Optical and Microwave Frequency Stability: Some Constraints*

L. Hollberg, S.A. Diddams, A. Bartels, J.J. McFerran, E.N. Ivanov, G. Wilpers and C.W. Oates
National Institute of Standards and Technology
325 Broadway, Boulder, CO 80305, hollberg@boulder.nist.gov

Abstract: Optical frequency references achieve the best frequency stability of any oscillators by taking advantage of high $Q = \nu_0/\Delta\nu$ optical resonances. These systems are beginning to run into fundamental and technical limitations which are discussed.

The stability advantage of optical atomic frequency references is well known, and stems primarily from the high operating frequencies ($\nu_0 \sim 500$ THz) and the high Qs that are achievable ($Q = \nu_0/\Delta\nu \sim 10^{14}$), where $\Delta\nu$ is the linewidth).[1] In the quantum-projection-noise limit, the fractional frequency instability (in terms of the Allan deviation) is approximately, $\sigma_y(\tau) = \frac{\delta\nu(\tau)_{rms}}{\nu_0} \approx \frac{\Delta\nu}{\pi\nu_0 C}\sqrt{\frac{T_C}{2N_0\tau}}$, where N_0 is the number of atoms, T_C is the measurement cycle time, C is the resonance contrast, and τ is the averaging time. In addition to the basic atomic stability, which is set by linewidth and signal size, other effects typically limit the frequency stability to levels that are significantly worse than predicted by the simple equation above. In particular, frequency noise on the laser local oscillator (LO) is a major concern. It is limited by several factors, including, optical shot-noise in locking to the cavity resonance, cavity thermal noise, and numerous technical noise sources that perturb the reference cavity frequency and the opto-electronic stabilization system. The deleterious effects of laser frequency noise are amplified by the Dick effect when the atomic clock-transition is probed with a low duty cycle as is typically the case with cold-atom frequency standards.[2] The result is that the frequency stability of the reference can be worse than both the LO and the atomic instability. For now, we ignore the many serious technical limitations to focus on the more fundamental noise sources: quantum projection noise, cavity thermal noise, and optical shot-noise. Numata et al.[3] recently suggested that the stability of the very best optical reference cavities (Bergquist et al.)[4] are actually limited by fundamental thermal-noise to a fractional frequency instability of about 4×10^{-16}. The dominant fundamental noise sources are plotted as an Allan deviation in Figure 1.

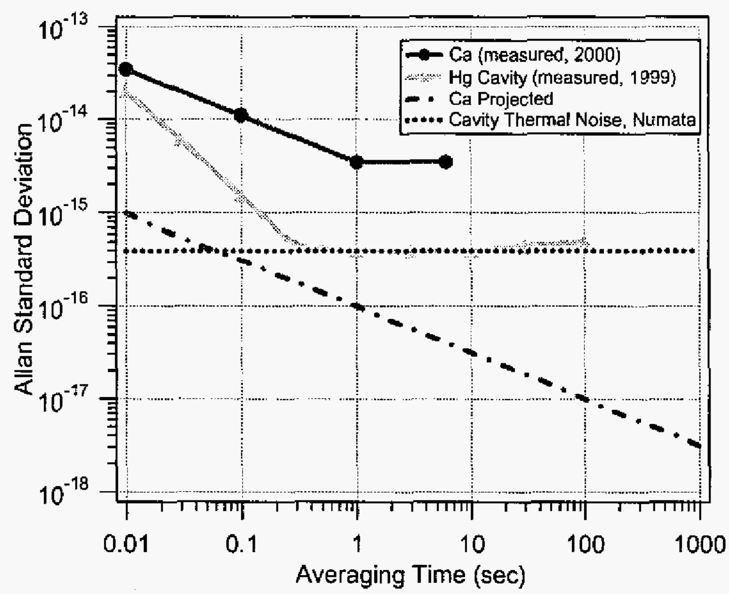

Fig. 1. Fractional frequency instabilities achieved with the Ca optical frequency standard and Hg+ optical reference cavities at NIST.[4,5] Also shown are the thermal noise of the optical cavities as calculated by Numata, and the projected stability for Ca if it was operated with a fast duty cycle .

Stable optical frequency sources can be converted to stable microwave signals (or electrical pulses) by using a "self-referenced" optical frequency comb based on a mode-locked Ti:sapphire laser. Those systems are nearly ideal optical frequency synthesizers[6] that can divide optical frequencies phase-coherently to the microwave domain. However, converting the ultra-short optical pulses to electronic pulses using photodiodes brings with it additional technical and fundamental noise.[7] In this case, photodiodes are usually saturated and nonlinear, and show time delays that are amplitude- and angle-dependent. At the same time, it is necessary to run the photodetectors with high photo-currents to extract large microwave signals and reduce the effects of photo-detection shot-noise.

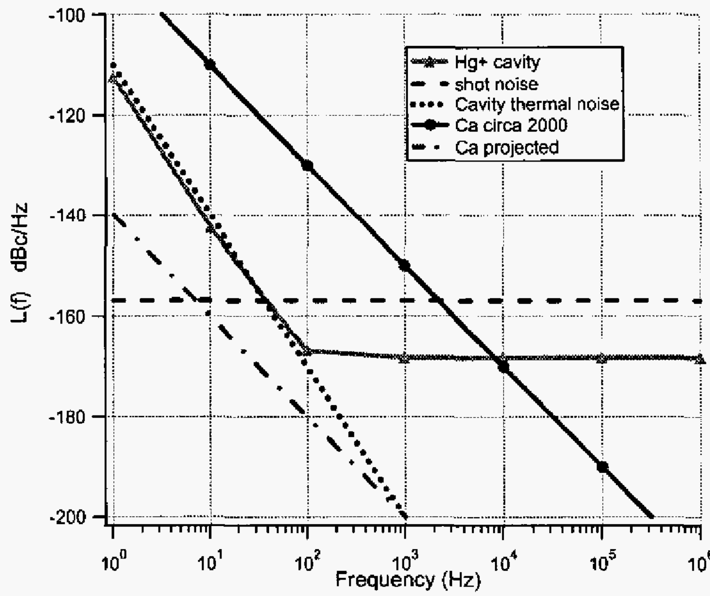

Fig. 2. Converting the noise sources discussed above (and shown in figure 1) into the frequency domain, and assuming perfect frequency division from optical (500 THz) to microwaves, we project the single-sideband phase-noise that might be achievable at 10 GHz. In this plot we have added the horizontal dashed-line that represents the photo detection shot-noise for a few milliamps of photocurrent.

Figure 2 shows how the effects of thermal-noise on the optical reference cavities, and the shot-noise on photo-detection set boundaries on the microwave phase-noise that is achievable. These traces represent very low noise levels (particularly close to the carrier) but some applications would benefit from microwaves with even lower phase-noise. Optical resonances in cold atoms could help push below these boundaries. For example, if a clock laser was locked to a narrow atomic resonance with a fast attach time (eg. millisecond) it should be possible bring the resulting laser stability quickly below the thermal noise floor for optical cavities (see Fig 1). This approach would also avoid problems associated with the Dick effect. The calcium optical-clock transition at 456 THz (657nm) with a 400 Hz natural linewidth might be a particularly good case for this application. It should be possible to operate that system with a ~1 kHz linewidth, a 2 ms cycle time and a few million atoms to achieve a stability similar to that shown as Ca projected in figures 1 and 2. Experiments will need to be done to fully test these ideas.

1. L. Hollberg, et al. IEEE Journal of Quantum Electronics **37,** 1502-1513 (2001).
2. A. Quessada, et al. Journal of Optics B Quantum and Semi-Class. Optics **5,** S150 - S154 (2003).
3. K. Numata, A. Kemery, and J. Camp, Phy.Rev.Letts. **93,** Article 250602 (2004).
4. B. C. Young, et al. Physical Review Letters **82,** 3799-3802 (1999).
5. C. W. Oates et al. Optics Letters **25,** 1603-1605 (2000).
6. L. S. Ma, et al. Science **303,** 1843-1845 (2004).
7. L. Hollberg, IEEE Proceed. Microwave Photonics, Ogunquit, Maine, (2004).