

A HIGH-RESOLUTION AMMONIA-MASER-
SPECTRUM ANALYZER

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Reprinted from IRE TRANSACTIONS
ON INSTRUMENTATION
Volume I-10, Number 1, June, 1961

PRINTED IN THE U.S.A.

A High-Resolution Ammonia-Maser-Spectrum Analyzer*

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INTRODUCTION

THE problem of precise frequency measurement can be understood only with a fairly complete knowledge of the frequency source and the effect of the measuring system. In particular, the problem becomes more difficult if different methods of frequency measurement are used (*e.g.*, counters and atomic resonances) and if it cannot be assumed that the frequency source is pure.

For almost all systems, the problem of amplitude modulation of the measured signal is of no concern since in practice the signal can be regulated quite well. The problem of frequency modulation, however, is quite another matter. In practice a quartz-crystal oscillator is used and it is assumed to be relatively free of modulation.

It can be shown [3] that in general a frequency counter will, on the average, measure the center of gravity of the frequency-modulation power spectrum. A spectral line in an atomic or molecular transition, however, will not in general measure the center of gravity of the power spectrum. Thus, for a meaningful comparison of frequency counter data and atomic or molecular resonance data, it is essential to know the spectral distribution of the signal being measured.

In practice, of course, one attempts to obtain a monochromatic source of radiation for the measurements. The results of the experiments made with the ammonia-maser-spectrum-analyzer system indicate how relatively pure signals in the microwave region can be obtained.

THEORY

In general, there are two methods of precise frequency measurement: 1) determining the total elapsed phase in an interval of time with an apparatus like a synchronous clock or a "frequency counter;" and 2) direct frequency measurement by a resonance method usually involving a molecular or atomic transition.

If the signal to be measured is composed of several sinusoidal components whose frequencies are near the main component, it is not immediately evident what frequency the two methods will "measure."

Since the frequency counter counts the number of times the signal passes through zero in a positive sense for an interval of time, both the synchronous clock and

the frequency counter indicate the total elapsed phase for the period of the measurement. The resonance method, however, must give some sort of an average of all the frequency components present and in general will *not* agree with the elapsed-phase method of measurement. Thus, for a meaningful comparison of these two types of measurement, it is essential that a detailed knowledge of the signal be available.

It is usually sufficient for this "detailed knowledge" to be given in terms of the *power spectrum*. The power spectrum, or spectral density, of a signal can be defined in the following way: If $P(\omega)$ is the power spectrum of a signal, then $P(\omega)d\omega$ is the element of power supplied by those frequency components of the signal in the frequency interval from ω to $\omega+d\omega$. Mathematically it can be shown [1] that if $f(t)$ is the signal, then its power spectrum $P(\omega)$ is given by the time average

$$P(\omega) = \lim_{T \rightarrow \infty} \frac{1}{2\pi T} \left| \int_{-T/2}^{T/2} f(t) e^{-i\omega t} dt \right|^2.$$

Thus the power spectrum shows the relative energy distribution for the various frequency components and can therefore be used as a measure of the purity of the signal.

For most precise atomic or molecular resonance measurements, a quartz-crystal oscillator at a frequency of a few megacycles is used as the source, either to excite an atomic transition or to compare with an atomic transition. This signal is multiplied in frequency by harmonic multipliers to the atomic or molecular resonance frequency which is usually in the microwave region. This multiplication process has a marked effect on the power spectrum of the signal.

Normally limiters are used throughout the electronic system so that amplitude modulation is not a problem. However, frequency modulation cannot so easily be removed. In fact, it can easily be shown that if a small amount of frequency jitter or modulation is present in the oscillator, it will be *enhanced* by the factor of multiplication.

As an example of the effect of frequency multiplication on an FM signal, consider just one stage of multiplication. Assume that the current $I(t)$ in the output tank of the multiplier is related to the input voltage $V(t)$ by the transfer function $g(V)$, which is a function of the input voltage; *i.e.*,

$$I(t) = g(V(t))V(t). \quad (1)$$

* Received by the PGI, June 13, 1960.

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If the input signal is of the form

$$V(t) = V_0 \cos \phi(t), \quad (2)$$

where $\phi(t)$ is some function of time, then the current becomes

$$I = g(V_0 \cos \phi) V_0 \cos \phi.$$

Since $\cos \phi$ is an even function of ϕ , $g(V_0 \cos \phi)$ is also an even function of ϕ , and therefore I is an even function of ϕ . Therefore I can be expanded as a Fourier cosine series in ϕ ; *i.e.*,

$$I = \sum_{n=0}^{\infty} a_n \cos n\phi. \quad (3)$$

To restrict the case to a simple FM wave, let

$$\phi(t) = \omega_0 t + \delta \sin \omega_m t, \quad (4)$$

where ω_0 is the carrier frequency, ω_m is the modulating frequency, and δ is the modulation index. Substitution of (4) into (3) yields

$$I(t) = a_0 + a_1 \cos(\omega_0 t + \delta \sin \omega_m t) + \dots \\ + a_N \cos(N\omega_0 t + N\delta \sin \omega_m t) + \dots$$

If the impedance $Z(\omega)$ of the output tank is sufficiently peaked about $\omega = N\omega_0$, but broader than $2N\delta\omega_m$, the output voltage $V'(t)$ is given approximately by

$$V'(t) \simeq a_N Z(N\omega_0) \cos(N\omega_0 t + N\delta \sin \omega_m t). \quad (5)$$

Typically, ω_m is very much smaller than ω_0 , and the condition that the bandwidth of the output tank be greater than $2N\delta\omega_m$ is easily satisfied. The condition that $Z(\omega)$ be sharp enough to reject $(N-1)\omega_0$ and $(N+1)\omega_0$ usually requires N to be less than 10.

Eq. (5) shows that the modulation index is multiplied by the factor of frequency multiplication but the frequency of modulation is unchanged. Extensive use is made of this fact in FM transmitters [6].

Fig. 1(a) shows the square root of the power spectrum ($\sqrt{P(\omega)}$) of a signal, while Fig. 1(b) and (c) show the same signal after being multiplied in frequency by 3 and 9, respectively.

To return now to the problem of frequency measurement, it can be shown [3] that a frequency counter or synchronous clock measures the center of gravity of the FM power spectrum. An atomic or molecular resonance device such as a cesium beam with Ramsey-type excitation will measure the center of gravity of the power spectrum *only* if the excitation and atomic spectra are both symmetric. (To see that power spectra can, in fact, be quite asymmetric, see Fig. 5.)

It is obvious that a spectrally "pure" signal is the most desirable source for precise frequency measurement. The experiments with the maser-spectrum analyzer indicate some means of obtaining relatively pure signals in the microwave region (see Figs. 2-8, pp. 5-7).

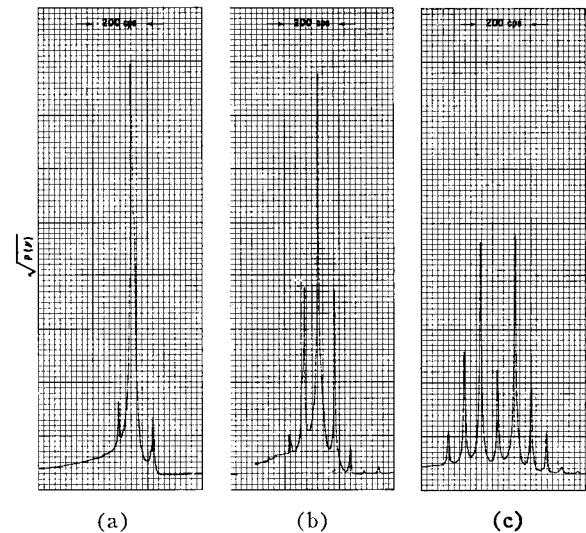


Fig. 1—Effects of frequency multiplication on a frequency-modulated signal. Fig. 1(a) shows the square root of the power spectrum for a 3.3-Mc signal. Fig. 1(b) and (c) show the same signal after being multiplied in frequency by factors of 3 and 9, respectively.

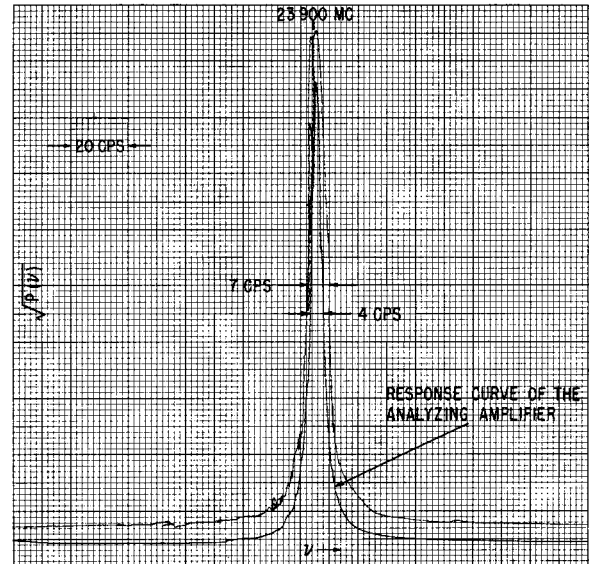


Fig. 2—Spectrum of free-running oscillator used in the spectrum-analyzer system. This spectrum was obtained by analyzing the direct beat note between the *free-running* oscillator-multiplier-chain system and the ammonia maser. It is this oscillator which is phase locked to the maser to give a relatively pure signal to be used in the analyzing of other oscillators. The response curve of the analyzer was obtained by replacing the maser-oscillator beat note by the signal from a high-quality signal generator.

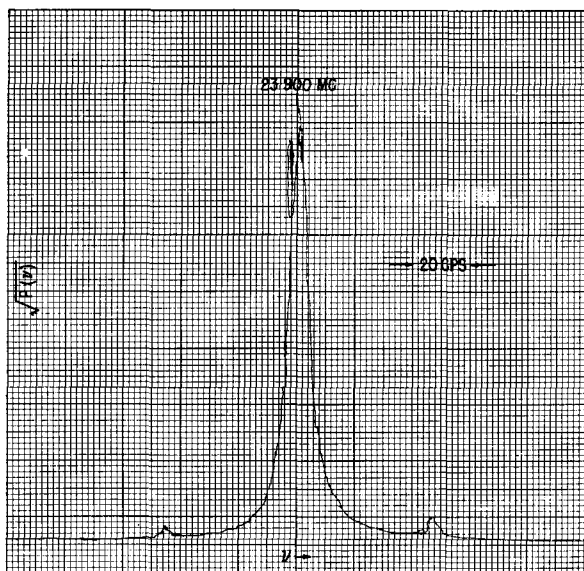


Fig. 3—This spectrum was obtained by intentionally frequency modulating the oscillator of Fig. 2 at a 50-cps rate. The total swing in frequency of the oscillator was about 2 parts in 10^{10} peak to peak. (Note the small sidebands at 50 cps on either side of the central peak.)

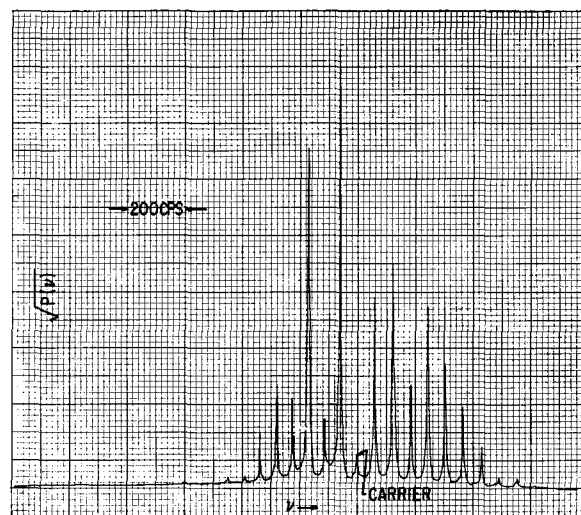


Fig. 5—Asymmetric spectrum of oscillator with ac filaments. This spectrum was obtained from the same oscillator as Fig. 4. At the time this trace was made, however, the oscillator was equipped with 60-cps ac filaments. (Note the different frequency scale.)

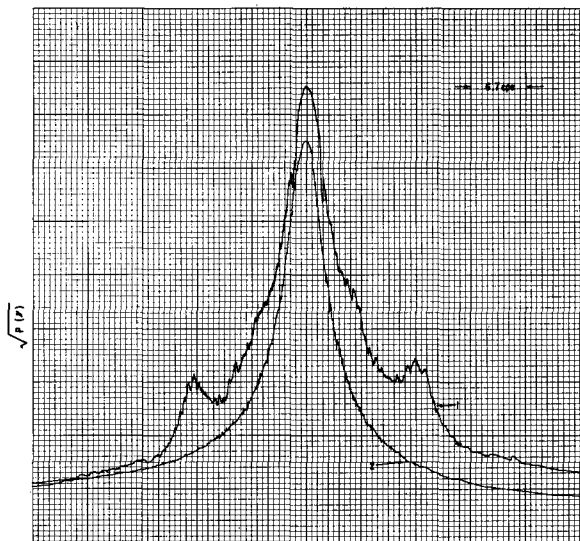


Fig. 4—High-resolution spectrum of liquid helium cooled quartz crystal oscillator. Trace 1 is a high-resolution spectrum of the central peak of a 10-Mc quartz-crystal oscillator whose crystal was thermostated in a liquid helium cryostat [4]. The oscillator was equipped with dc filaments but still exhibited 60-cps sidebands about 30 db below the central peak (not shown in this Figure). This oscillator operates at about 13.4 cps above 10 Mc and apparently some of the pickup of the standard is responsible for the sidebands shown in this trace. Trace 2 is the response curve of the spectrum analyzer.

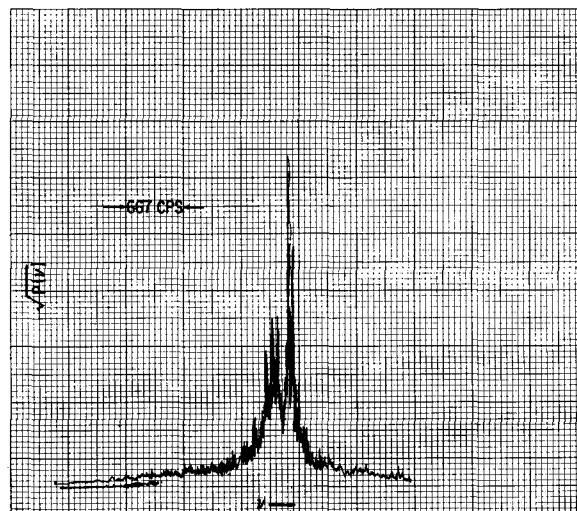


Fig. 6—This spectrum was made from a 100-kc Essen Ring oscillator multiplied in frequency by 145,800. This was the best spectrum ever observed for a 100-kc oscillator multiplied by this factor; however, this spectrum was not consistent, and would have markedly different shape from day to day. (Note the scale.)

EXPERIMENT

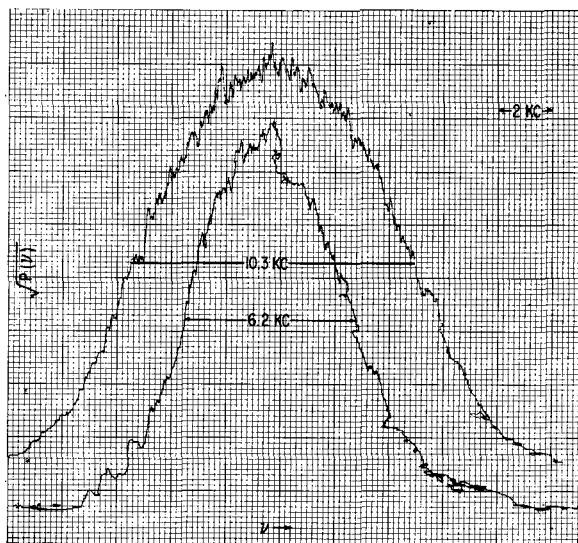


Fig. 7—Effects of warm-up time of multiplier chain on a 100-kc signal. This is a spectrum of a 100-kc oscillator, again multiplied 145,800 in frequency, located two floors above the room containing the maser-spectrum-analyzer system. The signal apparently picked up noise in the long cables connecting the two rooms. The upper trace was taken approximately one hour after the multiplier chain in the spectrum analyzer was first turned on, and the lower trace about 6 hours after the chain was turned on, showing the effect of warm-up time of a multiplier chain on a noisy signal. (Note the scale.)

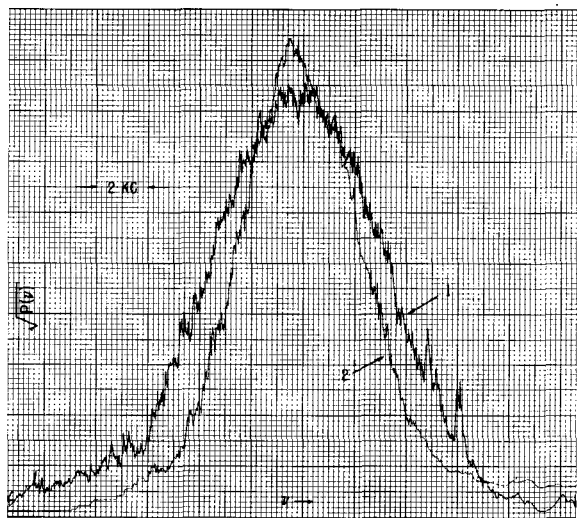


Fig. 8—Effect of buffer amplifier on a noisy 100-kc signal. Again these are the square roots of the power spectra for a 100-kc oscillator multiplied in frequency 145,800 times. Trace 1 employed a buffer amplifier of unit gain between the oscillator and first stage of multiplication, while trace 2 did not use this amplifier.

Since it is desired to resolve sidebands of a microwave signal which may be only a few parts in 10^{10} apart, it is essential to use a superheterodyne type of system such as that shown in Fig. 9 (next page). By heterodyning the measured signal with a relatively pure signal, the power spectrum is displaced (ideally without distortion) to lower frequencies (~ 79 kc) where filters of very narrow bandwidth (~ 4 cps) can be used.

The first problem is then to obtain a very "pure" signal to heterodyne with the measured signal.

The ammonia beam maser has been recognized as one of the most nearly monochromatic sources of radiation in the microwave region. In fact, the beat note between the two National Bureau of Standards masers demonstrates a bandwidth of somewhat less than about 0.5 cps at 23,870 Mc for a period of several minutes.

Since the maser puts out only about 10^{-9} watts of power and at the rather inconvenient frequency of 23,870 Mc, a crystal-oscillator-multiplier-chain system was phase locked to the maser. As mentioned previously, amplitude modulation can easily be avoided by the use of limiters throughout the multiplier chains. Thus the frequency jitter or modulation of the oscillator is the only problem. By phase locking the crystal oscillator to the maser, nearly the spectral purity of the maser should be imparted to the crystal oscillator.

The oscillator to be tested is multiplied to 270 Mc and 5 Mc and then heterodyned with the 265.551-Mc signal from the maser-stabilized oscillator-multiplier chain. (See Fig. 9.) The frequency multipliers throughout the apparatus are of conventional design operated from dc filament supplies to avoid modulating the signal in the system itself. (The effects of 60-cps filaments in the early stages of multiplication are quite severe.)

The action of this mixer is first to mix the two signals from the test oscillator at 270.0 Mc and 5.0 Mc to obtain the 265.0-Mc beat, and then to mix this signal with the "pure" 265.551-Mc signal from the maser-stabilized oscillator to give a 0.551-Mc beat. This displaces the power spectrum of the 265.0-Mc signal to 0.551 Mc. The fact that heterodyning an FM signal with a pure signal displaces the power spectrum to lower frequencies without distortion is often made use of in FM transmitters and receivers [6].

The 0.551-Mc output of the mixer was then sent to another multiplier with outputs at 3.3, 9.9, 29.7, and 89.3 Mc. The spectra shown in Figs. 4–8 were made by sending the 29.7-Mc signal to the spectrum analyzer shown in Figs. 10 and 11 (next page).

The spectrum analyzer is of rather conventional design. The 29.7-Mc signal to be analyzed is first mixed with a signal from a crystal-controlled frequency synthesizer and then passed through a 79-kc crystal filter whose bandwidth is about 4 cps. Thus by slowly sweep-

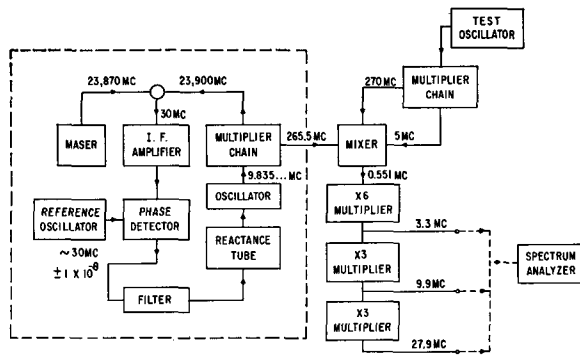


Fig. 9—Block diagram of ammonia-maser-spectrum-analyzer system.

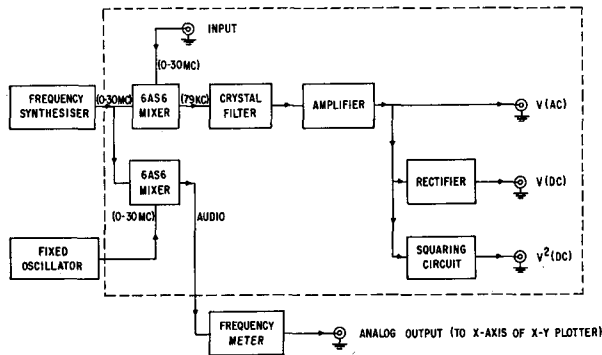


Fig. 10—Block diagram of spectrum analyzer.

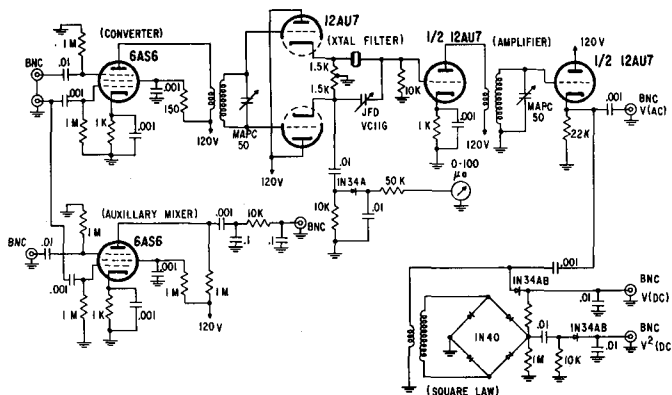


Fig. 11—Circuit diagram of spectrum analyzer.

ing the frequency of the synthesizer, the power spectrum was swept past the analyzing filter.

By separately mixing the signal from the frequency synthesizer with a fixed crystal-controlled oscillator, an analog voltage to the frequency was obtained to drive the x -axis of an x - y plotter. The y -axis was driven by the rectified output of the 79-kc filter. Thus the spectra shown are actually the square root of the power spectra, since the ordinate is nearly linear with voltage instead of power.

CONCLUSION

From the spectra shown, one is led to the conclusion that the most important things in obtaining a relatively pure signal in the microwave region are to keep the

electronics simple, and use dc filaments in the oscillator and early stages of multiplication. Probably the best oscillator would be a system involving a "master and a slave"; that is, a simple one- or two-tube crystal oscillator to drive the chain which is loosely phase locked to a more elaborate crystal oscillator which has good long-term stability. With this system, the amount of electronics affecting the signal directly is kept at a minimum without a sacrifice of stability [5].

The ammonia-maser-spectrum analyzer has proved to be a valuable instrument in the investigation of oscillator and multiplier-chain stability, and noise. In this respect, it is interesting to note an unusual effect which was observed in one of the crystal oscillators at the National Bureau of Standards.

The oscillator was a 5-Mc quartz crystal oscillator with the crystal thermostated in a liquid nitrogen bath. The temperature was maintained constant by regulating the pressure over the liquid nitrogen with a manostat. The oscillator was run on regulated dc filaments and with a well regulated B^+ supply. It was observed that the signal was slightly frequency modulated at about a 10-cps rate. Upon removing the manostat from the liquid nitrogen thermostat so that the temperature of the liquid nitrogen was determined by atmospheric pressure, it was observed that the 10-cps sidebands on the signal slowly reduced in amplitude. After several measurements were made, and the sidebands were observed, first slowly increasing after connecting the manostat and then decaying after disconnecting the manostat, a time constant of about 7 minutes for the build-up of the modulation was determined.

It should be mentioned that the connection of the manostat was virtually instantaneous (compared to 7 minutes). The probable explanation of this is a mechanical vibration of the crystal mount which is excited by the fluctuations in pressure.

In regard to precise frequency measurements, knowledge of the power spectra of the oscillators used to excite the two National Bureau of Standards cesium beam atomic standards has given added reliability to the measurements made with these cesium beams, as well as indicating when a signal is not suited to this form of measurement.

REFERENCES

- [1] See, for example, R. B. Blackman and J. W. Tukey, "The Measurement of Power Spectra," Dover Publications, Inc., New York, N. Y., pp. 84-88; 1958.
- [2] For an account of frequency modulation see F. E. Terman, *et al.*, "Electronic and Radio Engineering," McGraw-Hill Book Co., Inc., New York, N. Y., pp. 786-792; 1955.
- [3] J. Barnes and R. Mockler, "The power spectrum and its importance in precise frequency measurements," IRE TRANS. ON INSTRUMENTATION, vol. I-9, pp. 149-155; September, 1960.
- [4] This oscillator was designed and constructed by A. H. Morgan and his group at the National Bureau of Standards. The quartz crystal was made at the Bell Telephone Laboratories.
- [5] This method is presently used quite successfully with the National Bureau of Standards atomic frequency standards.
- [6] W. L. Everitt, "Frequency modulation," *Trans. AIEE*, vol. 59, p. 613; November, 1940.