Generation of Microwaves with Ultra-low Phase-Noise from an Optical Clock*

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Abstract -**Frequency stabilized lasers have significant advantages for generating microwave signals with unprecedented stability and ultralow phase-noise. We report the optical generation of 10 GHz microwave signals with phase-noise close to the carrier that is > 40 dB better than achieved with state-of-the-art microwave sources and high quality microwave synthesizers. The method uses stable lasers locked to optical resonators that have extremely high Q's and the frequency can be steered in the longer term to atomic resonances. An optical frequency divider is implemented with a femtosecond laser based optical frequency comb, which coherently divides the optical frequency to the microwave range.**

Optical Fabry-Perot cavities can have resonance Q's = $(f_{opt}/\Delta f) \geq 10^{11}$ which is much higher than is achieved with microwave resonators. Since the fractional frequency instability should scale roughly as the inverse of the Q, and the inverse of the power in the resonator, these optical cavities can provide exceptional frequency stability. This is especially true if the resonator is fabricated with low-expansion material (eg ULE), is evacuated to eliminate index of refraction effects, and isolated from environmental perturbations. This type of optical resonator is commonly used for laser frequency stabilization, and can produce lasers with low phase fluctuations and very narrow linewidths ($\Delta f_{laser} = 0.2$ Hz at 563 THz) [1]. For better long term stability and accuracy, the frequency of the cavity stabilized laser can be steered to a narrow atomic resonance. The appropriate cross-over time from cavity- to atom-control will depend strongly on the particulars of the cavity, the atoms and the environment. For present state-of-the-art systems this crossover time might be in the range of 1 to 10 seconds.

With the development of "self-referenced" femtosecond laser optical frequency combs (FLFC) generated by ultrafast mode-locked lasers we now have a convenient tool to divide optical frequencies down to the microwave range[2-6]. If the division process is phase-coherent and low noise we can take advantage of the high Q optical cavities and atomic resonances to produce stable microwaves. By self-referencing the mode-locked laser and phase-locking it to a stable optical frequency reference the resulting optical "clock" divides the optical frequency reference down(~ 500 THz) to produce an optical pulse train at the repetition rate (about 1 GHz) of the mode locked laser. In the appropriate configuration the resulting microwave signal is phase-coherent with the optical reference, but at a frequency $f_{rep} = f_{optical}/N$ where N is a large integer ($N \approx 5\times10^{5}$). Within measurement uncertainties, several recent experiments have now shown that these FLFC are nearly ideal frequency synthesizers and frequency dividers; and at least in the time average, and have shown frequency reproducibly approaching 10^{-19} fractionally [7-9]. Coherent frequency division preserves the fractional frequency instability, and reduces the phase fluctuations by the division factor $1/N=(\Delta\phi_{microwave}/\Delta\phi_{opt})$. In an ideal frequency divider the spectral density of the phase fluctuations at the lower frequency is reduced from the phase noise at the higher frequency as $L_{\varphi}^{rep} = L_{\varphi}^{opt}/N^{2}$ or $L_{\varphi}^{rep} = L_{\varphi}^{opt} - 20\log(N))$ in dB units, where L_{ϕ} is the single-sided phase noise spectral density. In our case, this means that the laser at 563 THz with a linewidth of 0.2 Hz (and assuming white FM noise) would correspond to a phase-noise spectral density at the divided down frequency of 10 GHz of $L(f) \approx [-110\ (1/f^{2})]$ dBc/Hz .

For comparison, the lowest phase-noise microwave sources today typically rely on some sort of low noise oscillator such as a YIG, DRO or microwave cavity stabilized oscillator that provides low noise at higher offset frequencies (f > 100 Hz) and which is locked to a quartz crystal oscillator to improve the phase-noise close to the carrier. The best phase-noise close to the carrier for a 10 GHz microwave source might come from perfect multiplication of a low phase-noise 5 MHz quartz crystal oscillator resulting in $L(f) \approx [-65\ (1/f^{2})]$ dBc/Hz near the carrier at 10 GHz. Optical sources based on high-Q references have clear performance advantages, and the potential to improve microwave phase-noise close to the carrier by 55 dB or more compared to quartz oscillator based sources.

Photo-detection of the optical pulses from the mode locked laser generates a repetitive train of current pulses and produces a microwave frequency comb in direct analogy with the optical frequency comb, but translated down in frequency by 1/N. The microwave frequency comb spans from the pulse repetition frequency to the bandwidth limit of the photodetector, typically tens of GHz for high speed detectors. We have studied some of the spectral characteristics of microwave frequency combs generated by stable femtosecond lasers and found their properties to be as expected, at least to first-order. However, the photodectection process does introduce some additional limitations in terms of microwave power, linearity and noise. In the present work we focus on the frequency stability and phase-noise of the 10 GHz signals extracted at the 10th harmonic of the 1 GHz repetition rate. A simplified experimental diagram is given in figure 1.

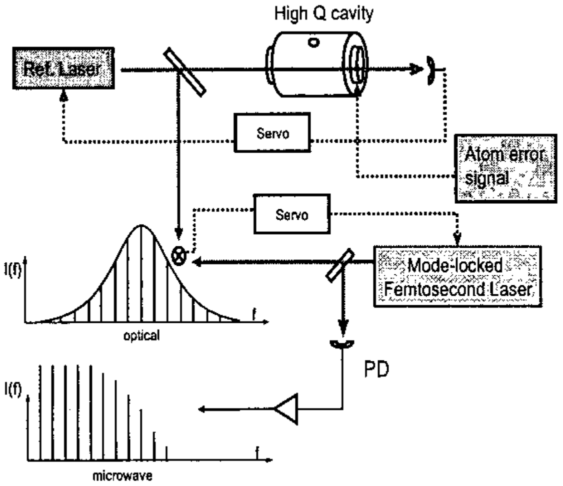

Fig. 1. An optical frequency divider/synthesizer that generates a stable microwave frequency comb starting from a stable optical frequency reference. The high-Q Fabry Perot cavity provides short-term frequency stability to the cw laser which can also be steered to an optical atomic reference on longer time scales. The optical frequency is then divided down coherently to the microwave range by a FLFC based on a mode-locked laser. The optical pulses detected in the photodiode PD produce an electrical pulse train whose spectral distribution is a comb of microwave frequencies spaced by the 1 GHz repetition rate of the mode-locked laser.

By configuring these systems in different ways (see Fig. 2) we have been able to test the components and identify several of the performance limiting factors in both the optical and microwave domains [9].

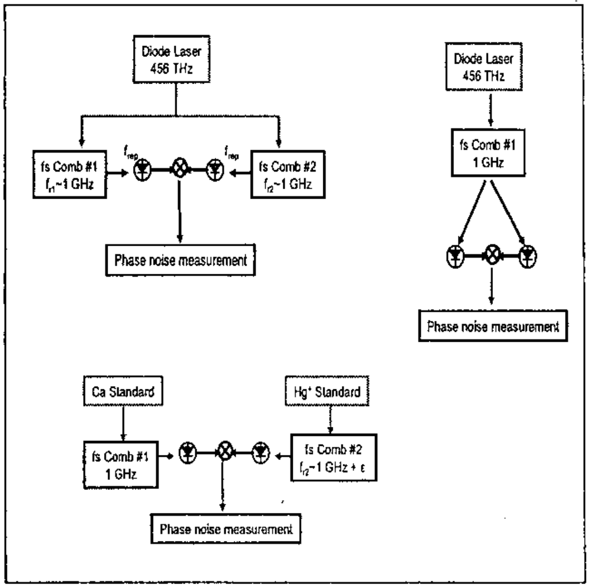

Fig. 2 Some configurations for measuring the phase-noise of the microwave signals divided down from optical frequency references by FLFCs.

Using one optical frequency reference and two FLFCs allows us to test the fidelity of the frequency division process from optical to microwave frequencies and most of the experiments described here were done in this configuration. Using one optical reference and one comb tests primarily the performance of the photodetectors and electronics, while using two different optical references with two optical combs combined with independent electronics gives information on the absolute stability, reproducibility and frequency accuracy. In figure 3 we show the microwave phase-noise from two independent optical frequency dividers when locked to a single stable optical reference. These results represent the residual phase-noise on the 10 GHz microwave signals and does not (to first order) contain the phase-noise of the optical reference. However, we have independently verified that the optical references cavities can have frequency stability that corresponds to phase-noise that is well below this level, as plotted in the lower curve of Fig. 3, labeled "Hg+ cavity" [1].

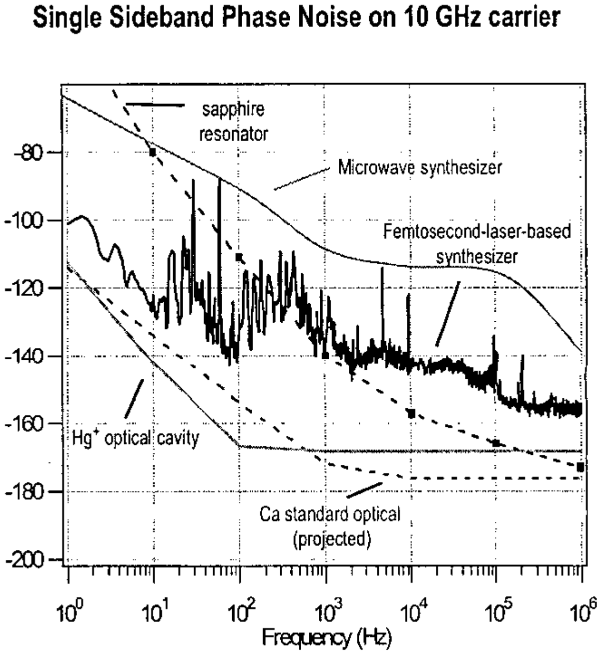

Fig. 3. Single-sided phase-noise spectral density (in units of dBc/Hz) at 10 GHz produced by the divided down optical frequency reference and compared to other state of the art microwave sources. This data was taken using one optical frequency reference and two independent optical frequency dividers based on FLFC. This data represents the residual phase-noise from the optical division process combined with phase-noise due to the optical to microwave conversion. The curve labeled microwave synthesizer represents the phase-noise of a high-quality microwave synthesizer that uses a state-of-the-art quartz crystal reference. The dotted curve labeled "sapphire resonator" corresponds to the phase-noise from an oscillator based on a whispering-gallery-mode sapphire resonator [11]. The "Hg+ optical cavity" curve is the phase-noise equivalent of the frequency stability demonstrated by two independent optical cavities [1]. The dotted curve labeled "Ca optical projected" is the estimated performance that should be achievable with a cold Ca optical frequency standard [10, 12].

On longer time scales than is relevant for the present experiments the cavities can be steered to optical atomic resonances to suppress the drift of the optical cavities (the two optical cavities used for the results presented here have drift rates of about 0.2 Hz/s and 10 Hz/s at the 500 THz optical frequency) [10]. Our results show that the cw lasers and optical cavities do provide exceptional frequency stability for short times ($\tau <$ a few seconds) and that it can be divided down to microwave frequencies with high fidelity. Environmental perturbations do affect the whole system at some level and thermal and optical path length changes can dominate the phase-noise in the 1 to 100 Hz range. Acoustical vibrations and electronic pickup introduce many bright lines into the amplitude and phase-noise spectra in the range form 20 Hz to 2 kHz, but are not a fundamental limitation. In some experiments microwave amplifiers were used after the photodetectors and they could limit the phase-noise floor in the intermediate frequency ranges. Amplifier phase-noise is an issue because of the limited microwave power extracted from the photodetectors (in the present experiments about -15 to -20 dBm was available at 10 GHz). With higher microwave power from the detectors the amplifiers could be eliminated. However, we've recently demonstrated, that interferometric measurement systems [13] are capable of making phase-noise measurements with the sensitivity approaching the standard thermal noise limit regardless on the signal power.

One of the more challenging issues is the performance of the photodiodes combined with the laser amplitude noise (AM) and pointing instabilities. In particular, the photodetectors limit the performance due to saturation, pulse distortion and the limited microwave power that they can deliver [14, 15]. Most of the present work was done with commercially available GaAs or InGaAs detectors that were not optimal for detecting 800 nm light from the mode-locked Ti:sapphire lasers. The detector problem is aggravated by significant cross coupling of the laser AM noise to the microwave phase. A typical AM to FM conversion factor for the photodetectors that are using is a few picoseconds of phase-delay for a change in average photocurrent of 1 mA. Unfortunately there is a mismatch in wavelength between the most stable femtosecond lasers and the best microwave photodetectors. The highest stability mode-locked lasers are Ti:sapphire lasers that operate around 800 nm, whereas the best microwave photodectors seem to be InGaAs devices optimized for the 1300-1600 nm range. These experiments would benefit from high power microwave photodetectors that perform well at 700-900 nm.

To reach the phase-noise performance provided by the optical frequency references the phase-noise of the free-running fs-laser must be strongly suppressed by the servo gain of the two phase-lock loops (the f_0 lock, and the lock to the optical reference). As performance improves this may not be trivial, especially at the relatively high Fourier frequencies (above 1 kHz), due to the use of the PZT transducers used to phase-lock the femtosecond laser to the optical frequency reference. For the highest performance systems we must consider all possible sources of excess phase noise. We will briefly consider just one of those components here.

The optical frequencies of the modes of the FLFC can be represented by the simple comb equation $f_{opt}(m) = mf_{rep} + f_0$, where f_{rep} is the pulse repetition frequency (~ 1 GHz) and f_0 is a common offset frequency ($f_0 \le f_{rep}/2$) for all modes when the comb is extrapolated back to zero frequency. The optical frequencies are thus known in terms of two RF frequencies and the large integer index m

By making use of a digital phase detector we measured the fluctuations in the offset frequency f_0. The results of those measurements are shown in Fig. 4. Here, the upper and lower traces correspond to the free-running and self-referenced laser, respectively. Assuming that the offset frequency fluctuations were the only noise mechanism influencing the frequency of the extracted microwave signal, its phase-noise spectrum can be evaluated from (1) using m = 45600. The phase-noise spectrum converted to the 10 GHz signal (at $10f_{rep}$) would then be at a level of about -160 dBc/Hz at 1 Hz offset.

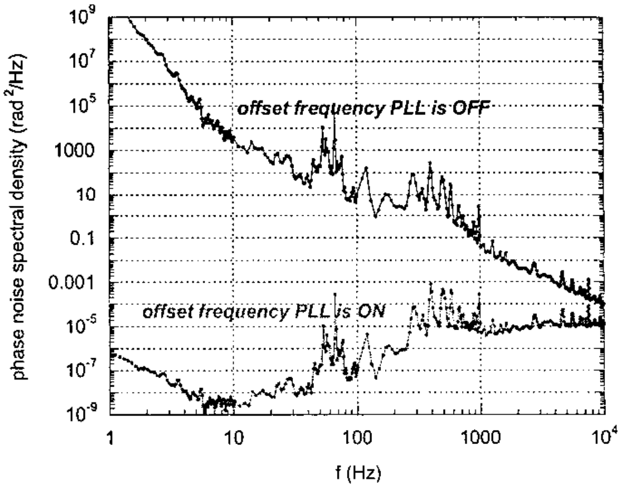

Fig.4 Phase-noise spectral density of the offset frequency (f_0 ~ 50 MHz) of the free running femtosecond mode-locked laser and when f_0 is phase-locked in the self-referenced configuration.

In principle, using cold atoms with even narrower optical transitions could provide some orders of magnitude improvement on the projected performance. For instance, assuming everything else works perfectly, in the quantum-projection-noise limit 10^6 cold atoms with a 1 Hz transition linewidth at 500 THz could give frequency stability sufficient to support a microwave phase-noise at 10 GHz of $L(f) = [-180\ (1/f^2)]$ dBc/Hz. This projected performance is well beyond the present reality, and other limitations both fundamental (such shot noise and thermal noise) and technical issues such as imperfect frequency division will likely prove to be more significant limitations than the atom limited stability.

Summary

We have demonstrated optical frequency dividers that generate microwave signals with much lower phase-noise close to the carrier than has been achieved with traditional microwave sources. For a 10 GHz signal and Fourier offset frequencies ranging from about 1 to 300 Hz the improvement in phase-noise is by about 40 dB compared to commercially available microwave oscillators or other published results. Over the entire spectral range from 1 Hz to 1 MHz the phase-noise from the optical frequency divider is 20 to 50 dB better than high quality microwave synthesizers.

Several applications could benefit from low phase-noise microwave and optical sources, including precise timing, radar, lidar, advanced clocks, and sensors. In the time domain, low phase-noise gives better timing resolution and reduced jitter. For example, the experimental microwave phase-noise plotted in fig.3 gives an equivalent timing jitter of only 1.1 fs when integrated from 1Hz to 1 MHz. A nice feature of these high repetition rate mode-locked lasers for microwave generation is that one automatically gets a comb of microwave frequencies as the output. The signals are separated by the pulse repetition rate (eg 1GHz) and the microwave comb extends out to the bandwidth of the photodetector (50+ GHz detectors now commercially available). In addition to low phase-noise and precise timing these microwave combs could prove useful in microwave synthesis and in time domain generation of well controlled ps electrical signals.

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