Testing Local Position Invariance with Four Cesium-Fountain Primary Frequency Standards and Four NIST Hydrogen Masers

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We report the most sensitive tests to date of the assumption of local position invariance (LPI) underlying general relativity, based on a 7 yr comparison of cesium and hydrogen atomic clocks (frequency standards). The latest results place an upper limit that is over 20 times smaller than the previous most sensitive tests; this is consistent with the null shift predicted by LPI. The result is based on precise comparisons of frequencies of four hydrogen masers maintained by NIST, with four independent Cs fountain clocks—one at NIST and three in Europe—as the Sun's gravitational potential at Earth's surface varies due to Earth's orbital eccentricity.

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Introduction.—Because of the many applications of the general theory of relativity in astronomy and cosmology, as well as to practical navigation in the Global Positioning System (GPS), it is important to push the experimental foundations of the theory as far as possible. One of the fundamental assumptions of general relativity is local position invariance (LPI). LPI asserts that syntonized atomic clocks ("syntonized" means "having equal frequencies"), no matter what their internal structure, will remain syntonized as the clocks experience a varying gravitational potential. In 1978 this idea was tested by comparing two hydrogen masers with three superconducting cavity-stabilized oscillators as the solar gravitational potential varied due to Earth's orbital motion over a period of 10 days [1]. Bauch and his co-workers [2,3] placed limits on the violation of LPI by comparing the frequencies of H masers and Cs primary standards located at PTB (Physikalish-Technische-BundesAnstalt, Braunschweig, Germany) and at NIST (National Institute of Standards and Technology, Boulder, CO) over a period of 30 months as the clocks experienced a varying solar gravitational potential due to Earth's eccentric orbit. Here we report results from a new study involving atomic clocks located at NIST as well as at other standards laboratories. The present analysis includes data over a span of 7 yr, as well as data from more accurate Cs fountain primary frequency standards.

General relativity predicts that the fractional frequency difference, $\Delta f/f$, between two identical clocks located at different gravitational potentials is

$$\frac{\Delta f}{f} = \frac{\Delta\Phi}{c^2}, \tag{1}$$

where $\Delta\Phi$ is the difference of Newtonian gravitational potential between the clocks and c is the speed of light. This was verified at the level of 140 ppm in the 1976 Gravity Probe-A mission by the launching of a hydrogen maser to an altitude of 10^4 km [4].

Explicit in Eq. (1) is the assumption of LPI: that the frequency shift does not depend on the clock's internal structure. If LPI is violated, then the fractional frequency difference will, to lowest order, be of the form

$$\frac{\Delta f}{f} = (1+\beta)\frac{\Delta\Phi}{c^2}, \tag{2}$$

where β may depend on the clock's internal structure [5]. By comparing the rates of different types of atomic clocks as they both experience the same varying gravitational potential, it is possible to place limits on the violation of LPI.

Two dissimilar clocks at the same location will generally have some measurable frequency difference. However, if the frequencies are syntonized and LPI is violated, and then the clocks move through a gravitational potential difference $\Delta\Phi$, there will be a fractional frequency difference

$$\frac{\Delta f}{f} = (\beta_2 - \beta_1)\frac{\Delta\Phi}{c^2}. \tag{3}$$

A possible cause of such a violation is based on the hypothesis that the fine structure constant α depends on the gravitational potential. The atomic clocks used in the present study all operate on the ground level hyperfine transition which is a function of $Z\alpha$, where Z is the atomic number of the element.

The best previous bound, from comparing an H maser with a Cs fountain was reported to be $|\beta_{\rm H} - \beta_{\rm Cs}| < 1.6 \times 10^{-5}$ [2]. However, the authors of Ref. [2] obtained their expression for the solar-potential variation from earlier papers [3,6] that referenced an expression [7] for the total fractional frequency variation of a GPS clock in a Keplerian orbit, due to orbital eccentricity e: $\Delta f/f = -2(GMe\cos\phi)/(ac^2)$, where a denotes the orbit's semimajor axis, ϕ is the true anomaly, and GM is the product of Earth's mass and the Newtonian gravitational constant. But GPS satellite clocks experience second-order Doppler shifts that contribute half of this frequency shift, whereas LPI violation is related only to the variation in the gravitational potential itself. Therefore the best previous result should be stated as 3.2×10^{-5} instead of 1.6×10^{-5}, as reported in [2]. The experiment reported here reduces this limit to $(0.1 \pm 1.4) \times 10^{-6}$, an improvement by a factor of about 23.

In the work reported here, comparisons of Cs frequencies from four different primary standards laboratories are made with an ensemble of four hydrogen masers at NIST in a manner similar to that of [2,3]. In the next section measurements of the frequency differences at NIST, using data from PTB, LNE-SYRTE (Laboratoire National de Métrologie et d'Essais—Systéme de Références Temps-Espace, Paris, France), and INRIM (Istituto Nazionale di Ricerca Metrologica in Turin, Italy, previously known as Istituto Elettrotecnico Nazionale Galileo Ferraris) are discussed. The data analysis and the combination of results from the four laboratories to obtain the final result are described below.

Data acquisition: NIST Cs fountain and hydrogen maser comparisons.—The four hydrogen masers (designated S2, S3, S4, and S5) used in this study are housed below ground level in environmentally controlled chambers that keep the temperature constant to within 0.1 K and the relative humidity to within 1%; thus frequency fluctuations due to varying environmental conditions have been reduced to a very low level. Most of the masers used here have been characterized for their frequency variations with fluctuations in temperature, humidity, barometric pressure, and magnetic field, and these sensitivities are used to remove any residual environmentally induced frequency fluctuations. The most important contribution to uncertainty, over and above intrinsic clock noise, comes from temperature variations [8]. Nevertheless the correlation of frequency variations of the hydrogen masers with temperature is very small and contributes to the final uncertainty in $|\beta_{\rm H} - \beta_{\rm Cs}|$ only in the third digit.

The NIST F-1 Cs fountain is located in the same building and is compared to one of the masers several times each year. At present 19 separate comparisons spread over 7 yr were used in the analysis. The internal measurement system for generating atomic time at NIST provides a continuous measurement of the relative frequencies of all the masers with a fractional frequency accuracy better than 2×10^{-17} in a 1 d averaging time. Thus the fountain clock comparison to one maser can be related to any of the other masers with essentially no degradation in accuracy. The effects of the environmental parameters on the Cs-H comparisons are discussed below.

Data acquisition: PTB, LNE-SYRTE, and INRIM.—In addition to measurements of the masers relative to NIST-F1, data from other Cs fountain primary frequency standards are available. These standards are all thousands of kilometers away so the comparison accuracy is reduced by frequency transfer instabilities. PTB-CSF1 was formally evaluated 16 times over the interval from August 2000 to July 2006 [9,10]. These data are available in the online publication *Circular T* [11] from the Bureau International des Poids et Mesures (BIPM) and can be related to the masers at NIST through internal measurements and from other data in *Circular T*. In addition to the use of *Circular T* data, with PTB-CSF1 and the INRIM fountain IT-CSF1, many of the formal report intervals of PTB-CSF1 and IT-CSF1 could be related directly to a NIST hydrogen maser via a joint effort involving two-way satellite time and frequency transfer (TWSTFT) [12]. The direct TWSTFT measurements have lower levels of uncertainty than the data from *Circular T* and were used where available. The PTB-CSF1 and IT-CSF1 data were processed in the same manner as the NIST-F1 data. Data from 11 comparisons of the INRIM fountain [13] from October 2003 to July 2006 were used.

LNE-SYRTE operates a Cs fountain (SYRTE-F02) [14] in Paris, and its data are also available from *Circular T*. Data from 11 comparisons of the SYRTE fountain were used for the interval July 2003 to February 2006. In the case of SYRTE-F02, only *Circular T* data were used.

Data analysis.—Our experiment measures the frequency of a hydrogen maser using a Cs fountain primary frequency standard and looks for an annual variation in that frequency with the same phase as the annual variation of the gravitational potential. Aging mechanisms within hydrogen masers cause the maser frequency to drift with time, typically $\Delta f/f \approx 2 \times 10^{-16}/\text{day}$ for the masers used in this experiment. This aging should not be correlated with the solar-potential variation; therefore we describe this aging phenomenologically with a cubic polynomial in the time.

The data for maser S5 are shown in Fig. 1, along with the cubic polynomial fit. [Modified Julian day (MJD) 51500 is 18 November 1999.] The curve represents the frequency of the maser measured by various Cs fountain primary frequency standards at specific times during the 7 yr span of the experiment. The Cs fountain NIST-F1 is colocated with the hydrogen maser, and the data from this fountain span the entire measurement interval. Data from the three remotely situated Cs fountains PTB-CSF1, SYRTE-F02, and IT-CSF1 are also shown in Fig. 1.

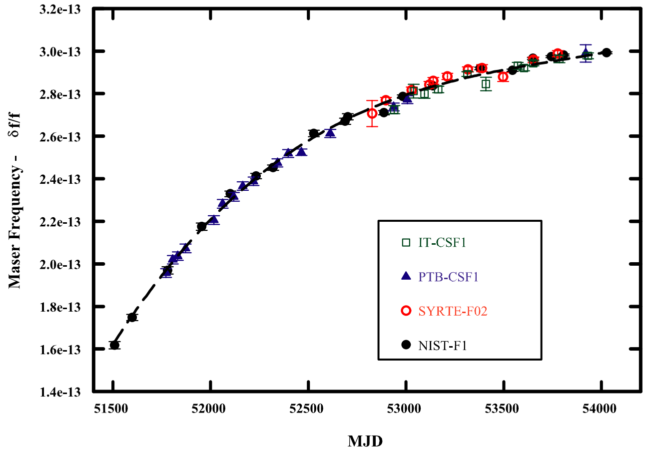

FIG. 1 (color online). Fountain comparisons with maser S5 for 7 yr, showing fractional frequency deviation of the maser from the NIST-F1, PTB-CSF1, SYRTE-FO2, and IT-CSF1 fountains. The horizontal axis is given in modified Julian days (MJD). The error bars represent the evaluated uncertainties of the measurements, including time-transfer errors.

The final uncertainty in this experiment is dominated by unmodeled maser instabilities. The uncertainties of the individual fountains and from the process of transferring information from the remote fountains are small. As a result, all fountains used in this study are weighted equally.

We can identify four distinct possible comparisons involving the four hydrogen masers vs the cesium-fountain primary frequency standards. The data plotted in Fig. 2 show the relative frequency of each of the four masers over the 7 yr of data obtained. Also shown in Fig. 2 is the variation of the solar potential $\Delta\Phi/(5000c^2)$. The factor 5000 was chosen to scale the solar-potential variation so it could be plotted on the same graph, to illustrate the time signature we are looking for.

The data analysis proceeds as follows. First, the masers' outputs are each corrected for any environmental disturbances. These corrections are generally quite small, with a total fractional frequency correction of about 2×10^{-15} being typical. The corrected data are then fit to a cubic polynomial that describes the frequency drift of a particular maser over the data span. The fit is subtracted from the data and the residuals are used in the search for the solar-potential induced frequency variation. The removal of a low-order polynomial from a 7-yr data set does not significantly affect the presence of an annual term in the frequency difference.

The expected signature of the LPI violating term depends on the gravitational potential experienced by the clocks, and therefore on the Earth-Sun (E-S) distance. We calculate the E-S distance using the VSOP-87 planetary ephemeris with an estimated inaccuracy in the E-S distance of 10^{-9} AU [15]. Because a typical measurement of the average maser frequency by a cesium fountain lasts from 20 to 30 days we take the average gravitational potential during the measurement. The frequency differences of the maser as measured by the cesium fountains with the maser drift removed are then fit to the expected signature of the gravitational potential for each data set. The data from Fig. 2 with the polynomials removed are shown in Fig. 3.

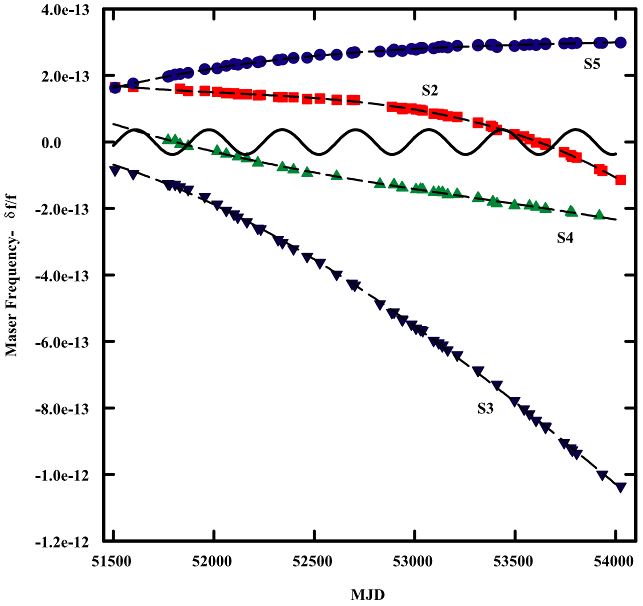

FIG. 2 (color online). The frequency of the hydrogen masers as measured by cesium-fountain standards over the course of this study. The four masers are represented by four different symbols, while the dashed lines are the cubic fits for a particular maser. The variation in the solar potential, divided by $5000 \times c^2$, is illustrated by the sinelike curve.

Many factors influence the frequency of hydrogen masers over long times. Slow ageing mechanisms, such as changes in the wall shift and ageing in electronic components, result in a drift largely removed by the cubic fit. Environmental sensitivities have been addressed to the extent that data is available. However, there are many other factors that are poorly understood and unmodeled. The residuals in Fig. 3 reflect these unmodeled, nonstochastic frequency shifts, which were not removed in this study. Residuals after removing the cubic fit clearly show an unremoved fourth order term. Although these instabilities are not entirely random, a histogram of the residuals is very nearly Gaussian. Thus a standard least-mean-square fit is used, and the standard deviation is a reasonable estimate of the uncertainty.

We fit each set of data in Fig. 3 to the gravitational potential (also shown in Fig. 3), with amplitude being the only free parameter. Upon fitting we obtain an amplitude of the effect and an uncertainty. The four amplitudes are then combined to give the final result,

$$|\beta_{\mathrm{H}} - \beta_{\mathrm{Cs}}| = (0.1 \pm 1.4) \times 10^{-6}, \tag{4}$$

consistent with zero and a factor of 23 improvement over

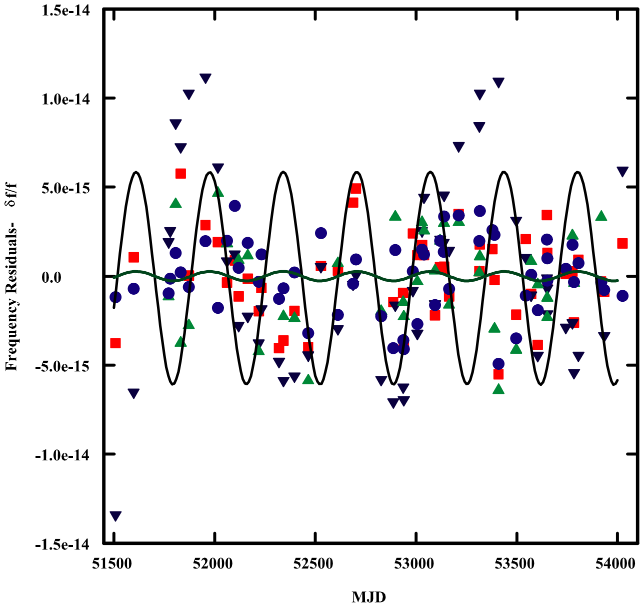

FIG. 3 (color online). Frequencies of the four masers after removal of the cubic polynomial. The large amplitude sinelike curve is the previous maximum solar-potential induced frequency modulation on the hydrogen-cesium system, while the 23 times smaller curve is the limit arrived at in this study. Most of the outliers are associated with maser S3.

the previous result. The smaller sinelike curve in Fig. 3 corresponds to the limit from this study, while the large amplitude sine wave in Fig. 3 corresponds to the limit from the earlier study [2]. Searches for a signal with period near 1 yr, and/or with different phase, also yielded no significant correlation. Similarly when Jupiter's gravitational potential was included in the total potential the result did not change significantly.

Conclusions.—The best previous limit on LPI violation with hydrogen and cesium corresponds to an uncertainty in $|\beta_{\rm H} - \beta_{\rm Cs}|$ of 3.2×10^{-5} [2]. The upper limit on the uncertainty of $|\beta_{\rm H} - \beta_{\rm Cs}|$ obtained in the present experiment is 1.4×10^{-6}, about 23 times smaller than this.

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