a spherical sample of cesium atoms gathered in the optical molasses with a diameter of about 1 cm roughly doubles its diameter in 1 second. As an alternative to the thermodynamic view presented above, laser-cooling can be viewed as mechanical effect of the light on the atom. If the laser is tuned

slightly lower in frequency than the optical resonance, the atom will, as a result of the Doppler-effect, preferentially absorb photons from the laser beam toward which it is moving. Each photon the atom absorbs carries momentum $\vec{p} = \hbar\vec{k}$, which is directed in opposition to the atomic motion. The atom reemits this photon in a random direction and, because the laser is tuned below resonance, the atom re-emits slightly more energy than it absorbed (the atom re-emits at the resonance frequency). This cycle of absorbing a photon of slightly lower energy than the reemitted photon is repeated many ($\sim 10^7$) times per second and provides the basic laser cooling cycle.

The basic operation of the cesium fountain proceeds in a sequence of steps: first a sample of $\sim 10^8$ cesium atoms is laser-cooled at the intersection of the six laser beams, as shown in Fig 4. These atoms are next “launched” upwards at approximately 4 m/s by detuning the frequency of the up and down laser beams to make a moving optical molasses. The laser light is then extinguished by shutters so that no laser light interacts with the cesium atoms along their ballistic flight path. The cloud of launched cesium atoms, about 1 cm in diameter, is typically in the $F = 4$ ground state, but all m_F levels are populated. The “ball” of cesium atoms is next state-selected with a short microwave pulse that drives the $|4, 0\rangle$ atoms into $|3, 0\rangle$ and leaves the other $F = 4$ atoms unperturbed. The remaining $F = 4$ atoms are removed from the cloud with a short optical blast. At this point the remaining cesium atoms, all in the $|3, 0\rangle$ state, enter the microwave cavity shown in Fig. 4 with a velocity of around 3 m/s. The passage through the cavity on the way up provides the first pulse of the two-pulse (Ramsey) microwave interrogation sequence. The atoms reach apogee about 1 m above the microwave cavity and begin to fall. The second passage through the cavity, in the opposite direction this time, occurs about 1 second after the first passage (typical fountains use more like ≈ 0.5 second interaction times, which leads to a 1 Hz linewidth). The atoms are detected optically with a laser tuned to the $F = 4 \rightarrow F' = 5$ optical transition, similar to the detection process in an optically pumped and detected thermal beam.

The fountain arrangement results in a line-width, Δf_a , of order 1 Hz (compared to the ~ 100 Hz linewidth of thermal beam standards). Most of the frequency uncertainties that limit the inaccuracy of the thermal beam standards scale with the inverse of the interaction time, allowing fountain-based frequency standards to achieve much lower inaccuracy than that of the older thermal beams. For example, the present systematic inaccuracy of NIST-F1 is $\delta f / f \cong 4 \times 10^{-16}$, with other fountain frequency standards having similar or only somewhat larger inaccuracies [11-16].

In NIST-F1, the frequency inaccuracy is limited by two distinct effects, a density shift and a blackbody shift [11, 15]. The blackbody shift is simply the result of the cesium atoms interacting with the thermal radiation emitted by the walls of the 300 K vacuum enclosure. The magnitude of this shift, $\delta f / f \cong 2.1 \times 10^{-14}$, is quite large; however, in the present generation of cesium fountains its uncertainty can be made as small as $\delta f / f \cong 2.6 \times 10^{-16}$. This represents a limit that will require either a great deal of theoretical calculation (to better understand the shift) along with difficult experimental determination of parameters for the improved theory, or a cryogenic vacuum system to reduce the magnitude of the effect. We are presently pursuing the second possibility at NIST. The density shift is caused by collisions between the cesium atoms in the

launched sample, and is quite large, as much as $\delta f / f \approx 4.0 \times 10^{-16}$ in NIST-F1, with an uncertainty in the correction of $\delta f / f \approx 1.0 \times 10^{-16}$. There are many proposals for lowering the uncertainties with this shift, but it remains a serious problem in the present generation of cesium fountains.

5. Second Generation Cesium Fountains

As mentioned previously, both the blackbody and cesium collision shifts are major sources of uncertainty in present cesium fountains. In the case of NIST-F1, these two effects have a magnitude of $\delta f / f \approx 2.6 \times 10^{-16}$ for the blackbody frequency bias and $\delta f / f \approx 1.0 \times 10^{-16}$ for the collision shift. We are presently assembling NIST-F2, which is designed to minimize the frequency uncertainty associated with these two frequency biases.

The blackbody frequency shift has been the source of some controversy of late with different groups calculating different frequency biases for the shift [17-21]. Even if one of these two

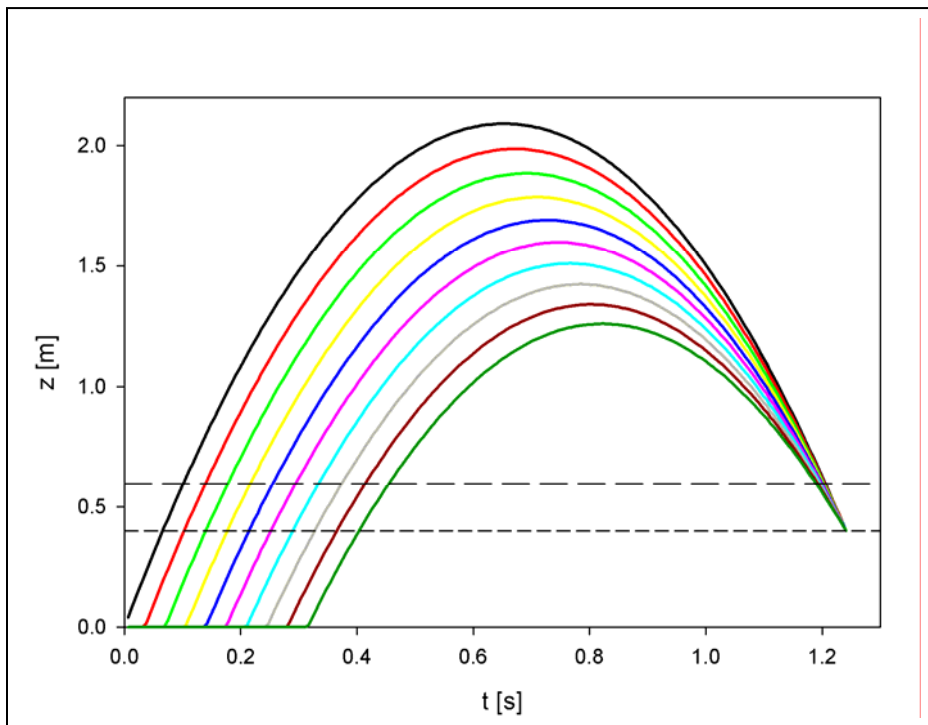


Figure 5 – The multiple ball toss scheme. The horizontal axis in the diagram is the time axis, while the vertical axis is the height above the launch region. As described in the text, the first launched ball goes the highest, while the next ball has an apogee slightly less, and so forth. Finally after a free flight for the first ball of about 1.2 s all of the atoms are in the detection region, where they are detected as a single ball.

competing results is shown to be correct, the present state of the art precludes an uncertainty in this bias of better than about $\delta f / f \approx 1.0 \times 10^{-16}$. In NIST-F2 we have chosen to avoid this difficulty by building a cryogenic ($T = 77$ K) vacuum structure that includes the microwave cavities and flight tube above them. This way the frequency bias associated with the blackbody shift becomes $\delta f / f \approx 7.6 \times 10^{-17}$, and the uncertainty of this bias is easily ten times less, removing the blackbody frequency shift as a major source of uncertainty.

The collisional frequency shift in NIST-F2 will be significantly reduced by use of a clever idea theoretically developed at INRM in Torino, Italy and experimentally demonstrated at NIST [22, 23]. Multiple balls of laser-cooled cesium atoms (as many as 10 in NIST-F2) are launched in quick succession (as shown in Fig. 5) with the first launched ball of cesium atoms having the highest apogee, while the second ball has an apogee just below the first, etc. The trajectories of all the balls intersect in the detection region. In this way the average cesium density is reduced by about a factor of 10, but the signal to noise ratio is preserved, as the same number of atoms is finally detected. We anticipate that we will most likely launch a greater total number of atoms in NIST-F2 in order to achieve a short term stability of $2-3 \times 10^{-13} \tau^{-1/2}$ at low densities, while achieving an uncertainty of $\delta f/f \approx 3.0 \times 10^{-17}$ for the collision shift.

We therefore expect to achieve a total systematic fractional frequency uncertainty in NIST-F2 of $\delta f/f < 1.0 \times 10^{-16}$, dominated by effects due to the microwave frequency of the clock. Smaller frequency uncertainties will probably be achieved only by the use of the quickly developing optical frequency standards.

6. Conclusions

The history of atomic clocks from the late 1940's to the present era shows a steady improvement in the fractional inaccuracy of the clocks from the $\delta f/f \approx 1 \times 10^{-10}$ level in 1950 to the $\delta f/f \leq 1 \times 10^{-15}$ level in 2000. The present state of the art in atomic clocks is defined by the accuracy of the cesium fountains with fractional frequency uncertainties of $\delta f/f \leq 1 \times 10^{-15}$ today and improvements will likely produce accuracies at the $\delta f/f \approx 2 \times 10^{-16}$ level before 2010.

Atomic clocks, currently based on atomic microwave transitions, will likely be replaced eventually by atomic clocks based on optical transitions. Atomic clocks using optical transitions with the resulting much higher frequency (10^{15} Hz) have the potential to reach inaccuracies of $\delta f/f \leq 10^{-17}$ and beyond. These optically based clocks are being investigated in many laboratories world-wide [24, 25], but are some years away from replacing cesium microwave clocks as the world's time and frequency standards.

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