Optical lattice polarization effects on magnetically induced optical atomic clock transitions

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We derive the frequency shift for a forbidden optical transition $J=0 \rightarrow J'=0$ caused by the simultaneous actions of an elliptically polarized lattice field and a static magnetic field. We find that a simple configuration of lattice and magnetic fields leads to a cancellation of this shift to first order in lattice intensity and magnetic field. In this geometry, the second-order lattice intensity shift can be minimized as well by use of optimal lattice polarization. Suppression of these shifts could considerably enhance the performance of the next generation of atomic clocks.

I. INTRODUCTION

Optical clocks based on large numbers of neutral atoms tightly confined in optical lattices are attracting considerable attention due to their potential for extremely high stability and accuracy [1]. Recently, lattice-based systems have demonstrated record line Q's ($\sim 2 \times 10^{15}$) and have achieved inaccuracies as low as 1 part in 10^{15}, with considerable room for still further improvement [2–5]. With the potential inaccuracy for such systems projected to be 10^{-17} or lower, it is critical to examine potential barriers and find ways, if necessary, to overcome them.

For lattice-based clocks, one of the most serious concerns arises from the lattice itself. The confinement from the lattices results from tightly focused red-detuned, high power laser beams. The resulting standing-wave potential wells have depths of approximately 1 MHz, which affect both the ground and excited states of the clock transition. Achieving a fractional frequency inaccuracy of below 10^{-17} requires suppression of the net shift for the clock transition by more than eight orders of magnitude, so optimization of the lattice parameters (such as wavelength, polarization, and intensity) plays a critical role in the design of lattice-based atomic clocks. Fortunately, fairly straightforward techniques have been developed that allow one to minimize the net lattice shift on the clock transition to a value at or below the desired level of fractional frequency inaccuracy (e.g., less than 10^{-17}).

In this work we address the following question: How does the presence of a magnetic field modify these minimization procedures? This question is significant because magnetic-field-induced spectroscopy of forbidden transitions has opened up an important new path for the lattice-based clocks, as it enables the use of transitions in even isotopes [5–7]. The even isotopes have several attractive features, including experimental simplicity due to lack of magnetic substructure (i.e., no optical pumping effects), and reduced magnetic sensitivity. However, if the presence of a magnetic bias field compromises our ability to minimize the effects of the lattice fields on the clock transition, then we must understand the limitations. Fortunately, as we will show, while the presence of the magnetic field indeed leads to new effects, the resulting frequency shifts are very manageable. Moreover, we find that even in the presence of the magnetic field, lattice clocks based on even isotopes still have an additional degree of freedom (lattice polarization) that enables further minimization of intensity-dependent shifts of the clock transition frequency.

To understand these issues more fully, let us first consider the net frequency shift of the clock transition induced by the lattice in the absence of a magnetic field. We write the shift, $\Delta\omega$, in a series of increasing powers of I, the intensity of the optical lattice:

$$\Delta\omega = \alpha_1(\omega,\mathbf{e})I + \alpha_2(\omega,\mathbf{e})I^2 + \cdots , \tag{1}$$

where the polarizability coefficients α_i represent the difference in polarizability for the ground and excited states of the clock transition, and depend on the lattice frequency, ω, and the lattice polarization, $\mathbf{e}$. We note that the terms in this series decrease rapidly, so the most critical are those for the lowest powers of intensity [8,9]. Thus, it is essential that the lowest-order coefficients in Eq. (1) be reduced to the point where the resulting frequency shifts do not limit clock performance through sensitivity to lattice intensity effects (e.g., lattice light fluctuations, atom location in the Gaussian lattice beams, or an atom's motional state in its potential well).

The reduction of these coefficients has thus far been achieved through a judicious choice of clock transitions and values for the lattice parameters. Specifically, to remove what might otherwise be an unmanageable polarization sensitivity in the first term, α_1, lattice clocks use forbidden $J=0 \rightarrow J'=0$ transitions (usually ones found in alkaline-earth–like atoms) [1]. Then α_1 can be reduced to a near-negligible value by tuning the lattice frequency to a value for which the ground and excited states experience equal shifts. At this so-called "magic wavelength," the clock transition is effectively unperturbed by the lattice to first order in the lattice intensity [8]. The use of the magic wavelength is now a

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standard part in existing lattice clock apparatus, and requires an absolute frequency accuracy for the lattice of only 1 MHz to achieve less than 10^{-17} uncertainty for the clock. It has been proposed that the second-order term, α_2, can be be minimized in a similar way, but this time by tuning the polarization of the lattice [10]. This approach works because even for $J=0 \rightarrow J'=0$ clock transitions, α_2 has a polarization dependence (associated with two-photon transitions to higher lying states) that can be exploited for its minimization; for some atoms, the various contributions to α_2 actually have opposite signs, leading to the existence of a magic ellipticity, for which α_2 vanishes.

For realizable clocks, however, it is of course necessary for the $J=0 \rightarrow J'=0$ transition to be at least weakly allowed in order to be able to transfer measurable population to the excited state. In the odd isotopes of alkaline-earth–like atoms, a nonzero nuclear spin mixes a sufficient fraction of $J=1$ states into the upper $J'=0$ state to make the spectroscopy feasible. However, the hyperfine interaction leads to a non-negligible polarization dependence for α_1, the first-order coefficient in Eq. (1), so **e** is usually chosen to minimize α_1. (This dependence can be further suppressed by alternating between oppositely polarized spin states [11].) With both ω and **e** fixed, however, there are no degrees of freedom left for minimizing α_2, thus the procedure described in [10] cannot be applied. In this case, the I^2 term in Eq. (1) can be reduced only by lowering the lattice intensity, although this approach is limited by the minimum lattice depth required to sufficiently confine the atoms [12]. Even with this limitation, however, resulting lattice shifts below a part in 10^{17} have been shown to be achievable for the case of clocks based on Sr [9].

For the even isotopes, however, the situation is quite different. Since the nuclear spin of these isotopes is zero, the required level mixing must be generated by an additional field for single photon excitation (additionally several multiphoton schemes have been devised [13–15]). It was proposed in [16] that a single light field could generate the required mixing, although considerable intensity is needed ($I \sim 500 \text{ kW/cm}^2$). Alternatively, a modest magnetic bias field ($B \sim 1$ mT) can be used—this approach has been successfully demonstrated for the case of ^{174}Yb [5,6]. In this case, however, the expression (1) becomes more complicated, due to the possible dependence of the coefficients α_i on the magnetic field, **B**:

$$\Delta\omega = \alpha_1(\omega, \mathbf{e}, \mathbf{B})I + \alpha_2(\omega, \mathbf{e}, \mathbf{B})I^2 + \cdots . \qquad (2)$$

In this paper, we investigate the possible consequences of this additional dependence by deriving the frequency shift for a forbidden optical $J=0 \rightarrow J'=0$ transition caused by the combination of static magnetic and elliptically polarized lattice fields. Indeed, when combined with an elliptically polarized lattice field, the possibility of non-negligible first-order (in lattice intensity) frequency shifts can arise. However, we find the existence of geometrical conditions for the mutual orientation of polarization vector and magnetic field direction for which this shift practically vanishes. Fortunately, these conditions are easily realizable and in fact correspond to the preferred one-dimensional geometry already in use in

the experiments. Additionally and significantly, we find that under these conditions, we have enough freedom in our choice of polarization to minimize α_2 in the one-dimensional case. For two- and three-dimensional lattices, optimization of the lattice polarization is certainly more challenging, but we find that the presence of the magnetic field should not significantly complicate the situation.

II. DERIVATION OF THE FREQUENCY SHIFTS

To understand the origin of these results, let us consider an atom under the influence of a monochromatic, elliptically polarized field with frequency ω

$$\mathbf{E}(t) = \text{Re}\{\mathbf{E}e^{-i\omega t}\}, \quad \mathbf{E} = E\mathbf{e}, \qquad (3)$$

where **E** is the vector amplitude, which can be written as a product of the scalar amplitude E and the complex unit polarization vector **e**, i.e., $(\mathbf{e}^* \cdot \mathbf{e}) = 1$. If the quantization axis Oz is oriented orthogonally to the polarization ellipse, then we have the following expansions in Cartesian $\{\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z\}$ and spherical $\{\mathbf{e}_0 = \mathbf{e}_z, \mathbf{e}_{\pm 1} = \mp (\mathbf{e}_x \pm i\mathbf{e}_y)/\sqrt{2}\}$ bases:

$$\mathbf{e} = \cos(\varepsilon)\mathbf{e}_x + i\sin(\varepsilon)\mathbf{e}_y$$
$$= -\sin(\varepsilon - \pi/4)\mathbf{e}_{-1} - \cos(\varepsilon - \pi/4)\mathbf{e}_{+1}. \qquad (4)$$

Here ε is the ellipticity angle, which is defined in such a way that $|\tan(\varepsilon)|$ is equal to the ratio of minor and major axes of the ellipse, and the sign of ε defines the helicity. Note that $\varepsilon = 0$ corresponds to linearly polarized fields, while $\varepsilon = \pm \pi/4$ corresponds to circularly polarized fields. The lattice potential is formed by the spatially inhomogeneous light shifts in the field (3), with the vector amplitude **E** dependent on the position $\mathbf{E}(\mathbf{r})$.

The atom-field interaction will be considered in the dipole approximation $-(\hat{\mathbf{d}} \cdot \mathbf{E})$. The atom also interacts with a static magnetic field $\mathbf{B} = B\mathbf{b}$, where B is the field amplitude and **b** is the unit vector in the direction **B**. Such a problem statement corresponds, for example, to the method of magnetic-field-induced spectroscopy of forbidden optical transitions $^1S_0 \rightarrow {}^3P_0$ for alkaline-earth–like atoms in lattice-based optical atomic frequency standards [6]. Under the action of the field (3) each level of the forbidden transition $J_g = 0 \rightarrow J_e = 0$ acquires the energy shift ΔE_j ($j = e, g$). In the case of alkaline earth-like atoms the levels with $J_g = 0$ and $J_e = 0$ are the states $^1S_0$ and $^3P_0$, respectively. Let us now focus on the lowest-order shifts (i.e., quadratic Stark shifts), which are proportional to the lattice intensity, $I = |E|^2$. In the presence of a static magnetic field **B** these shifts can be written as a series in powers of B:

$$\Delta\omega_j = \{\kappa_j^{(0)}(\omega) + \kappa_j^{(1)}(\omega, \mathbf{e}, \mathbf{b})B + \cdots\}|E|^2. \qquad (5)$$

The first term in this expansion describes the quadratic Stark shift in the absence of a magnetic field. For levels with $J=0$ this term does not depend on the polarization, i.e., the coefficients $\kappa_j^{(0)}(\omega)$ are governed completely by the field frequency ω. The second and higher terms of the expansion proportional to $\kappa_j^{(n)}(\omega, \mathbf{e}, \mathbf{b})B^n$ in Eq. (5) at $n \geq 1$ describe the influence of the magnetic field **B** on the quadratic Stark shifts

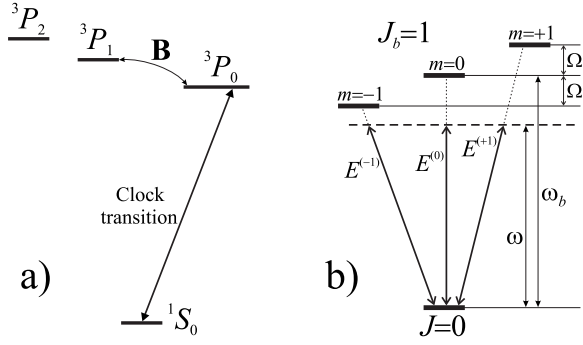FIG. 1. (a) The general level structure for alkaline earthlike atoms, where the levels $^3P_1$ and $^3P_0$ are mixed by the magnetic field **B**. (b) The scheme of the transition $J=0 \rightarrow J_b=1$ in the magnetic field for calculation of the light shift (16).

of the energy levels. The coefficients $\kappa_j^{(n)}(\omega, \mathbf{e}, \mathbf{b})$ depend on the frequency ω as well as on the polarization vector $\mathbf{e}$ and the magnetic field direction $\mathbf{b}$. From Eq. (5) it follows that the forbidden transition frequency ω_0 is shifted by the amount

$$\Delta\omega \equiv \Delta\omega_e - \Delta\omega_g$$
$$= \{\widetilde{\kappa}^{(0)}(\omega) + \widetilde{\kappa}^{(1)}(\omega, \mathbf{e}, \mathbf{b})B + \widetilde{\kappa}^{(2)}(\omega, \mathbf{e}, \mathbf{b})B^2 + \cdots\}|E|^2,$$

$$\widetilde{\kappa}^{(0)}(\omega) = \kappa_e^{(0)} - \kappa_g^{(0)}, \quad \widetilde{\kappa}^{(1)}(\omega, \mathbf{e}, \mathbf{b}) = \kappa_e^{(1)} - \kappa_g^{(1)}, \dots . \quad (6)$$

From comparison with Eq. (2), we see that the bracketed expansion in Eq. (5) is equivalent to $\alpha_1(\omega, \mathbf{e}, \mathbf{B})$. At the magic frequency $\omega_m = 2\pi c/\lambda_m$, in the absence of a magnetic field the quadratic Stark shift vanishes, i.e., $\widetilde{\kappa}^{(0)}(\omega_m) = 0$. Here the dangerous term (from the metrological point of view) in Eq. (6) is the first-order correction with respect to the magnetic field $\widetilde{\kappa}^{(1)}(\omega_m, \mathbf{e}, \mathbf{b})B|E|^2$, which will be studied below.

Based upon a general analysis, we find that this correction consists of two contributions:

$$\widetilde{\kappa}^{(1)}(\omega, \mathbf{e}, \mathbf{b})B|E|^2 = A_1 + A_2. \quad (7)$$

The first contribution, A_1, results from the admixture of levels with $J=1$ to levels with $J_{g,e}=0$. In particular, for alkali-earth–like atoms the mixing of the levels $^3P_1$ and $^3P_0$ [see in Fig. 1(a)] plays a crucial role in the method of magnetic-field-induced spectroscopy [6]. In a manner similar to that of an external magnetic field, the elliptically polarized lattice field (3) can also cause mixing. As was shown in [16], in respect to the level mixing the lattice field is equivalent to an additional static magnetic field $\mathbf{B}_{\text{lat}}^{(a)}$:

$$\mathbf{B}_{\text{lat}}^{(a)} = \eta_a i[\mathbf{E}^* \times \mathbf{E}] = \eta_a i[\mathbf{e}^* \times \mathbf{e}]|E|^2, \quad (8)$$

where the real coefficient η_a is an individual characteristic of the chosen element and of the specific level with $J_a=1$. The direction of the vector $\mathbf{B}_{\text{lat}}^{(a)}$ is governed by the real vector $i[\mathbf{e}^* \times \mathbf{e}]$. Its value, in view of Eq. (4), is $|i[\mathbf{e}^* \times \mathbf{e}]| = |\sin(2\varepsilon)|$, and its direction is orthogonal to the polarization ellipse $\mathbf{e}$. Note that in the general case of 2D and 3D lattice fields with inhomogeneous polarization $\mathbf{e}(\mathbf{r})$, the orientation of the cross product $i[\mathbf{e}^* \times \mathbf{e}]$ and the ellipticity parameter ε

are also spatially nonuniform, i.e., they are local values.

In the case of a 1D standing wave with the spatially uniform elliptical polarization $\mathbf{e}$ and the scalar amplitude $E = E_0 \cos(\mathbf{k} \cdot \mathbf{r})$, the additional magnetic field (8) is directed along the lattice wave vector $\mathbf{k}$, and its value is modulated in space with the lattice spatial period

$$\mathbf{B}_{\text{lat}}^{(a)} = \eta_a \sin(2\varepsilon)\cos^2(\mathbf{k} \cdot \mathbf{r})|E_0|^2 \mathbf{k}/|\mathbf{k}|. \quad (9)$$

Obviously, in the case of linear polarization ($\varepsilon=0$) the effective magnetic field is absent $\mathbf{B}_{\text{lat}}^{(a)}=0$.

Thus, considering the mixing of the specific level $J_a=1$ with levels $J_{g,e}=0$, we can formally suppose that an atom is in an effective static magnetic field $\mathbf{B}_{\text{eff}}^{(a)}$:

$$\mathbf{B}_{\text{eff}}^{(a)} = \mathbf{B} + \mathbf{B}_{\text{lat}}^{(a)}. \quad (10)$$

This mixing leads, in particular, to the quadratic Zeeman frequency shift of the transition $J_g=0 \rightarrow J_e=0$, which can be written as

$$\Delta_Z^{(a)} = \beta_a(\mathbf{B}_{\text{eff}}^{(a)} \cdot \mathbf{B}_{\text{eff}}^{(a)}). \quad (11)$$

In accordance with Eq. (10) this shift can be presented in the form

$$\Delta_Z^{(a)} = \beta_a(\mathbf{B} \cdot \mathbf{B}) + \beta_a(\mathbf{B}_{\text{lat}}^{(a)} \cdot \mathbf{B}_{\text{lat}}^{(a)}) + 2\beta_a(\mathbf{B} \cdot \mathbf{B}_{\text{lat}}^{(a)}). \quad (12)$$

Here the first term corresponds to the standard quadratic Zeeman shift in the external magnetic field $\mathbf{B}$. The second term, proportional to $|E|^4$, is one of the contributions into the frequency shift due to the atomic hyperpolarizability [10]. The last term in Eq. (12) is proportional to $B|E|^2$:

$$2\beta_a(\mathbf{B} \cdot \mathbf{B}_{\text{lat}}^{(a)}) = 2\beta_a \eta_a (i[\mathbf{E}^* \times \mathbf{E}] \cdot \mathbf{B})$$
$$= 2\beta_a \eta_a (i[\mathbf{e}^* \times \mathbf{e}] \cdot \mathbf{b})B|E|^2, \quad (13)$$

and, consequently, we identify it as the first contributor in Eq. (7), A_1. Physically, this shift results from the interaction of the small fraction (about 10^{-6} for the Yb experiments in [5]) of $^3P_1$ mixed into $^3P_0$ with higher lying states. Summing over all the possible levels $J_a=1$, we can write a general expression for A_1:

$$A_1 = \xi_1(i[\mathbf{E}^* \times \mathbf{E}] \cdot \mathbf{B}) = \xi_1(i[\mathbf{e}^* \times \mathbf{e}] \cdot \mathbf{b})B|E|^2,$$

$$\xi_1 = \sum_a 2\beta_a \eta_a, \quad (14)$$

where the real parameter ξ_1 is characteristic of the chosen element.

Let us consider now the term A_2 in Eq. (7). This contribution is determined by the linear Zeeman splitting of various degenerate levels with the angular momentum $J=1$, which participate in the quadratic Stark shift of the levels $J_{g,e}=0$. This leads to the dependence of Stark shift on the magnitude and direction of the magnetic field $\mathbf{B}$. Indeed, consider the shift of the nondegenerate level with $J=0$ through the interaction with the level $J_b=1$ [see in Fig. 1(b)]. The frequency of the transition $J=0 \rightarrow J_b=1$ is ω_b. Directing the quantization axis z along the magnetic field, we have the following decomposition for the polarization vector (3):

$$\mathbf{E} = \sum_{q=0,\pm1} E^{(q)}\mathbf{e}_q, \tag{15}$$

where $E^{(q)}$ are contravariant components of the vector $\mathbf{E}$ in the spherical basis. In these conditions the quadratic Stark shift of the level with $J=0$ has the form

$$\Delta = \frac{|d_b|^2}{4\hbar^2} \sum_{q=0,\pm1} \left[\frac{|E^{(q)}|^2}{\omega - \omega_b - q\Omega} - \frac{|E^{(-q)}|^2}{\omega + \omega_b + q\Omega} \right]. \tag{16}$$

Here d_b is the reduced matrix element of the dipole moment for the transition $J=0 \rightarrow J_b=1$; $\Omega = \mu_b B$ is the linear Zeeman splitting of the level $J_b=1$ [see in Fig. 1(b)]. The first and second terms on the right-hand side of Eq. (16) describe the resonant and off-resonant contributions, respectively.

Let us expand the expression (16) with respect to the small parameter $\Omega = \mu_b B$, to the zeroth and first orders:

$$\Delta = \Delta_0 + \Delta_1 + \cdots,$$

$$\Delta_0 = \frac{|d_b|^2 \omega_b}{2\hbar^2(\omega^2 - \omega_b^2)}|\mathbf{E}|^2, \tag{17}$$

$$\Delta_1 = \frac{|d_b|^2 \mu_b \omega_b \omega}{\hbar^2(\omega^2 - \omega_b^2)^2}(|E^{(+1)}|^2 - |E^{(-1)}|^2)B. \tag{18}$$

The term Δ_0 contributes into the usual shift $\widetilde{\kappa}^{(0)}(\omega)|E|^2$ of the transition frequency $J_g=0 \rightarrow J_e=0$ (6). Here we are interested in the second term Δ_1, which is proportional to $B|E|^2$. The multiplier $(|E^{(+1)}|^2 - |E^{(-1)}|^2)B$ in Eq. (18) can be expressed in the invariant vector form:

$$(|E^{(+1)}|^2 - |E^{(-1)}|^2)B = -(i[\mathbf{E}^* \times \mathbf{E}] \cdot \mathbf{B}). \tag{19}$$

Thus the contribution $\Delta_1^{(b)}$ due to the nonresonant excitation of the dipole transition $J_{g,e}=0 \rightarrow J_b=1$ can be written as

$$\Delta_1^{(b)} = \chi_b(i[\mathbf{E}^* \times \mathbf{E}] \cdot \mathbf{B}). \tag{20}$$

Summing over all the possible levels $J_b=1$, we can write the expression for the term A_2 in Eq. (7),

$$A_2 = \xi_2(i[\mathbf{E}^* \times \mathbf{E}] \cdot \mathbf{B}) = \xi_2(i[\mathbf{e}^* \times \mathbf{e}] \cdot \mathbf{b})B|E|^2,$$

$$\xi_2 = \sum_b \chi_b, \tag{21}$$

i.e., the terms A_2 and A_1 (14) have the same vector dependences. Thus the total shift (7) has the following universal form:

$$A_1 + A_2 = \widetilde{\xi}(i[\mathbf{E}^* \times \mathbf{E}] \cdot \mathbf{B}) = \widetilde{\xi}(i[\mathbf{e}^* \times \mathbf{e}] \cdot \mathbf{b})B|E|^2,$$

$$\widetilde{\xi} = \xi_1 + \xi_2, \tag{22}$$

and, consequently,

$$\widetilde{\kappa}^{(1)}(\omega, \mathbf{e}, \mathbf{b}) = \widetilde{\xi}(\omega)(i[\mathbf{e}^* \times \mathbf{e}] \cdot \mathbf{b}), \tag{23}$$

where $\widetilde{\xi}(\omega)$ is characteristic of the chosen element. The shift is maximal when the light field polarization $\mathbf{e}$ is circular ($\varepsilon = \pm\pi/4$) and the magnetic field is orthogonal to the polarization ellipse:

$$\max|\widetilde{\kappa}^{(1)}(\omega, \mathbf{e}, \mathbf{b})| = |\widetilde{\xi}(\omega)|. \tag{24}$$

Since the magic frequency ω_m for all alkaline-earth–like atoms is sufficiently far from any dipole transition $J=0 \rightarrow J=1$, the term A_1 in Eq. (7) dominates. The main contribution is due to the mixing of the levels $^3P_1$ and $^3P_0$ [see in Fig. 1(a)]. Using the results of the paper [16] for various atoms, we estimate the maximal value of the frequency shift of forbidden transition $^1S_0 \rightarrow {}^3P_0$ to be on the level of 0.1–10 Hz for a magnetic field of order of 0.1–1 mT (1–10 G) and typical lattice field intensities [1,2,5,9]. Since lattice-based clocks are striving for millihertz-level frequency uncertainty, these frequency shifts could be of considerable concern for optical frequency standards based on the method of magnetic-field-induced spectroscopy [6]. In the next section we consider how to minimize these shifts.

III. MINIMIZATION OF THE FREQUENCY SHIFT

Starting from the expansion (4), the vector product in Eq. (23) can be written as

$$i[\mathbf{e}^* \times \mathbf{e}] = \sin(2\varepsilon)\mathbf{n}_\mathbf{e}, \tag{25}$$

where $\mathbf{n}_\mathbf{e}$ is the unit vector orthogonal to the polarization ellipse $\mathbf{e}$. Then the expression (23) can rewritten as

$$\widetilde{\kappa}^{(1)}(\omega, \mathbf{e}, \mathbf{b}) = \widetilde{\xi}(\omega)\sin(2\varepsilon)(\mathbf{n}_\mathbf{e} \cdot \mathbf{b}). \tag{26}$$

As is seen, to nullify this coefficient it is sufficient that the magnetic field vector $\mathbf{b}$ lies in the plane of polarization ellipse $\mathbf{e}$, i.e., $\mathbf{b} \perp \mathbf{n}_\mathbf{e}$. In the case of linear polarization ($\varepsilon=0$) the coefficient (26) equals zero for arbitrary direction of the magnetic field, because $\sin(2\varepsilon)=0$.

Thus, for a lattice consisting of a 1D standing wave in the presence of a magnetic field, the problem of the induced frequency shift is, in principle, readily solved. All that is required is to direct the magnetic field orthogonal to the lattice field wave vector $\mathbf{k}$ (in such a case the vectors $\mathbf{e}$ and $\mathbf{b}$ are coplanar), as is done in existing experiments. In this case the magic wavelength is independent of polarization. (For the example of Yb, this orthogonality would need to be set with an uncertainty of less than 50 mrad for circularly polarized lattice beams to reach a fractional clock uncertainty of 10^{-17}.) What is exciting about this result is that it opens the door for the use of the method of magnetic-field-induced spectroscopy [6] in combination with a standing wave lattice whose polarization is set to the optimal ellipticity ε_{opt} [10], the value at which $\alpha_2(\omega, \mathbf{e})$ is minimized (or even vanishes for the magic ellipticity ε_m). This is particularly important for the case of ^{174}Yb, currently under investigation, since the $\alpha_2 I^2$ term in Eq. (1) could have a value as large as a few hertz (at typical lattice intensity) due a near coincidence between the magic wavelength and a $J=0 \rightarrow J'=0$ two-photon transition [5]. The actual value remains to be measured.

However in the general case of 2D and 3D lattices with spatially inhomogeneous polarization $\mathbf{e}(\mathbf{r})$, the coefficient (26) will differ from zero if the vector $\mathbf{n}_\mathbf{e}$ (25) is spatially nonuniform. This circumstance limits (from the metrology viewpoint) the choice of field configurations for the forma-

tion of 2D and 3D lattice potential in the method of magnetic-field-induced spectroscopy of forbidden optical transitions $J_g = 0 \rightarrow J_e = 0$. These configurations should be chosen now in such a way that the vector $\mathbf{n_e}$ is spatially uniform, i.e., the local polarization vector $\mathbf{e}(\mathbf{r})$ should lie in the same plane independently of the position $\mathbf{r}$. The magnetic field vector $\mathbf{B}$ should also be in the same plane. Alternatively, the configuration should be chosen in such a way that at least in all the potential minima $\{\mathbf{r}_{\min}\}$ the corresponding local polarizations $\mathbf{e}(\mathbf{r}_{\min})$ belong to the same common plane. In this case the coefficient (26) is small due to the strong localization in the lower vibrational levels of the lattice potential for atoms confined to the Lamb-Dicke regime. Alternatively one can work with a lattice for which the polarization vector $\mathbf{e}$ is linear everywhere. In this case the orientation of the magnetic field is consistent with the rest of the paper.

Thus far we have considered the effect of the magnetic field on only the first-order term in Eq. (2). In fact, the $\mathbf{B}$ field can have a small effect on α_2 as well:

$$\alpha_2(\omega, \mathbf{e}, \mathbf{B}) = \alpha_2^{(0)}(\omega, \mathbf{e}) + \alpha_2^{(1)}(\omega, \mathbf{e}, \mathbf{b})B + \cdots, \quad (27)$$

where $\alpha_2^{(0)}(\omega, \mathbf{e}) \equiv \alpha_2(\omega, \mathbf{e})$ [see Eq. (1)]. The $\mathbf{B}$-dependent term in Eq. (27) could, in principle, change the polarization $\mathbf{e}$ required to minimize α_2 according to the prescription in [10]. The magnetic field $\mathbf{B}$ affects $\alpha_2^{(1)}(\omega, \mathbf{e}, \mathbf{b})B$ through shifting (by the Zeeman effect) of the intermediate and final levels of two-photon Raman and ladder transitions induced by the lattice laser. We estimate the importance of $\alpha_2^{(1)}(\omega, \mathbf{e}, \mathbf{b})$ by considering the ratio $\alpha_2^{(1)}(\omega, \mathbf{e}, \mathbf{b})B / \alpha_2^{(0)}(\omega, \mathbf{e})$:

$$\frac{\alpha_2^{(1)}(\omega, \mathbf{e}, \mathbf{b})B}{\alpha_2^{(0)}(\omega, \mathbf{e})} \approx \max\left\{ \frac{\Omega}{\delta_{1\mathrm{ph}}}, \frac{\Omega}{\delta_{2\mathrm{ph}}}, \frac{\Omega}{\Delta_{\mathrm{fs}}} \right\}, \quad (28)$$

where $\Omega \approx \mu_B |\mathbf{B}|$ is the characteristic value of the Zeeman splitting for levels with $J = 1, 2$; $\delta_{1\mathrm{ph}}$ is the characteristic value for the one-photon detuning for transitions $J = 0 \rightarrow J = 1$; $\delta_{2\mathrm{ph}}$ is the characteristic value for the two-photon detuning for transitions $J = 0 \rightarrow J = 2$; and Δ_{fs} is the fine structure splitting between $^3P_1$ and $^3P_0$. Since the Zeeman shifts (≈ 10 MHz) are scaled against the relevant one- and two-photon detunings and fine-structure splittings (≈ 1 to 100 THz), the contribution of $\alpha_2^{(1)}(\omega, \mathbf{e}, \mathbf{b})B$ is negligible for most atoms, including the important lattice candidates Sr and Yb [7,9,17]. For example, we estimate that for Yb,

$$\frac{\alpha_2^{(1)}(\omega, \mathbf{e}, \mathbf{b})B}{\alpha_2^{(0)}(\omega, \mathbf{e})} < 10^{-5}. \quad (29)$$

Even for the worse case the $\alpha_2^{(1)}$ term would modify the shift by less than 100 μHz, or below 10^{-18} fractionally for typical intensities in a lattice-based optical clock. Moreover, it can be shown that when the vector $\mathbf{B}$ lies in the plane of the polarization ellipse $\mathbf{e}$ (the usual operating case), we actually have $\alpha_2^{(1)}(\omega, \mathbf{e}, \mathbf{b}) = 0$. Therefore, practically speaking, the value for the magic ellipticity ε_m [10] is basically unchanged by the presence of the magnetic field.

IV. MODIFICATION OF THE INDUCED RABI FREQUENCY

Apart from the frequency shift (2), the combination of elliptically polarized lattice field and magnetic bias field leads to another consequence. The expression for the Rabi frequency of the probe field on the clock transition $J_g = 0 \rightarrow J_e = 0$ now takes the following general form:

$$\Omega_R = \eta(\mathbf{B} \cdot \mathbf{E}_p) + \vartheta(i[\mathbf{E}^* \times \mathbf{E}] \cdot \mathbf{E}_p), \quad (30)$$

where $\mathbf{E}_p$ is the polarization vector of the probe field $\mathbf{E}_p(t) = \mathrm{Re}\{\mathbf{E}_p e^{-i(\omega_p t - \mathbf{k}_p \mathbf{r})}\}$, and the probe field frequency ω_p is scanned around the frequency of the forbidden transition $J_g = 0 \rightarrow J_e = 0$. The difference from the expression for Ω_R in the paper [6] consists in the appearance of an additional term ($\propto \vartheta$) in Eq. (30). Note that this term is essential for the spectroscopic method proposed in [16].

For a 1D standing wave with arbitrary elliptical polarization, the last term in Eq. (30) vanishes when the wave vector $\mathbf{k}_p$ of the probe field is parallel to the lattice field wave vector $\mathbf{k}$. In this case $i[\mathbf{E}^* \times \mathbf{E}] \propto \mathbf{k} \| \mathbf{k}_p$, and because of this ($i[\mathbf{E}^* \times \mathbf{E}] \cdot \mathbf{E}_p) = 0$, since $(\mathbf{E}_p \cdot \mathbf{k}_p) = 0$ due to the transverse nature of electromagnetic waves. However, in the general case of 2D and 3D lattices with spatially inhomogeneous polarization $\mathbf{e}(\mathbf{r})$ the term $\propto \vartheta$ in Eq. (30) should be taken into account in the analysis of various polarization configurations of the lattice field.

V. CONCLUSIONS

In the present paper, we have investigated the frequency shift of the forbidden optical transition $J = 0 \rightarrow J' = 0$ caused by the simultaneous action of an elliptically polarized lattice field with magic frequency ω_m and a static magnetic field. The analytical vector dependence for this shift and has a simple universal form [Eq. (22)]. Estimates show that under certain conditions this shift could have non-negligible consequences for lattice-based optical atomic standards. However, we have identified experimental geometries for which the undesired shift can be significantly suppressed or even completely cancelled. The basic requirement is that the magnetic field needs to lie in the plane of the polarization ellipse of the lattice field. In the case of 1D standing wave this condition is automatically fulfilled when the magnetic field is directed orthogonally to the lattice field wave vector, the usual experimental case. For a linearly polarized lattice field, the frequency shift under investigation vanishes for an arbitrary direction of the magnetic field. As applied to the cases of 2D and 3D lattice fields, we have indicated the limitations on the choice of possible polarization configurations.

The obtained results open, in particular, the possibility of using the method of magnetic-field-induced spectroscopy [6] of the strictly forbidden transition $^1S_0 \rightarrow {}^3P_0$ in even isotopes of alkaline earth-like atoms confined to an optical lattice that is formed by an elliptically polarized standing wave with the optimal ellipticity $\varepsilon_{\mathrm{opt}}$ [10]. For this ellipticity the second-order light shift is minimal (or even cancelled). Such a combination of methods could enable significant improvement of the metrology characteristics of lattice-based frequency standards.

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