

# Spectral Dependence of Phase Noise of Stabilized Optical Frequency Combs

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**Abstract.** We present the spectral dependence of the phase noise between two optical frequency combs that are frequency shifted from each other by their offset frequencies and both stabilized to the same optical reference.

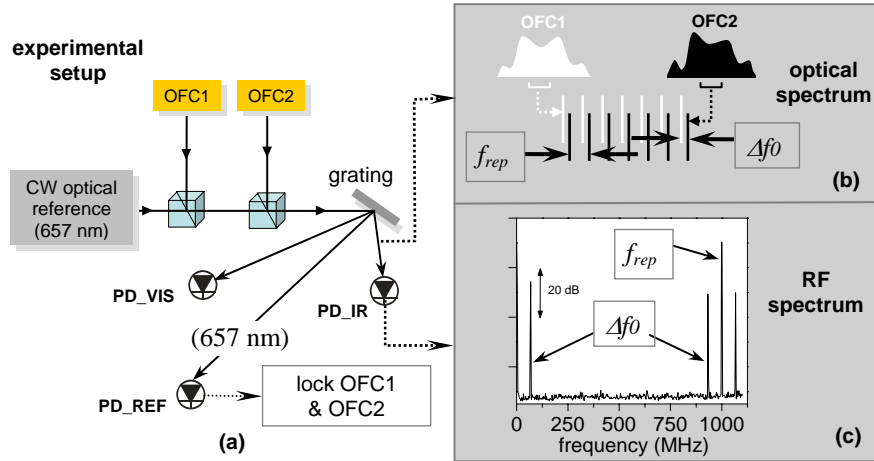
## 1. Introduction

The advent of stabilized optical frequency combs, based on femtosecond mode-locked lasers, has served to replace large scale harmonic frequency chains, thus revolutionizing optical frequency metrology. The frequency comb can be engineered to have excellent frequency stability and indeed, is now currently being integrated into optical-atomic clock schemes. Recent work on the femtosecond comb itself has involved studying the fundamental limiting factors in the linewidth of each optical frequency comb (OFC) mode. Some studies suggest a particular scaling in fluctuations of an OFC mode away from the optical frequency lock point of the comb and the relevant frequency comb mode [1,2]. For long measurement time scales ( $>1$  sec) the optical combs modes scale with the RF equation predicting the optical comb mode position and indeed, can have extremely good frequency reproducibility ( $10^{-19}$  level) [3]. However, for short measurement time scales ( $<1$  sec), relevant say for the fast times of atomic and molecular dynamics, the factors contributing to the linewidth are still uncharacterized. In our work, we study the spectral phase noise dependence away from the lock point by measuring the residual phase noise between two OFCs that are phase locked to the same CW optical reference.

## 2. Phase noise measurement scheme

We use two Titanium:Sapphire modelocked femtosecond lasers having a 1 GHz repetition rate, placed on one optical table and physically separated by several meters and [4]. They are independently pumped with different frequency doubled Nd:YVO<sub>4</sub> lasers and their offset frequencies are independently stabilized [4]. We derive the offset frequencies  $f_{01}$  ( $f_{02}$ ) of each OFC by employing a self-referencing technique where  $2f$  light is optically mixed with  $3f$  light (the bandwidth of the lasers is sufficient to support this method of deriving the offset frequencies and hence, microstructure fiber is not used). The  $f_{01}$  ( $f_{02}$ ) error signal is fed into an acousto-optic modulator placed in the pump beam which is the actuator phase-locking  $f_{01}$  ( $f_{02}$ ). We then

stabilize each OFC to a common CW optical reference laser at 657 nm ( $f_{657}$ ), where the RF beat  $f_{beat1}$  ( $f_{beat2}$ ) between the optical reference and OFC1 (OFC2) is stabilized using a piezo-electric transducer in the femtosecond laser's optical cavity.



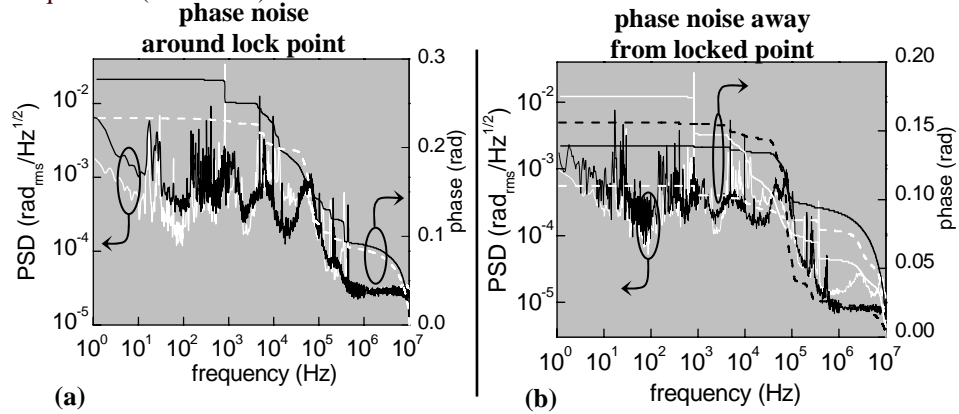
**Fig. 1.** (a) Experimental setup for the measurement of the spectral dependence of the relative phase noise between two independently  $f_0$  stabilized optical frequency combs (OFC) both phase locked to the same CW optical reference at 657 nm. This configuration was chosen to optimize common mode optical paths. (b) Optical spectrum demonstrating that the OFC repetition rates  $f_{rep}$  are set equal and are also given a relative offset from one another of  $\Delta f_0 = f_{01} - f_{02}$ . (c) Characteristic RF spectrum at each photodiode (PD) showing  $\Delta f_0$  and  $f_{rep}$ .

After both  $f_{01}$  ( $f_{02}$ ) and  $f_{beat1}$  ( $f_{beat2}$ ) are phase-locked, a stable phase-coherent beat between the two OFCs will be observed provided that repetition rates are set equal to one another and the pulses emitted by the femtosecond laser arrive synchronously at the photodiode (PD). For OFC1 (OFC2), the laser's repetition rate is given by the frequency comb equation,  $f_{rep1,2} = (f_{657} \pm f_{beat1,2} \pm f_{01,2})/n_{1,2}$ . If we now select the same mode number for each laser,  $n_1 = n_2$  and select the proper phase lock sign, we obtain,  $f_{beat1} + f_{01} = f_{beat2} + f_{02}$ . We can readily select  $f_{beat}$  and  $f_0$  for each frequency comb so as to satisfy this relation. In this case, at each PD we observe heterodyne RF beats (where the fundamental beat is at the frequency  $\Delta f_0 = f_{01} - f_{02}$ ) between the two OFC's when the pulses arrive synchronously at the PD. In our case, instead of building an optical delay line between the two lasers, we correct for the relative pulses delays by temporarily tuning the synthesizer locking  $f_{02}$  to obtain temporal overlap at the PD.

### 3. Results and Discussion

Using the setup shown in Fig. 1 (a), we have compared the noise on the relative offset frequency  $\Delta f_0$  by mixing together  $\Delta f_0$  centered at the spectral range of 631 nm (621 nm) (with PD\_VIS) with 704 nm (716 nm) (with PD\_IR) [Fig. 2(a)]. With this scheme, we can also observe the relative noise between the two OFCs when compared with their lock point at 657 nm in order to observe the scaling of the noise with frequency away from the lock point [Fig. 2 (b)]. For the data shown in Fig. 2, we

observe that in the lowest frequency range ( $<100$  Hz) there is approximately a  $1/f^2$  rise in the phase noise. Additionally, mechanical vibration and amplitude fluctuations are known to be contributing factors to the phase noise profile. We emphasize that the phase noise density observed here is in the milli-rad/Hz $^{1/2}$  level for these optical frequencies ( $\sim 400$  THz).



**Fig. 2.** (a) Phase noise spectral density PSD ( $\text{rad}_{\text{rms}}/\text{Hz}^{1/2}$ ) and integrated phase noise (rad) between opposite sides of the lock point. The black (white) trace is the  $\Delta f_0$  signal centered at 631 nm (621 nm) (from PD\_VIS) electronically mixed with the  $\Delta f_0$  signal centered at 704 nm (721 nm) (from PD\_IR). (b) Integrated phase noise of all the  $\Delta f_0$  specified in (a) when each is electronically mixed with the  $\Delta f_0$  centered at the lock point at 657 nm (from PD\_REF):  $\Delta f_0$  centered at 621 nm (721 nm) is in white dashed (white) and 631 nm (704 nm) in black dashed (black). PSD trace of two signals (for clarity):  $\Delta f_0$  centered at 621 nm (704 nm) in black (white).

## 4. Conclusions

In summary, we have observed the relative phase noise of two optical frequency combs stabilized to the same CW optical reference. We have compared different spectral regions of the frequency combs to determine both the relative phase noise and the scaling of the phase noise with frequency away from the optical lock point of the frequency comb, all done at the milli-radian level. Observing the phase spectral density for these different spectral regions on time scales  $<1$  sec reveals features common to frequency comb sources as well as provides information on the relative fluctuation of the frequency comb from the ideal equation frequency comb equation used to describe each comb mode.

## References

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