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**A PRACTICAL LIMITATION TO THE LENGTH
OF AN ATOMIC BEAM MACHINE**

by

R. C. Mockler, R. E. Beehler and J. A. Barnes



**U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS
BOULDER LABORATORIES
Boulder, Colorado**

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ABSTRACT

A logical way of increasing the precision and accuracy of an atomic beam machine is to increase its length with a corresponding increase in beam intensity. However, practical problems arise as the transit time of the atoms of the beam between the two oscillating fields is increased. The Ramsey pattern will tend to be smeared out as a result of the frequency instability of the excitation. A quantitative estimate of this effect has been made using short time frequency stability data obtained from a reasonably good crystal oscillator. The data was obtained by a comparison of a crystal oscillator - frequency multiplier chain with an ammonia Maser. The results indicate that, at least for the crystal oscillator tested, the decrease in amplitude of the Ramsey pattern can become serious for oscillator field separations of 20 feet or more (for a cesium beam). In order to avoid this problem of short period frequency instability, it is suggested to stabilize the excitation chain of the beam apparatus with an ammonia maser for very long beams.

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1. INTRODUCTION

The rapid development of techniques for the precise measurement of time and frequency have brought a number of significant physical experiments within the realm of possibilities. The most popular of these is an experimental test of Einstein's general theory of relativity. The performance of this experiment requires a precision of measurement 1 to 3 orders of magnitude better than the precision provided by present day atomic clocks^{*}, depending upon the circumstances of the experiment. It seems appropriate to increase the precision of frequency and time measurements for a "red shift" experiment and for other experiments - also important - in astronomy, geophysics, perhaps in atomic and nuclear structure, and for space vehicle guidance systems.

The most highly developed atomic clocks at the present time are atomic beam devices. A logical way of increasing the accuracy of such a device would be simply to increase the separation of the two oscillating fields^{**} with a corresponding increase in the atomic beam intensity. To be sure, this should yield a more precise machine - but with limitations. Practical problems arise as the transit time of the atoms of the beam between the two oscillating fields is increased. Greater demands are made upon the frequency stability of the exciting radiation. Eventually, when the transit time becomes large, the Ramsey pattern [1] will seriously decrease in amplitude - depending upon the degree of instability of the excitation^{***}. This smearing out of the Ramsey pattern cannot be

* Presently existing atomic clocks have a precision and accuracy approaching 1×10^{-10} .

** We consider here Ramsey type excitation of the atomic transition where the atomic beam passes through two separated oscillating fields.

*** The decrease in intensity of which we speak here is not due to the inverse square relationship of intensity. We consider that the detector current remains fixed as the beam is lengthened by whatever means suitable.

completely avoided. It might first be supposed that a correction signal could be derived from the atomic transition and - by means of a servo system - eliminate the fluctuations adequately. This technique becomes less useful as transit times become large. The servo requires a modulation of the signal, and obviously the period of the modulation cannot be less than the flight time of an atom between the two oscillating fields. As the spectral line is narrowed by increasing the flight time of the atoms, the modulation frequency required for the operation of the servo must be reduced. There is a corresponding increase in the time constant of the servo network. The crystal oscillator* will have its free running instability for periods within this time constant.

Qualitatively the effect may be very briefly described in the following way:

Suppose that during the time that it takes an atom of the beam to move from the first oscillating field to the second, the exciting radiation changes frequency slightly due to some sort of instability. The atom sees a relative phase shift between the two fields as a result of this frequency variation. Atoms that see no relative phase difference - that is atoms that see a phase in step with their own precessional motion in both oscillating field regions - have a maximum probability of transition. Atoms that see a phase difference of $\pm \pi$ radians between the two fields have a minimum probability of transition. In general, a distribution of phase differences can be expected for longer transit times. Under conditions of very poor stability or very long transit times (or both) the Ramsey pattern will be smeared out so badly that only the broad Rabi line shape will remain.

* Existing atomic clocks derive their beam excitation from a crystal oscillator and a frequency multiplier chain. The correction signal is applied to the quartz crystal circuit of the oscillator. Other sources of noise in the system can usually be reduced below that of the oscillator.

The phase distribution for one crystal oscillator and frequency multiplier chain has been investigated with an ammonia maser. If the single crystal oscillator tested can be considered representative then for oscillating field separations of 20 feet or more, we might expect the instability to become serious. However, some of our crystal oscillators behave so erratically at times that only the Rabi pattern is observed - and the oscillating fields are separated by only 50 cm in our atomic beam machine.

2. THEORY

In order to make a reasonable estimate of the effect of frequency (or phase) instability on the observed Ramsey signal from an atomic beam apparatus it is necessary to average the transition probability over the appropriate phase distribution (in addition to the usual average over the velocity distribution). If P_{ij} is the probability of transition between states i and j , we wish its average over the distribution in phase, $\rho(\delta)$. More specifically, we require

$$\overline{(P_{ij})}_{\delta} = \int_{-\infty}^{+\infty} \rho(\delta) P_{ij} d\delta \quad (1)$$

where δ is the phase difference that - to the atom - appears to exist between the two oscillating fields. $\rho(\delta) d\delta$ is the normalized probability that the relative phase has a value lying between δ and $\delta + d\delta$. In order to properly represent $\rho(\delta)$ it is necessary to learn some new things about oscillator short time instability. The results and techniques of our investigation of the phase distribution of a crystal oscillator and an associated frequency multiplier chain, with an ammonia maser, are discussed in Section 3. Let us assume some particularly simple forms

for $p(\delta)$ just to demonstrate the point. First, however, we need the explicit form for P_{ij} (see reference 1, page 128). We are interested in the frequency region near resonance so we can simplify the transition probability for our purposes to the following form

$$P_{ij} = \sin^2 2 \frac{\mu_{ij}}{\hbar} H_0 \tau \cos^2 \frac{1}{2} \left\{ \left[\left(\frac{\overline{W}_j - \overline{W}_i}{\hbar} \right) - \omega \right] T - \delta \right\} \quad (2)$$

or

$$P_{ij} = \sin^2 2b\tau \cos^2 \frac{1}{2} \left\{ [\omega_0 - \omega] T - \delta \right\} \quad (3)$$

where $b = \frac{\mu_{ij} H_0}{\hbar}$,

μ_{ij} is the dipole matrix element between states i and j ,

H_0 is the magnitude of the oscillating magnetic field,

τ is the time it takes an atom to pass through either of the two oscillating fields,

\overline{W}_j and \overline{W}_i are the average energies of the field dependent states j and i -- these are averaged energies since in general the "C" field is not uniform,

ω_0 is the average resonant frequency,

T is the time it takes an atom to go from the first oscillating field to the second,

ω is the angular frequency of the radiation exciting the transition.

The above form of P_{ij} is an adequate approximation near resonance or for $|\omega_0 - \omega| \ll 2b$.

As an example suppose $\rho(\delta) = \frac{1}{2\pi}$, in the interval $-\pi \leq \delta \leq +\pi$ and is zero outside this interval.

Then

$$\overline{(P_{ij})}_{\delta} = \frac{\sin^2 ab\tau}{2\pi} \int_{-\pi}^{+\pi} \cos^2 \frac{1}{2} [(\omega_0 - \omega) T - \delta] d\delta$$

and

$$\overline{(P_{ij})}_{\delta} = \sin^2 2b\tau. \quad (4)$$

This is simply the Rabi line shape in the region of the resonance peak. The Ramsey pattern has been completely smeared out by this phase distribution.

As a more reasonable choice for the phase distribution function let us assume that it is a Gaussian distribution. That is

$$\rho = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(\delta - \bar{\delta})^2}{2\sigma^2}} \quad (5)$$

where $\sigma^2 = \overline{(\delta - \bar{\delta})^2}$ and $\bar{\delta} = 0$ for our particular problem.

The average of the transition probability over a Gaussian distribution is given by the following integral

$$\overline{(P_{ij})}_{\delta} = \frac{2}{\sqrt{2\pi\sigma^2}} \sin^2 2b\tau \int_0^{\infty} e^{-\frac{\delta^2}{2\sigma^2}} \cos^2 \frac{1}{2} [(\omega_0 - \omega) T - \delta] d\delta \quad (6)$$

The result of the integration is

$$\overline{(P_{ij})}_{\delta} = \frac{1}{2} \sin^2 2b\tau [1 + e^{-\frac{\sigma^2}{2}} \cos(\omega_0 - \omega) T]$$

$$\overline{(P_{ij})}_{\delta} = \frac{A}{2} [1 + e^{-\frac{\sigma^2}{2}} \cos(\omega_0 - \omega) T] \quad (7)$$

where $A = \sin^2 2b\tau$.

Maser measurements of the phase instabilities (see Section 3) sometimes give a value of approximately 1 radian for σ for a transit time of 50 milliseconds. Using this value of σ in Equation 7 the amplitude of the Ramsey signal will be 0.6A as compared to the value A for $\sigma = 0$. If $\sigma = \pi$, $(P_{ij})_{\delta}$ reduces - as in the previous example - to the form for a Rabi "flop," i.e., the Ramsey pattern is completely smeared out.

The atomic beam contains atoms with many velocities. The distribution of these velocities is determined by the equilibrium conditions in the oven source. Averaging $(P_{ij})_{\delta}$ over the velocity distribution in the beam, we have

$$[(P_{ij})_{\delta}] = 2 \int_0^{\infty} y^3 e^{-y} (P_{ij})_{\delta} dy \quad (8)$$

where $y = v/a$, a is the most probable velocity, and - from Equation 7 -

$$\begin{aligned} (P_{ij})_{\delta} &= \frac{1}{2} \sin^2 2b \frac{l}{v} \left\{ \left[\cos \left(\omega_0 - \omega \right) \frac{L}{v} \right] e^{-\frac{\sigma^2}{2}} + 1 \right\} \\ &= \frac{1}{2} \sin^2 \left(\frac{2bl}{ay} \right) \left\{ 1 + e^{-\frac{\sigma^2}{2}} \cos \left(\omega_0 - \omega \right) \frac{L}{ay} \right\} \end{aligned}$$

Here $l/v = \tau$,

l is the length of each of the oscillating field regions,

$L/v = T$,

L is the distance between the two oscillating fields, and the other symbols have the same meaning as before.

After some rearrangement and integration, we have

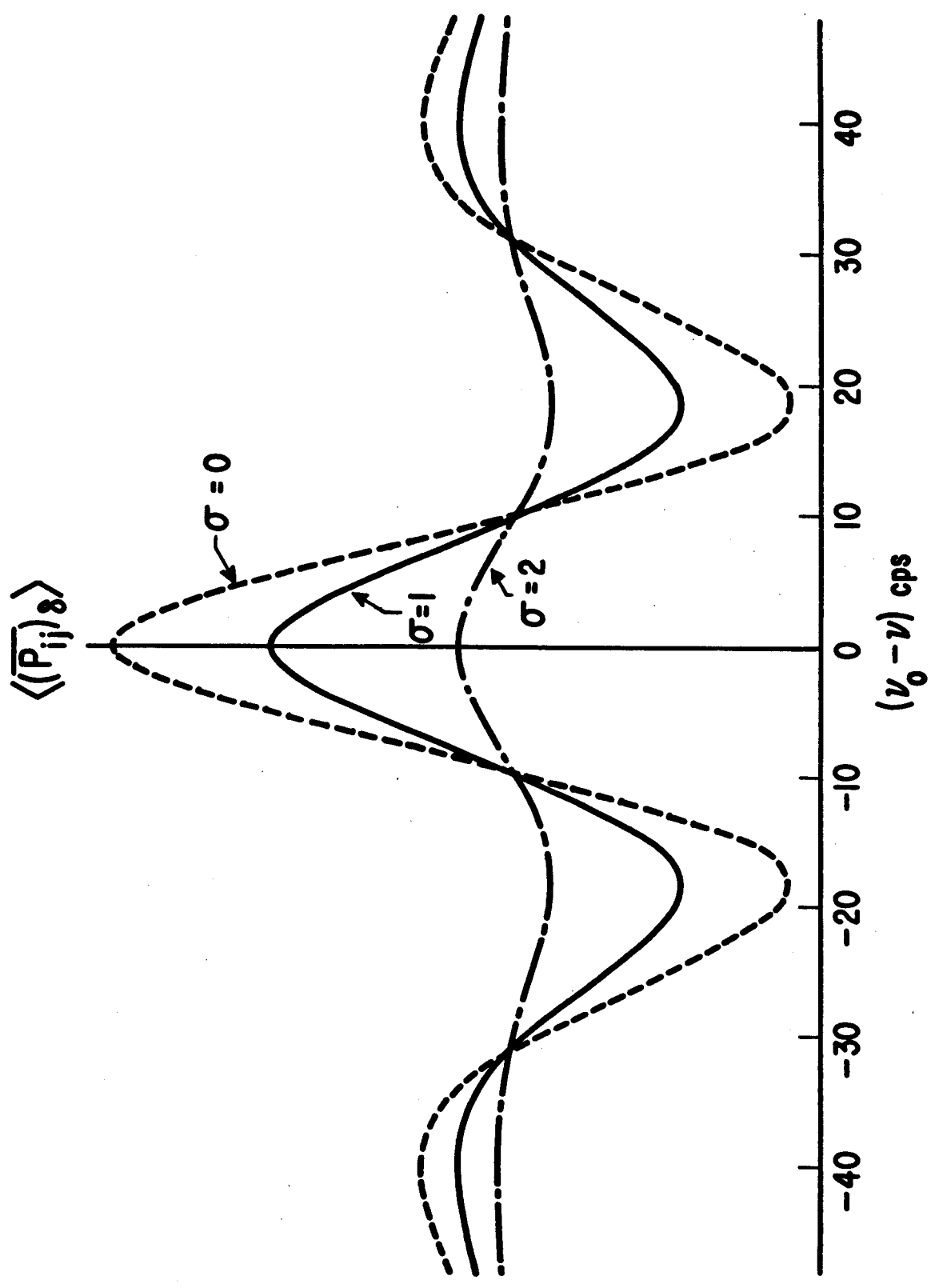


Figure 1

The predicted effect of excitation instability on the Ramsey pattern of an atomic beam machine.

$$\begin{aligned}
 [(\overline{P}_{ij})_0] &= \frac{1}{2} - \frac{1}{2} e^{-\frac{\sigma^2}{2}} I \left[\frac{4bf + (\omega_0 - \omega) L}{a} \right] \\
 &- \frac{1}{2} e^{-\frac{\sigma^2}{2}} I \left[\frac{4bf - (\omega_0 - \omega) L}{a} \right] \\
 &+ \frac{1}{2} e^{-\frac{\sigma^2}{2}} I \left[\frac{(\omega_0 - \omega) L}{a} \right] \\
 &- \frac{1}{2} I \left(\frac{4bf}{a} \right)
 \end{aligned} \tag{9}$$

In the above expression we follow Ramsey [2] in defining

$$I(x) = \int_0^{\infty} e^{-y^2} y^3 \cos \frac{x}{y} dy \tag{10}$$

Tables of this integral are given in reference 2. For $\sigma = 0$ Equation 9 reduces to*

$$\begin{aligned}
 (P_{ij}) &= \frac{1}{2} - \frac{1}{2} I \left[\frac{4bf + (\omega_0 - \omega) L}{a} \right] - \frac{1}{2} I \left[\frac{4bf - (\omega_0 - \omega) L}{a} \right] \\
 &+ \frac{1}{2} I \left[\frac{(\omega_0 - \omega) L}{a} \right] - \frac{1}{2} I \left(\frac{4bf}{a} \right).
 \end{aligned} \tag{11}$$

Figure 1 is a plot of Equation 9 versus $(\nu_0 - \nu) = \frac{\omega_0 - \omega}{2\pi}$. The various parameters are given the values - $L = 1000$ cm, $l = 1$ cm, $a = 2.4 \times 10^4 \frac{\text{cm}}{\text{sec}}$, and $\frac{2bf}{a} = 0.600\pi$ for optimum transition probability. The value of σ for each curve is indicated in the figure. The most probable velocity, a , chosen above is for cesium at an oven temperature of 140°C . Most cesium clocks using pure cesium, are operated at lower temperatures. The smaller values of a that would obtain at lower oven temperatures only make the effect under discussion more troublesome.

* This is identical to Equation V40, 129 of Ramsey's book.

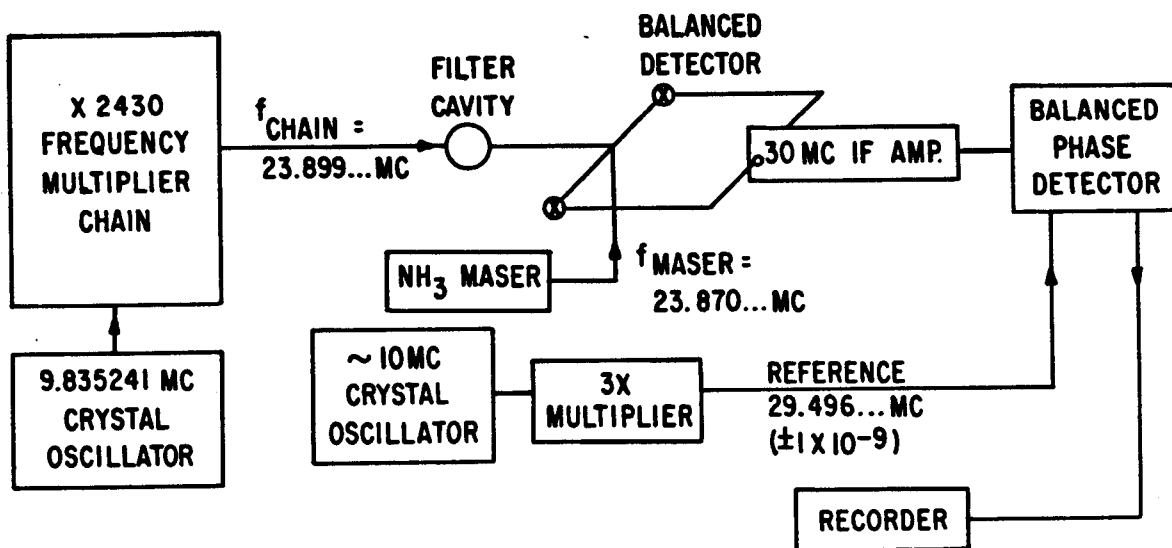


Figure 2

Schematic of method used for measuring the short period instabilities of a crystal oscillator - frequency multiplier chain combination.

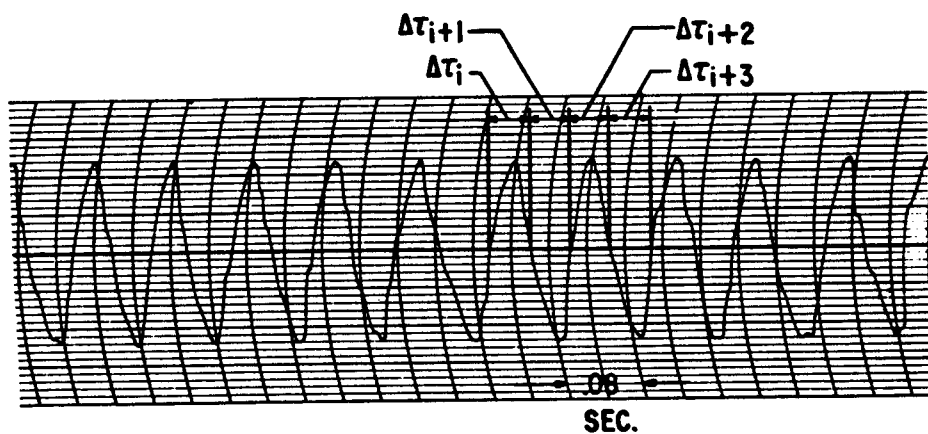


Figure 3

Sample of a recording used to determine the short term stability of a crystal oscillator - frequency multiplier chain combination. The trace was obtained using the circuit arrangement of Figure 2.

3. THE MEASUREMENT OF PHASE VARIATIONS

The ammonia maser - because of its remarkable stability - provides an ideal spectrum analyzer for signal generators. Cast in this role, the maser has provided data on the short time instability of a crystal oscillator-frequency multiplier system that permits us to make reasonable predictions about the behavior of very long atomic beam machines.

The block diagram of Figure 2 shows the method of measurement. The 30 Mc beat note between a frequency multiplier chain and a maser was phase detected. The output of the phase detector was displayed on a recorder with a time constant of about 0.01 second. A sample of a recording is shown in Figure 3. The analysis of the recordings was made by measuring the separation in time of the successive points where the curve passes through zero. Let $\Delta\tau_i$ be the i th such time interval, then $\frac{1}{2\Delta\tau_i}$ is the mean difference frequency (for the time interval $\Delta\tau_i$) between the 30 Mc reference signal, f_{ref} , and the 30 Mc maser-chain difference signal, $(f_{chain} - f_{maser})$. The difference frequency $[(f_{chain} - f_{maser}) - f_{ref}]$, was set such that the average of the $\Delta\tau_i$'s - which we will call $\overline{\Delta\tau}$ - equal the atomic transit time of interest. The average of the difference frequency over the time interval $\Delta\tau_i$ is simply $\frac{1}{2\Delta\tau_i}$, and the progression in phase in the time $\overline{\Delta\tau}$ is $\pi(1/\Delta\tau_i)\overline{\Delta\tau} = \phi_i$. The "discrepancy" in the phase change that occurs in the transit time $\overline{\Delta\tau}$, during the time interval $\Delta\tau_i$, is given by $\delta_i = |\phi_{i-1} - \phi_i|$. The average of the δ_i^2 's was used as an estimate for σ^2 in Equation 9. If the results of the analysis are to be applied to a cesium beam then one must take into account the fact that the measurements were made at about 24,000 Mc instead of at the cesium transition frequency of 9200 Mc. Assuming that the instrumentation above 9200 Mc contributes

TABLE I

Standard Deviation, σ , of Phase Differences
Experienced by Atoms of the Beam.

σ' measured in radians at 23,900 Mc.	σ predicted for 9193 Mc { $\sigma = \frac{9193}{23,900} \sigma'$ } (in radians)	Transit time, $\Delta\tau$ (in seconds)	Corresponding oscillator field separation, L (cm). for $a = 2 \times 10^4 \frac{\text{cm}}{\text{sec}}$.	Total time over which σ is averaged, T. (in seconds)
0.51	0.20	.030	600	85
0.54	0.21	.069	1380	85
0.67	0.26	.092	1840	85
0.80	0.31	.14	2800	85
1.4	0.52	.28	5600	85

TABLE II

Multiplier Chain Frequency Stability.*

RMS value of the 1st differences in frequency, $\sqrt{\frac{1}{N-1} \sum_{i=1}^N (f_i - f_{i+1})^2}$, measured in parts in 10^{10} .	Mean time, $\Delta\tau$. $\Delta\tau = \frac{1}{N} \sum_{i=1}^N \Delta\tau_i$. (in seconds).	Total time interval of measurement, N $T = \sum_{i=1}^N \Delta\tau$, (in seconds).
2.0	.030	85
1.0	.069	85
0.83	.092	85
0.76	.14	85
0.64	.28	85

*The frequency stability of the multiplier chain - for different time intervals - was measured by comparing with an ammonia maser. In Table II, f_i is the average frequency, $[(f_{\text{chain}} - f_{\text{maser}}) - f_{\text{reference}}]$, for the particular time interval $\Delta\tau_i$ (see Figure 2) f_{maser} and f_{ref} have been shown, experimentally, to be sufficiently stable so that they can be considered fixed in frequency. N is the total number of time intervals, $\Delta\tau_i$. In Column I the RMS value of the first differences between successive frequency values are recorded. The values give an indication of the continuity and smoothness of the frequency variations between periods of length $\Delta\tau$. The standard deviation of the frequency (not the 1st differences) was about 8×10^{-11} for the 85 seconds. The same data has been used throughout for both Tables I and II.

nothing to the phase instability, we may reduce the results to 9200 Mc by multiplying by the factor 92/240. Table I gives the σ values determined for various $\overline{\Delta\tau}$ values. The frequency stability is also shown for different time intervals. The average frequency over a period

$T = \sum_{i=1}^N \Delta\tau_i$ is simply $\frac{1}{N} \sum_{i=1}^N \left(\frac{1}{2\Delta\tau_i} \right)$ where N is the number of time

intervals in the time T. $\overline{\Delta f}$ is the average of the absolute values of the first differences $|\overline{f}_i - \overline{f}_{i+1}|, |\overline{f}_{i+1} - \overline{f}_{i+2}|, \dots$. Values of σ are given for a frequency of 24,000 Mc, the frequency at which they were actually measured. The values of σ for 9200 Mc are estimated by multiplying by 92/240. The frequency stability was about 8×10^{-11} for periods of about 1 minute. The oscillator is rather good for periods of several minutes or less but does not compare with the best crystal oscillators for long periods (hours). Exceptional stability is required only for time intervals comparable to the sweep time over the spectral line. This may range from 10 minutes to less than a second depending upon the type of detector - whether an electron multiplier or electrometer circuit - and on the spectral line breadth.

The results of Table I are reproducible but they do represent the best behavior of the crystal oscillator. Values of σ ranging from 1 to $2(\overline{\Delta\tau} \sim .05 \text{ second})$ are not unusual and were the measured values at the beginning of the experiments. Improvements of the oscillator during the course of the experiment account for the somewhat better results of Table I.

Strictly speaking, in averaging $(\overline{P_{ij}})_\delta$ over a velocity distribution, σ should be written as a function of velocity. For a fixed oscillator field separation, L, the variation of σ with velocity can be seen from Table I if one considers the various $\overline{\Delta\tau}$'s to be the transit times

for different velocities, instead of different L values. Equation 8 was integrated without regard for this - i.e., σ was assumed fixed - for the sake of simplicity.

4. DISCUSSION

From the results of Table I and the curves of Figure 1 we would expect that the best present day crystal oscillators would be satisfactory for beams as long as $L = 200$ feet. This conclusion, drawn from Table I and Figure 1 is very optimistic, however, at least for the particular oscillator tested. One month of continuous operation of the oscillator was required to obtain these "best" values of σ . Also some care was taken to avoid mechanical vibration. It was not unusual to obtain values of σ 5 to 10 times larger than those given in Table I. Crystal oscillators are quite microphonic and deviate substantially under varied conditions. In view of this non-uniform behavior of the beam exciting radiation, a more realistic estimate of a limit to the practical length of an atomic beam machine is 20 to 30 feet (oscillating field separation, L). Excitation instability becomes more troublesome for transitions of higher frequency. The thallium transition occurs at 21,300 Mc. Column 1 of Table I will provide more appropriate values of σ for atomic beam machines employing thallium. A stabilizing device with a rapid response controlling the beam excitation - such as a maser - would probably be very helpful in obtaining good signal to noise ratio for machines longer than 20 feet. It is perhaps fitting to remark once again that from time to time our own cesium beam machine (for which the most probable transit time is about 2.1 milliseconds and $L \approx 50$ cm) gives no observable Ramsey pattern - only a Rabi pattern. The reason seems to be instability in the crystal oscillator.

The Ramsey patterns of Figure 1 should be considered as an average of many experimental line shape traces. This is because a symmetrical phase distribution was assumed. The phase distribution will not, in general, be symmetrical for the time interval required to sweep over a line shape. Consequently, one would expect that any single trace will be distorted from a symmetrical curve. Too, it may turn out that the distribution in phase is even unsymmetrical for long periods in which case there will be a shift in the frequency of the central peak even when an average of many traces is made. Clarification of this point will only come after the analysis of more data.

The authors wish to express their thanks to Dr. Paul Wacker, Mr. Alfonso Sandoval and Mr. Joseph Ballard for help with the multitudinous data.

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- [2] Reference 1, Appendix D; V. Kruse and N. F. Ramsey, J. Math. Phys. 30, 40 (1951).