A SYNCHRONOUS SATELLITE TIME DELAY COMPUTER

A special purpose slide rule designed to compute the free space propagation delay between a synchronous satellite and points on the earth's surface is discussed. The slide rule was developed to provide users of time information relayed by geostationary satellites a means of computing the propagation delays without dealing directly with the satellite's orbital elements. The delays computed with the slide rule are compared with the values obtained from orbital elements using a high precision digital computer. The limitations and accuracy of the slide rule are discussed. A sample slide rule which may be cut out and used is included in the report.

Key words; Satellite timing; slant range; synchronous satellites; time delay.

1. INTRODUCTION

The usefulness of potentially highly accurate timing signals relayed through geostationary satellites is complicated by the computation of the propagation delay from the transmitter through the satellite back down to the receiver. The computation of this delay is usually accomplished through the manipulation of orbital elements. From the orbital elements, six constants which describe the satellite's position and velocity at a given instant of time, and a complete description of perturbing forces, it is possible to compute the satellite's position at other times. Once satellite position is known, the free space propagation delay to any reciever follows directly from simple geometric considerations. The computation of satellite position from orbital elements, however, is complicated and includes the solution of a transcendental equation best accomplished by iterative techniques using a digital computer. As has been mentioned, an accurate prediction of the delay using orbital elements must also

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account for the forces which perturb the two-body orbit such as the sun's and the moon's gravitational fields, the non-uniformity of the earth's gravitational field, and solar radiation pressure, making the computation all the more laborious.

This type of computational burden would be unmanageable for most of the expected users of satellite time and frequency signals. Consequently, the National Bureau of Standards has developed a special-purpose computer in the form of a circular slide rule which, utilizing the satellite's position, allows easy computation of the delay to high accuracy.

An operational procedure for time synchronization using a geostationary satellite and this special purpose slide rule may be envisioned along the following line. Using satellite tracking data which could be in the form of orbital elements, the satellite's position in terms of its sub-satellite point and radius (the distance from the center of the earth) is computed at the transmitter in advance of the time broadcast. This position information is incorporated into the time broadcast. The information might be conveyed in digital form or as a voice announcement. The receiver uses the slide rule to compute his delay from the satellite's position and his own longitude and latitude. The computed delay, valid only during a few minutes before and after the announcement because of satellite motion, is then used in conjunction with his time measurements to synchronize the receiver's clock.

The slide rule was designed to work with satellites in geosynchronous, near circular, near equatorial orbits. For such satellites, one may estimate the orbit dimensional quantities to be: a semi-major axis of 6.61 earth radii, an eccentricity of 0.005 or less, and an inclination angle between the equatorial and the orbit plane of 5° or less.

2. SLIDE RULE DESIGN

The satellite's position at any given time may be specified by its sub-satellite point and radius. The sub-satellite point is the point on the surface of a spherical earth which intersects the line joining the satellite and the center of the earth. The satellite and any point on the surface of the earth define the end points of a free space propagation path for which the slide rule was designed to solve.

The problem of determining the delay from the satellite to an arbitrary site may be approached in several ways. The method used in this analysis is to solve the triangle formed by straight lines joining the satellite, the center of the earth and the site (fig. 1). This solution from plane trigonometry is

$$r = \sqrt{R^2 + h^2 - 2Rh \cos \beta}, \qquad (1)$$

where r is the range from the receiver to the satellite, R is the distance from the satellite to the center of the earth, h is the distance from the receiver to the center of the earth, and β is the central angle between the sub-satellite point and the receiver. The quantity R is a component of the satellite's position and is assumed to be available via the satellite broadcast. The quantity h is related to the geodetic latitude, φ, of a site by the following equation

$$h = a\sqrt{\frac{1 + \frac{b^4}{a^4} \tan^2 \varphi}{1 + \frac{b^2}{a^2} \tan^2 \varphi}}, \qquad (2)$$

where a = 6378.2064 km, the earth's semi-major axis; and b = 6356.5838 km, the earth's semi-minor axis [1].

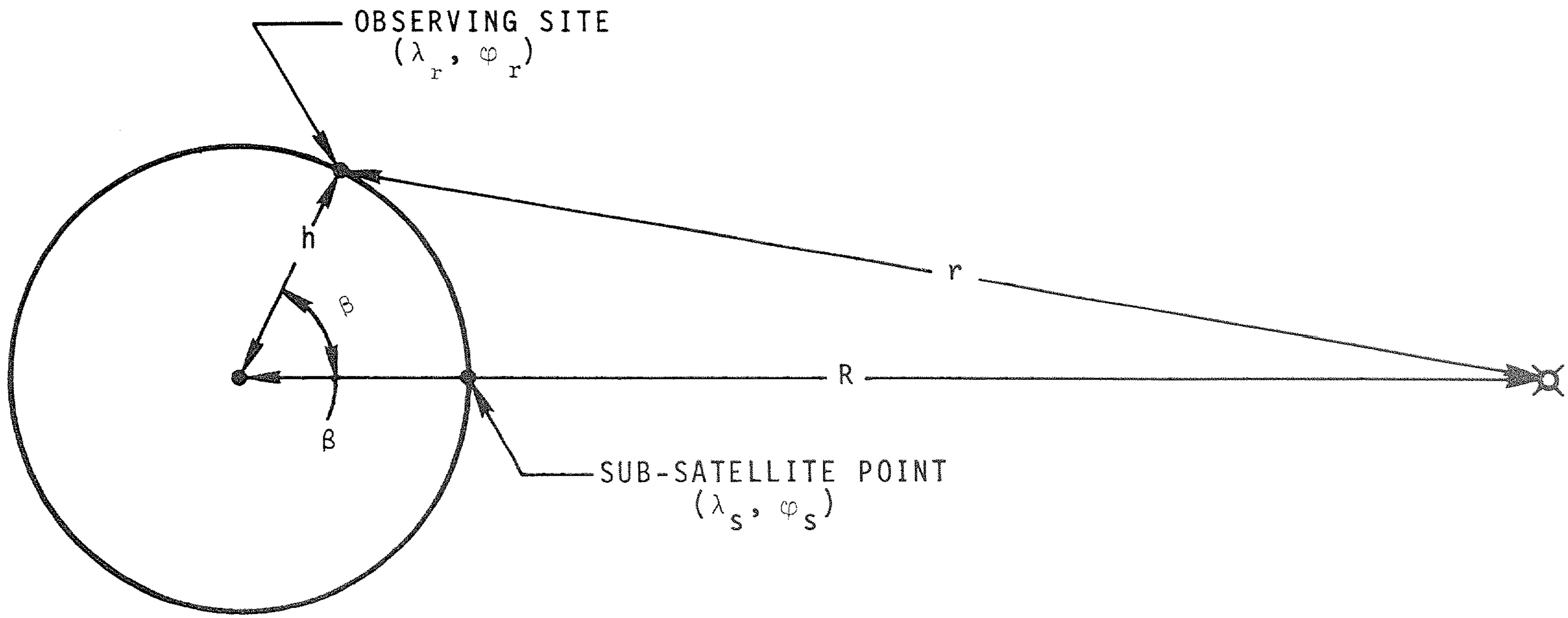

Figure 1. Earth satellite geometry for slant range calculation.

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For use in the equations below, the geocentric latitude, φ', is computed from the geodetic latitude, φ, by the following equation.

$$\tan \varphi' = (b^2 / a^2) \tan \varphi. \tag{3}$$

The sub-satellite latitude is already referenced to the center of the earth and does not need to undergo this transformation. In the following discussion, λ indicates longitude and subscripts s and r denote sub-satellite point and receiver site respectively.

All that is left then is the computation of $\cos \beta$. The direct solution may be obtained from the triangle consisting of the sub-satellite point, the site, and the intersection of the z axis with the spherical earth (i.e., the North Pole) using spherical trigonometry as follows

$$\cos \beta = \sin \varphi'_r \sin \varphi_s + \cos \varphi'_r \cos \varphi_s \cos |\lambda_s - \lambda_r|. \tag{4}$$

This method, however, does not lend itself to a simple slide rule computation. Instead, the more complicated approach of solving for α and δ, the two constructed angles shown in figure 2, leads to the elimination of terms in the formulation and a simplified slide rule procedure. The angle δ, the perpendicular from the sub-satellite point to the meridian passing through the receiver, may be calculated by

$$\sin \delta = \cos \varphi_s \sin |\lambda_s - \lambda_r|. \tag{5}$$

The angle α, the "corrected satellite latitude" or merely the arc length from the intersection of the perpendicular and the meridian to the equator, is given by

$$\cos \alpha = \cos \varphi_s \cos |\lambda_s - \lambda_r| / \cos \delta. \tag{6}$$

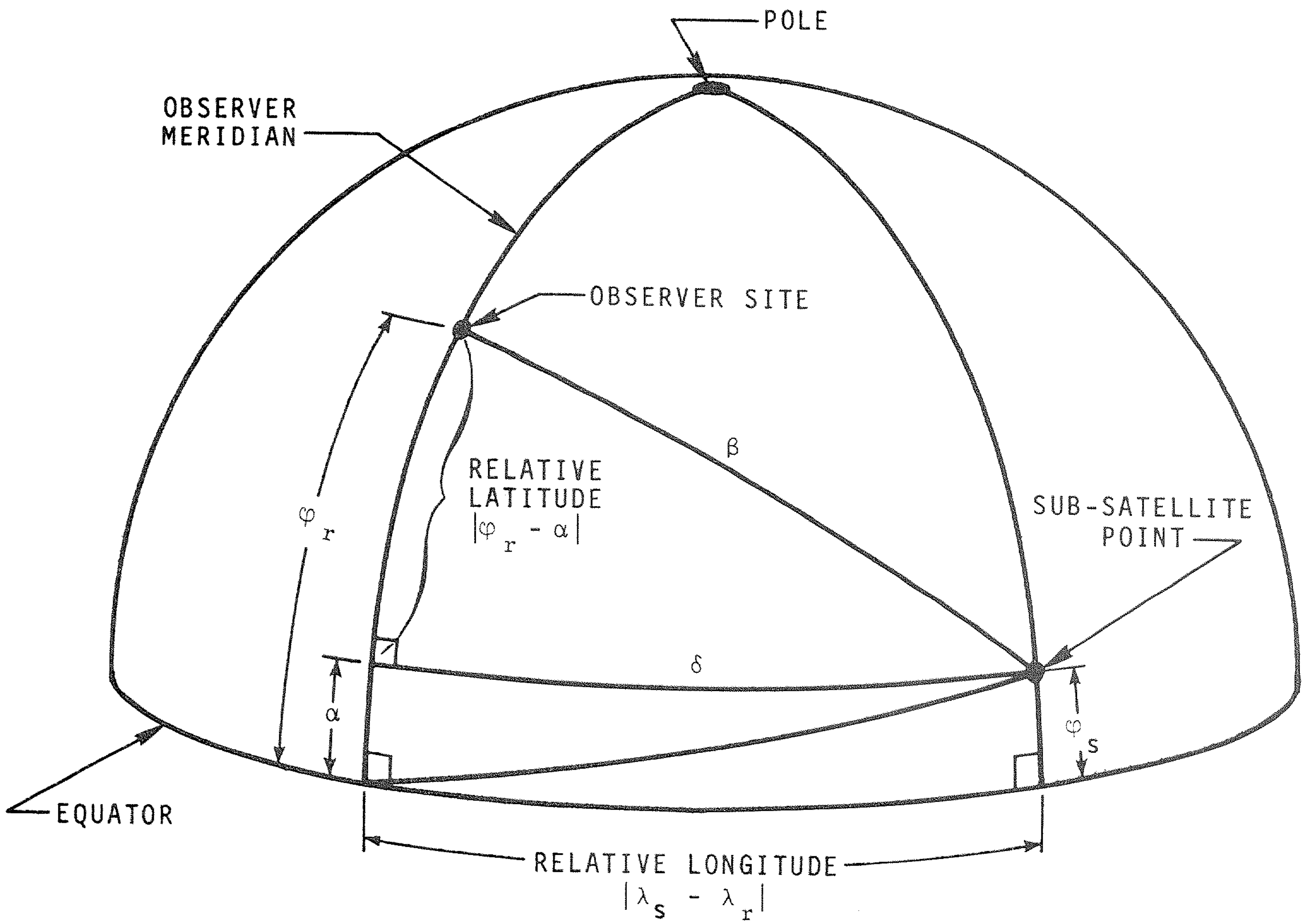

Figure 2. Geometry used to calculate the central angle, β.

Equation (4) then becomes

$$\cos \beta = \cos (\varphi_r' - \alpha) \cos \delta . \qquad (7)$$

Using eqs (1) through (7), the "down-link" free space propagation delay from the satellite to the receiver is easily determined by dividing the range by the velocity of free space propagation (0.2997925 km/μs). The procedure must be repeated substituting the transmitter for the receiver location to determine the "up-link" delay. The total free space propagation delay, then, is the sum of the delays computed using the transmitter and receiver locations. The change in signal velocity through the troposphere and ionosphere and the accompanying ray bending can be shown to introduce only a few microseconds difference in the round-trip free space propagation time when operating above 100 MHz [2].

In order to incorporate these computations on a slide rule, several approximations were made. Most of the approximations ultimately affected the accuracy of the calculation of β. Consequently, it was necessary to know the relationship between errors in β and errors in delay. Figure 3 shows this delay error for several error values of β as a function of β. Here it may be seen that in all cases an error of 0.05° in β will cause less than 20 μs of error, which is acceptable for most timing applications.

Two approximations dealing with the computation of δ and α were possible because we were dealing with geostationary satellites. For these satellites, the inclination of the orbit is small, usually less than 3°. The resultant sub-satellite latitude is a function that is periodic in time about the equator and has an amplitude equal to the inclination. The sub-satellite latitude, φ_s, therefore, is normally less than 3°. Referring to eq (5), it may be seen that since $\cos \varphi_s$ is very nearly unity, $\sin \delta \approx \sin |\lambda_s - \lambda_r|$ or $\delta \approx |\lambda_s - \lambda_r|$. The slide rule design sets $\delta = |\lambda_s - \lambda_r|$. Table 1 gives the

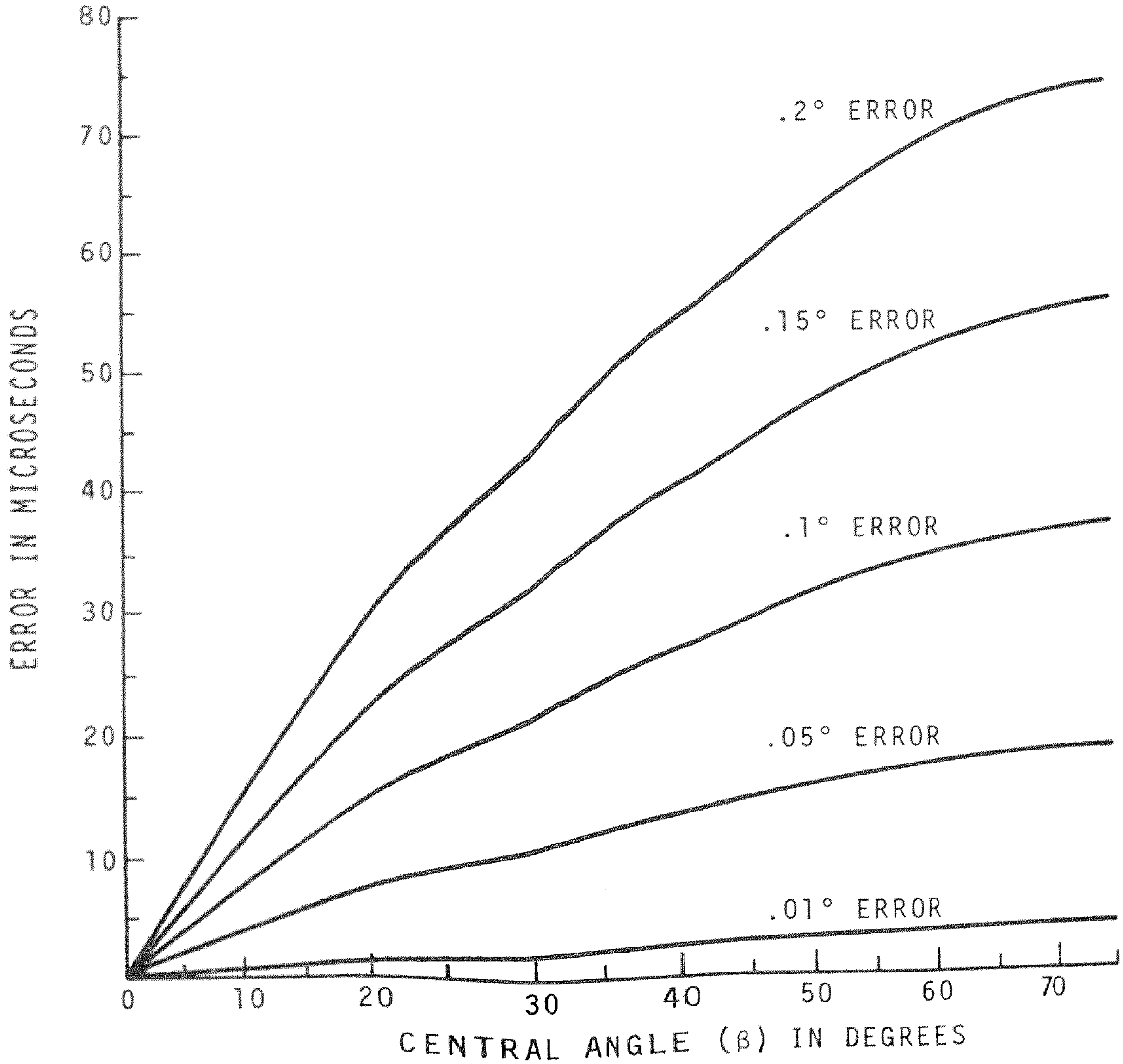

Figure 3. Delay error as a function of error in central angle.

Table 1. Values of δ for various values of φ_s and $|\lambda_s - \lambda_r|$ showing $\delta \approx |\lambda_s - \lambda_r|$.

Relative Longitude $\lvert\lambda_s - \lambda_r\rvert$	Satellite Latitude φ_s 0	.5	1.0	1.5	2.0	2.5	3.0
0	0	0	0	0	0	0	0
5	5.000	5.000	4.999	4.998	4.997	4.995	4.993
10	10.000	10.000	9.999	9.997	9.994	9.991	9.986
15	15.000	14.999	14.998	14.995	14.991	14.985	14.979
20	20.000	19.999	19.997	19.993	19.987	19.980	19.972
25	25.000	24.999	24.996	24.991	24.984	24.975	24.963
30	30.000	29.999	29.995	29.989	29.980	29.969	29.955
35	35.000	34.999	34.994	34.986	34.976	34.962	34.945
40	40.000	39.998	39.993	39.984	39.971	39.954	39.934
45	45.000	44.998	44.991	44.980	44.965	44.946	44.922
50	50.000	49.997	49.990	49.977	49.958	49.935	49.907
55	55.000	54.997	54.988	54.972	54.950	54.922	54.888
60	60.000	59.996	59.985	59.966	59.940	59.906	59.864
65	65.000	64.995	64.981	64.958	64.925	64.883	64.832
70	70.000	69.994	69.976	69.946	69.904	69.851	69.785
75	75.000	74.992	74.968	74.927	74.870	74.798	74.710

actual values of δ for various values of φ_s and $|\lambda_s - \lambda_r|$. As may be seen from eq (7) the error in central angle introduced by this approximation depends also upon the value of $|\varphi_r' - \alpha|$ or the relative latitude. Figure 4 shows this error in central angle for the various parameters involved. As will be shown in example 4 of section 4 of this report, it may be desirable to compute the error introduced by this assumption in order to reduce the overall delay computation error for some sites.

Because $\cos \varphi_s$ is very nearly linear over the small range of φ_s, one may make an approximation in the calculation of α. From this linearity and the near equality of δ and $|\lambda_s - \lambda_r|$, it may be seen in eq (6) that $\cos \alpha$ must also be nearly linear. Therefore, if one knows α for some non-zero value of φ_s, he may approximate any other α by extrapolating ($\varphi_s = 0^\circ$ implies $\alpha = 0^\circ$). Taking an assumed value for φ_s of 1.25 then, the slide rule calculates α as

$$\alpha \approx \frac{\varphi_s}{1.25} \cos^{-1} (\cos 1.25 \cos |\lambda_s - \lambda_r| / \cos (\sin^{-1} (\cos 1.25 \sin |\lambda_s - \lambda_r|))). \quad (8)$$

Naturally, this approximation introduces some error. Table 2 shows the difference between this method of caluclation of α and that of eq (6). By interchanging longitude and latitude in figure 4, one may determine the error in central angle caused by approximation 8. This error amounts to less than 0.01° of error in central angle or less than 5 μs of range error for better than 90% of the table.

Since the orbits of geostationary satellites are somewhat elliptical, the value of the radius, R, varies as a function of time. Referring to figure 1, it can be envisioned that these small changes in radius will cause nearly the same change in slant range (r). For typical eccentricities, the peak-to-peak variation in R amounts to about 300 km or roughly

Table 2. Difference between exact and approximate method for computing α in degrees exact approximate

Relative Longitude $\|\lambda_s - \lambda_r\|$	Satellite Latitude φ_s .5	1.0	1.5	2.0	2.5	3.0
0	0.00000	0.00000	-0.00000	-0.00000	-0.00000	-0.00000
5	0.00000	0.00000	-0.00000	-0.00000	-0.00001	-0.00002
10	0.00000	0.00000	-0.00000	-0.00002	-0.00004	-0.00007
15	0.00000	0.00000	-0.00001	-0.00004	-0.00009	-0.00017
20	0.00001	0.00001	-0.00001	-0.00007	-0.00017	-0.00032
25	0.00002	0.00001	-0.00003	-0.00012	-0.00029	-0.00054
30	0.00003	0.00002	-0.00004	-0.00019	-0.00046	-0.00087
35	0.00004	0.00003	-0.00006	-0.00030	-0.00071	-0.00135
40	0.00006	0.00005	-0.00010	-0.00045	-0.00109	-0.00208
45	0.00009	0.00008	-0.00015	-0.00070	-0.00168	-0.00320
50	0.00015	0.00013	-0.00023	-0.00109	-0.00262	-0.00499
55	0.00024	0.00020	-0.00037	-0.00176	-0.00422	-0.00802
60	0.00040	0.00034	-0.00063	-0.00296	-0.00711	-0.01351
65	0.00072	0.00062	-0.00114	-0.00536	-0.01286	-0.02442
70	0.00147	0.00126	-0.00230	-0.01084	-0.02597	-0.04925
75	0.00357	0.00305	-0.00558	-0.02626	-0.06276	-0.11866

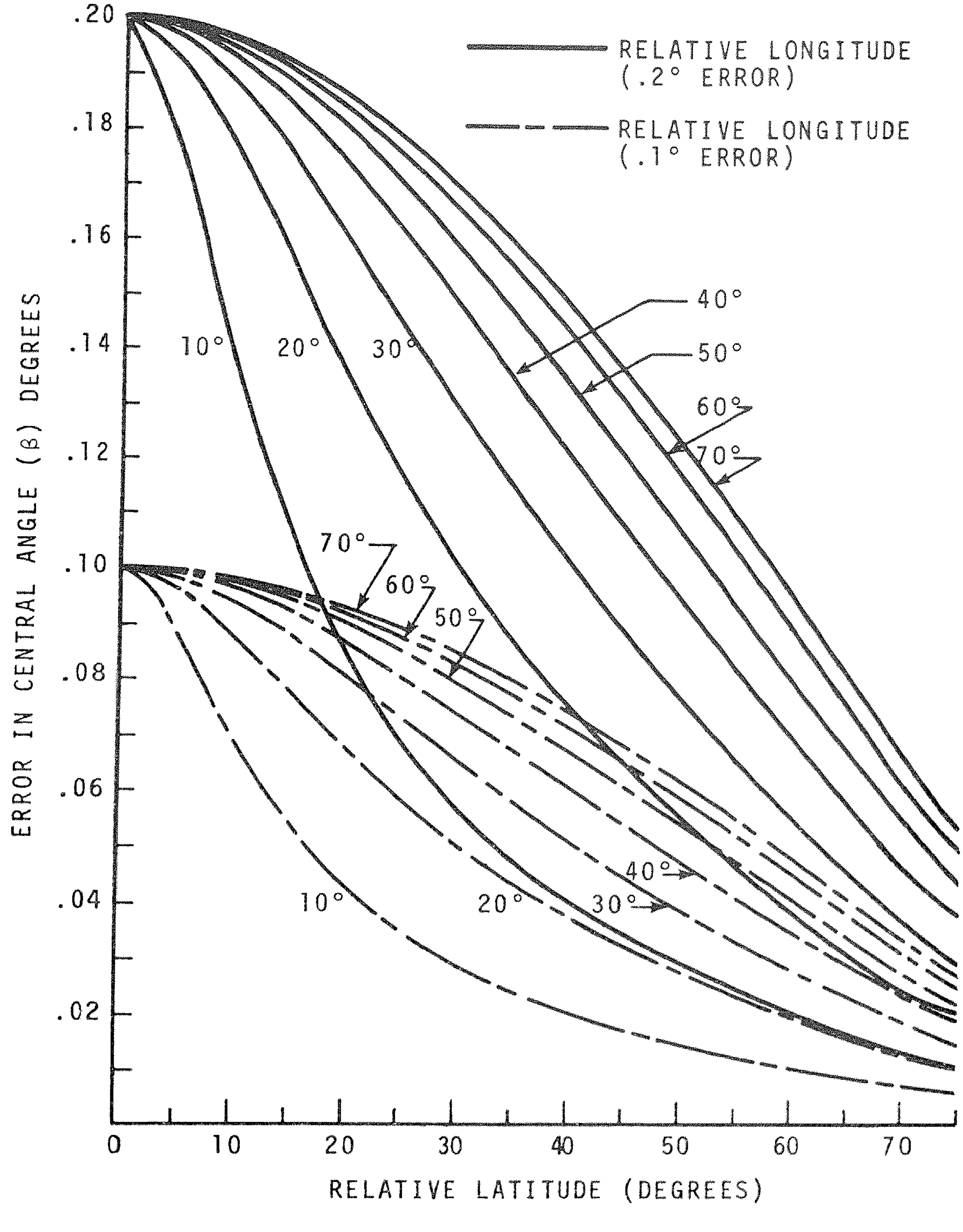

Figure 4. Error in central angle as a function of relative latitude and relative longitude for two values of error in relative longitude.

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1 ms. In the extreme case, the variation should amount to little more than twice this amount or 2 ms. The slide rule computes the delay for a value of R equal to 42,143.4517 km. The difference between the actual radius and this value has been termed the "radius correction." The value of the radius correction is one of the three parameters defining the satellite's position and is given in microseconds. This value is added to the slant range delay computed by the slide rule; i.e., the computation procedure used by the slide rule sets $\Delta r = \Delta R$. The error introduced by this approximation may be calculated from

$$\frac{\partial r}{\partial R} = (R - h \cos \beta)\, / r. \tag{9}$$

Evaluating this shows a zero error when $\beta = 0$, increasing to 1.2% of the value of the radius correction when $\beta = 75^{\circ}$. Table 3 gives the value of $\partial r/\partial R$ and the accompanying delay error for the normal (± 500 μs and the extreme (± 1000 μs) values of the radius correction. This table indicates a maximum error of 12 μs introduced by this method of dealing with variations in R.

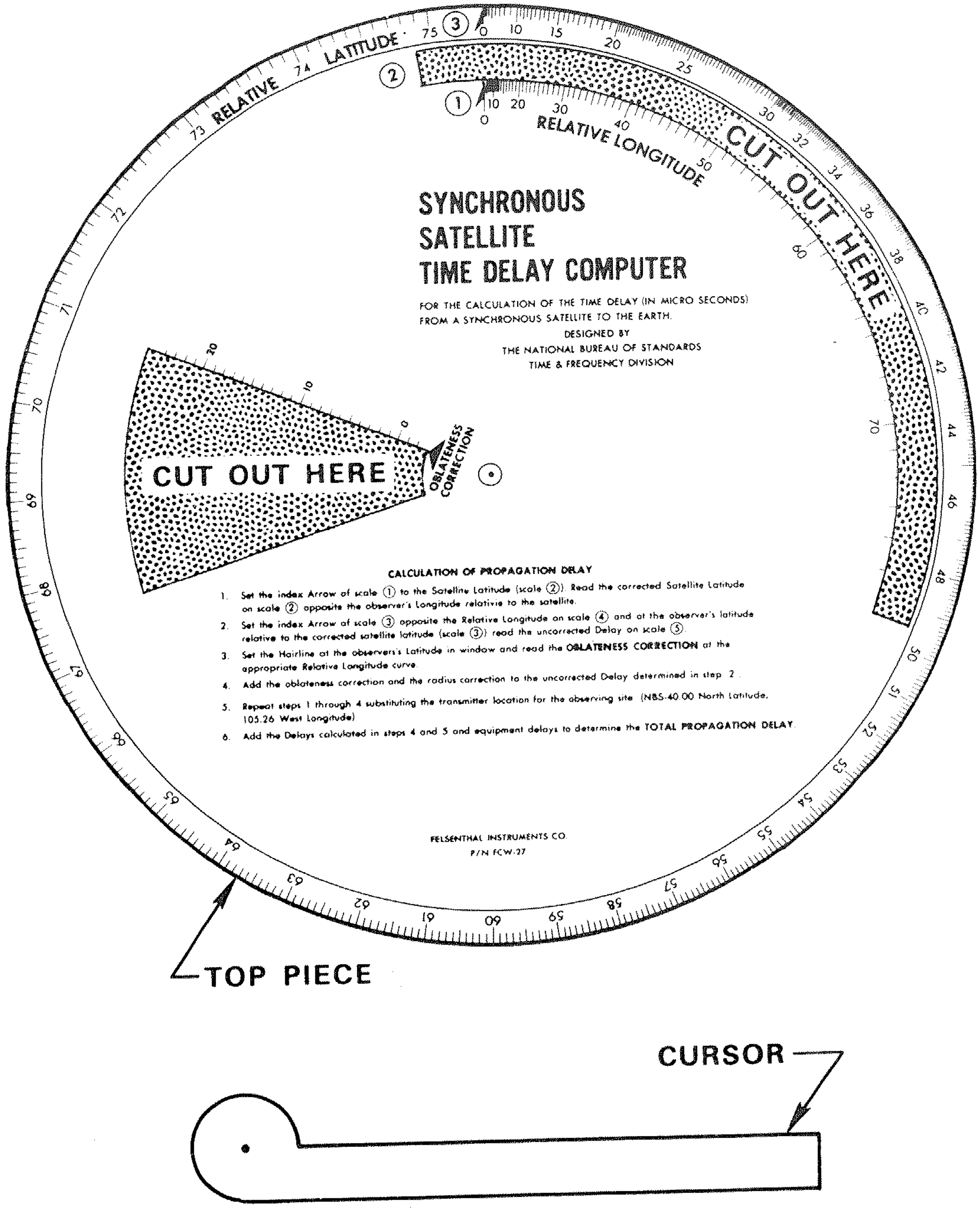
Finally, the variations of h introduced by eq (2) are not accounted for in the initial calculation of the delay. Instead, the value of h in eq (1) is set to the semi-major axis, a. The error introduced by this procedure is then removed. Figure 5 gives this error as a function of relative longitude and relative latitude. This graph, called the "oblateness correction," is included on the face of the slide rule.

3. COMPARISON WITH THEORY AND EXPERIMENT

In order for the slide rule to be useful for the computation of the delay from satellite to ground, two factors must be valid. First, the slide rule delay must agree with the theoretical delay as computed

Table 3. Evaluation of the partial derivative of equation 9. Also shown are the delay errors for values of radius correction equal to 500 and 1000μs caused by setting this partial derivative equal to 1.0.

β	∂r/∂R	Error with 500 μs radius correction	Error with 1000 μs radius correction
0	1.000000	0	0
5	.999881	.059366	.118732
10	.999526	.237167	.474334
15	.998952	.524044	1.04809
20	.998187	.906289	1.81258
25	.997268	1.36608	2.73216
30	.996235	1.88273	3.76546
35	.995132	2.43396	4.86791
40	.994007	2.99656	5.99313
45	.992903	3.54844	7.09689
50	.991862	4.06903	8.13806
55	.990919	4.54038	9.08077
60	.990105	4.9473	9.89461
65	.989443	5.27841	10.5568
70	.988949	5.52535	11.0507
75	.988633	5.68348	11.367



Assembly Instructions (figs. 12 and 13).

1. Cut out top piece, cursor, and base of the slide rule.
2. Cut out the two windows on the top piece.
3. Using a straight pin or other suitable fastner, assemble the cursor, top piece, and base at the central point, indicated with a dot in the middle of a small circle.

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Figure 12. Sample slide rule top and cursor.

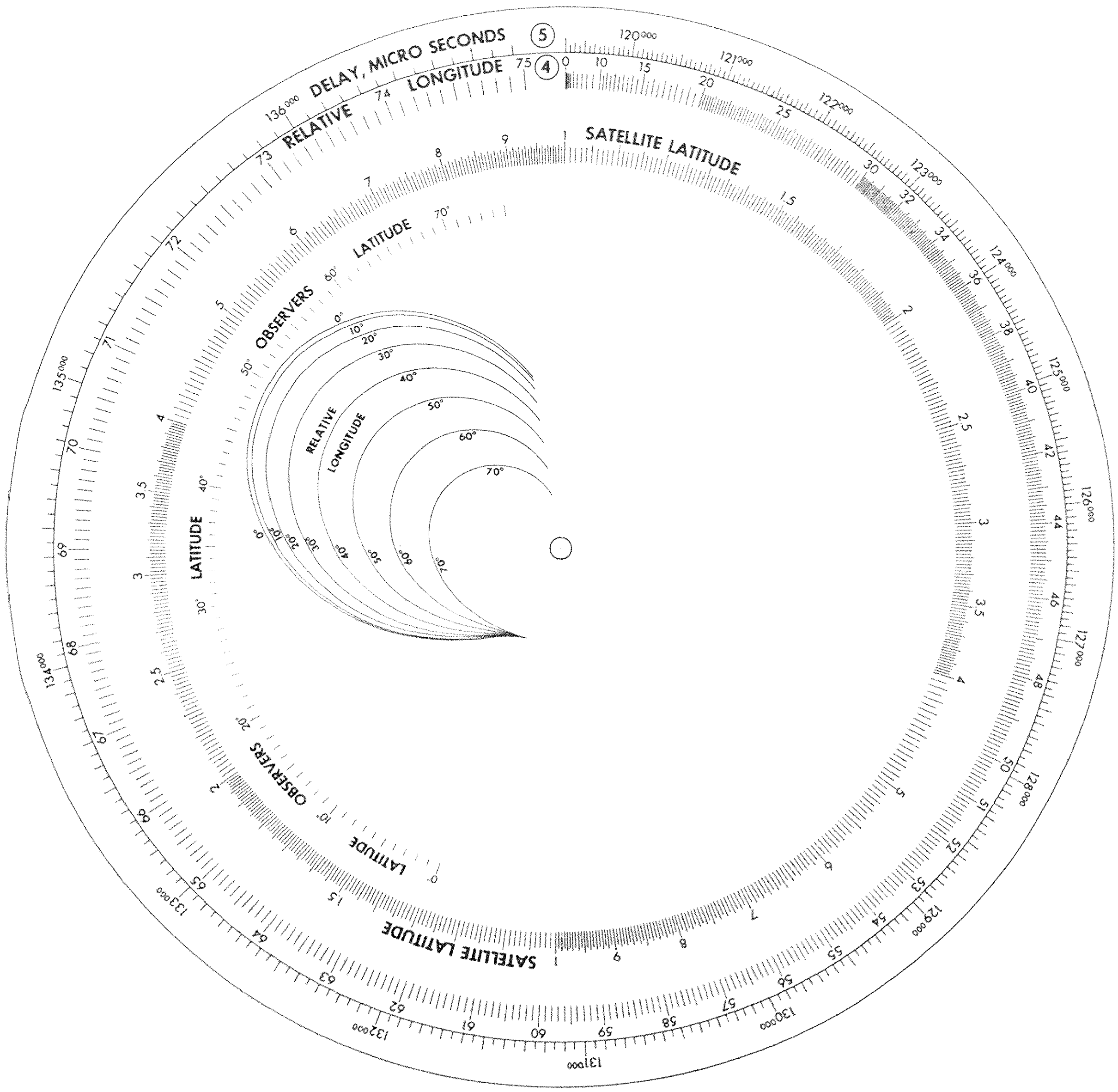

Figure 13. Sample slide rule base.

The oblateness correction in microseconds computed as the difference between the range to a satellite using the oblate Clarke spheroid (R_c) and a spherical earth (R_s).

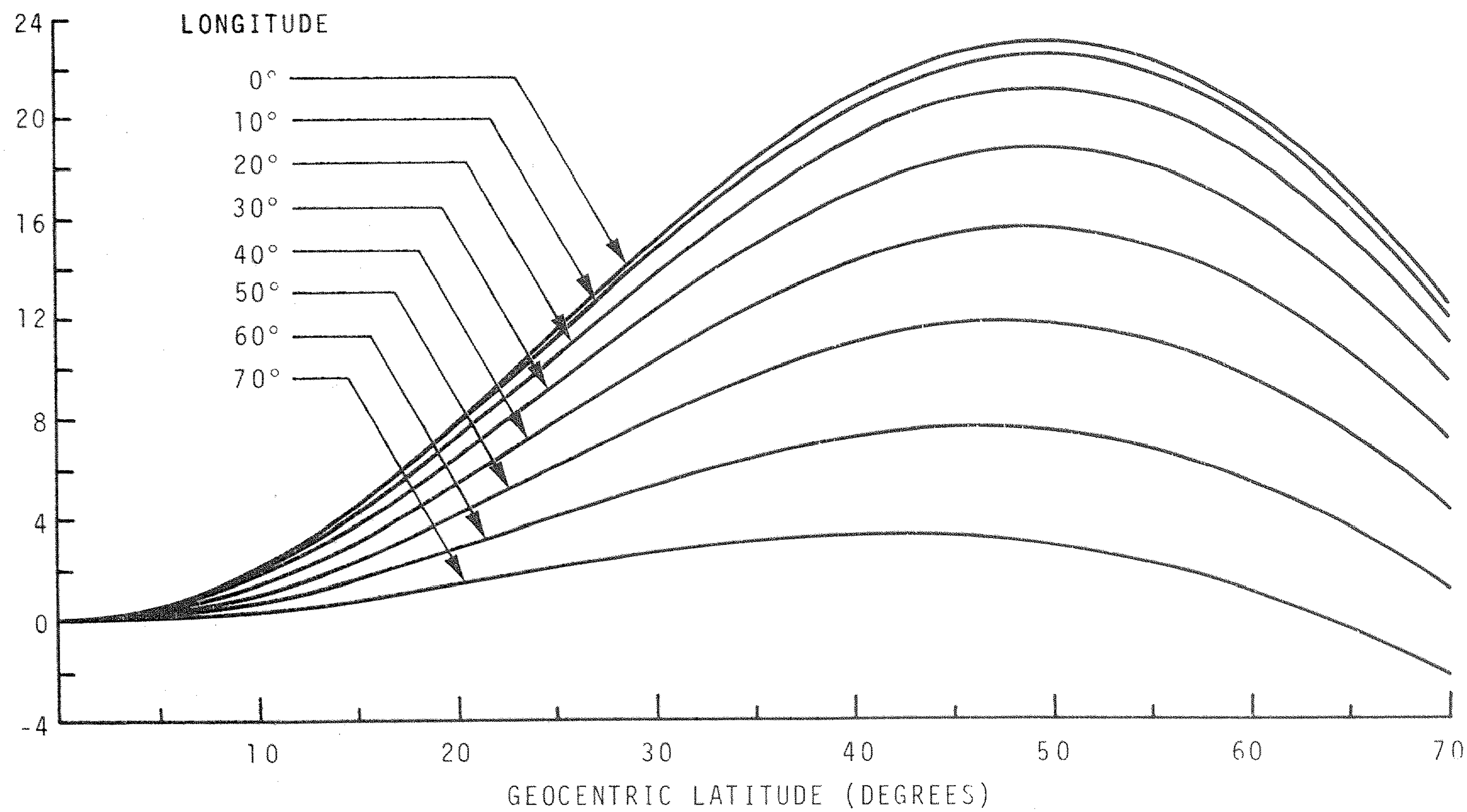

Figure 5. The oblateness correction.

by a high precision digital computer using orbital elements. Second, it must be shown that the prediction of satellite position (and the resultant delay) from orbital elements agrees with actual delay measurements. Remember that the orbital elements resulted from high resolution tracking of the satellite. From the orbital elements and a complete description of the orbit perturbations, the satellite's position at any other time is predicted. Satellite position in terms of sub-satellite point and radius is given to the user who in turn uses the slide rule to calculate the total free space delay. The process is illustrated in figure 6.

A comparison of slide rule delay and the delay derived from the orbital elements was accomplished using over 500 different sites and three different satellite positions. The sites were chosen in 1° increments of relative longitude from 0° to 70° for a fixed relative latitude. Relative latitude was varied from 0° to 70° in 10° increments. No computations are shown for central angles greater than 75° which is considered the limit of usefulness of the slide rule. Figure 7 shows the error of the delay computed using the slide rule. Figure 8 gives the distribution of this error and shows that the slide rule is weighted towards giving a high delay prediction. Figures 9 and 10 give the corresponding results after the errors in relative longitude listed in table 1 have been removed. (The method of correction for this longitudinal error is illustrated in example 4 below.) The distribution now is nearly normal and the rms error has been reduced from 9.88 μs to 6.79 μs. This longitudinal error is large with respect to this rms error for only about 15% of the satellite coverage, but for this region (the shaded area of table 1), it is suggested that the error be removed.

The question of how well the orbital elements describe or can be used to predict satellite position is answered in figure 11. Over a period of several months, it is seen that measured and predicted values

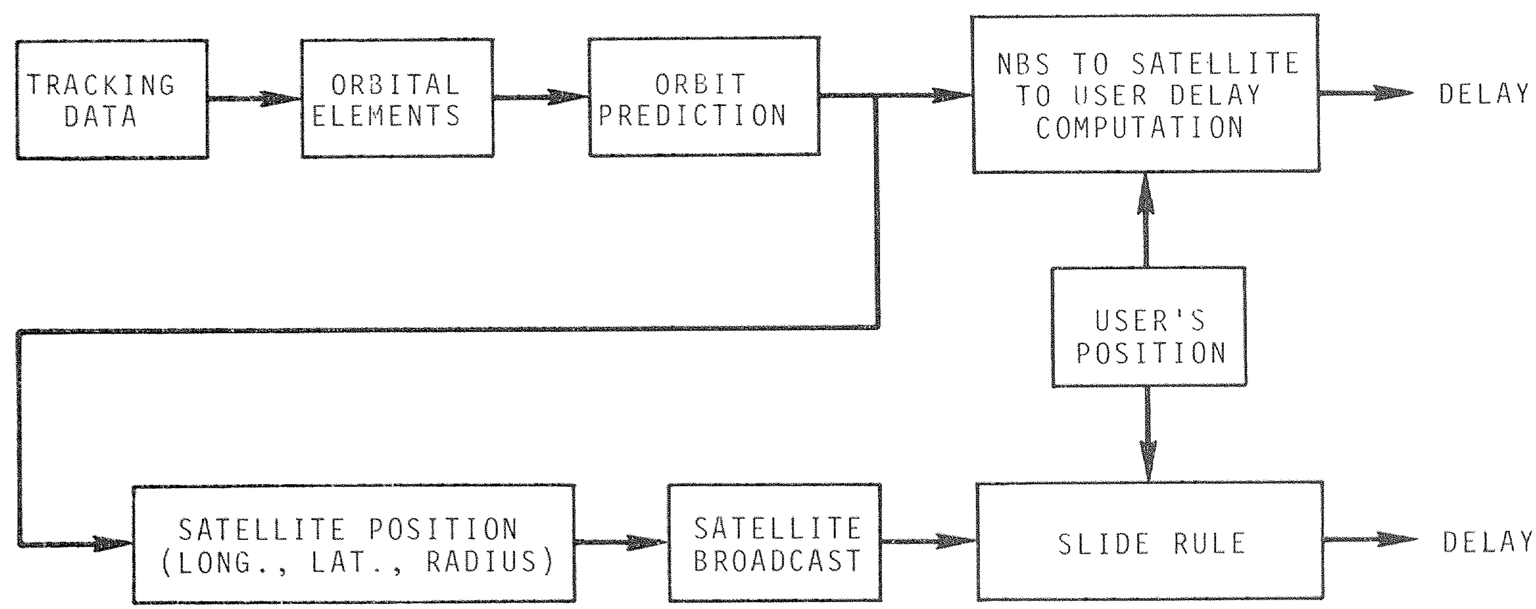

Figure 6. Block diagram of the delay computation procedure.

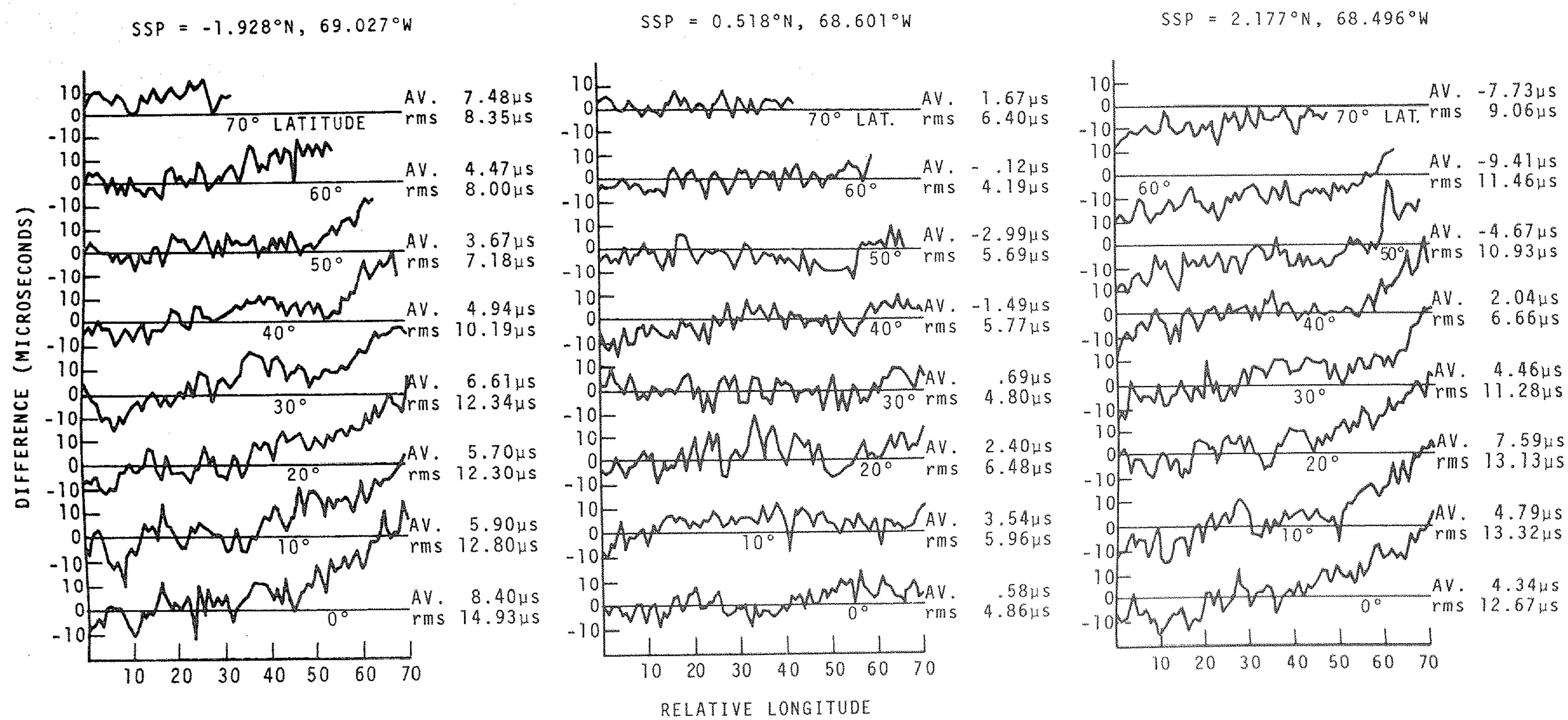

Figure 7. Slide rule delay error. Theoretical delay minus slide rule delay.

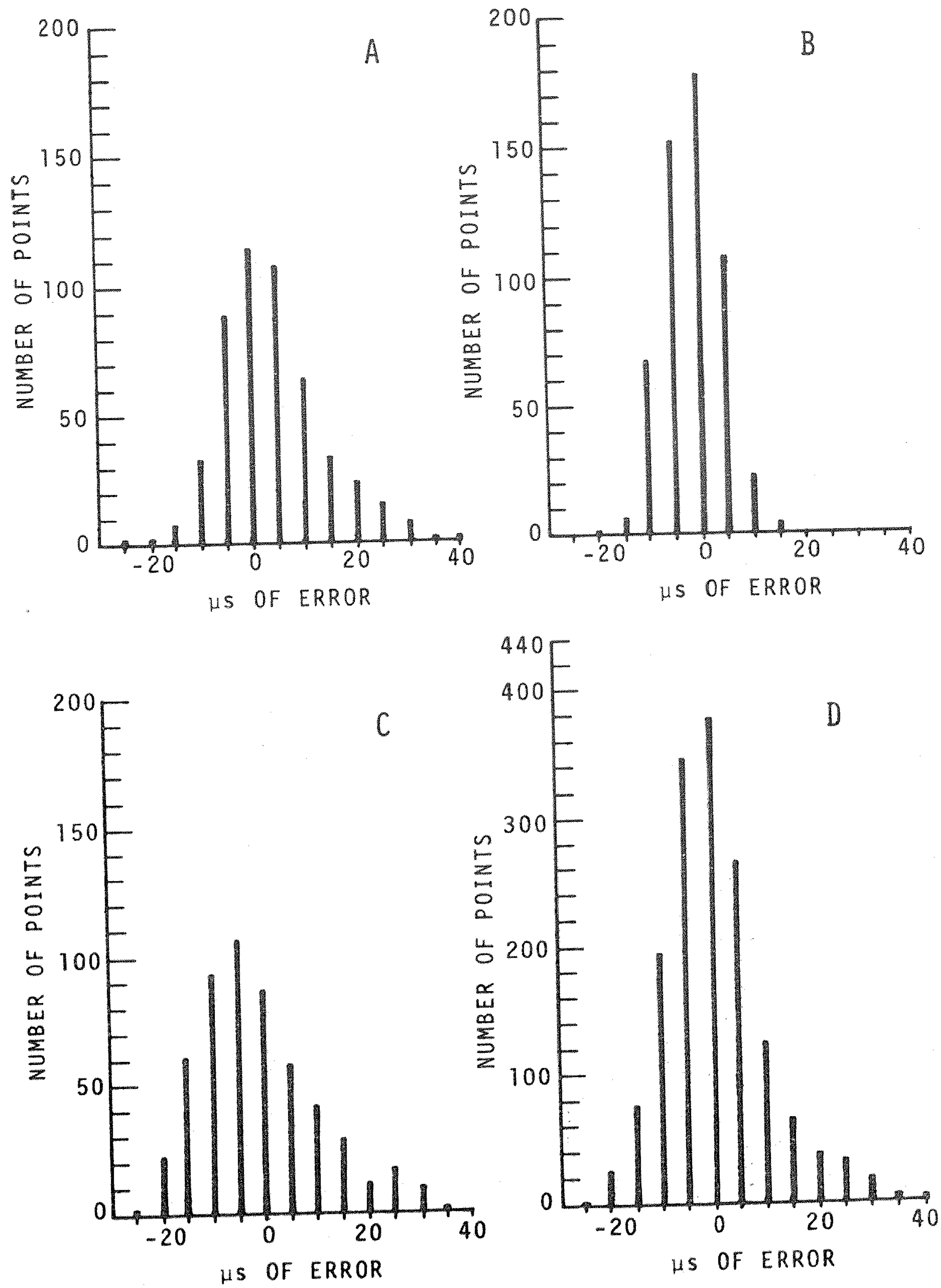

Figure 8. Distribution of slide rule delay errors.

A. Satellite at -1.928°N, 69.027°W
B. Satellite at 0.518°N, 68.601°W
C. Satellite at 2.177°N, 68.496°W
D. Total of A, B, and C.

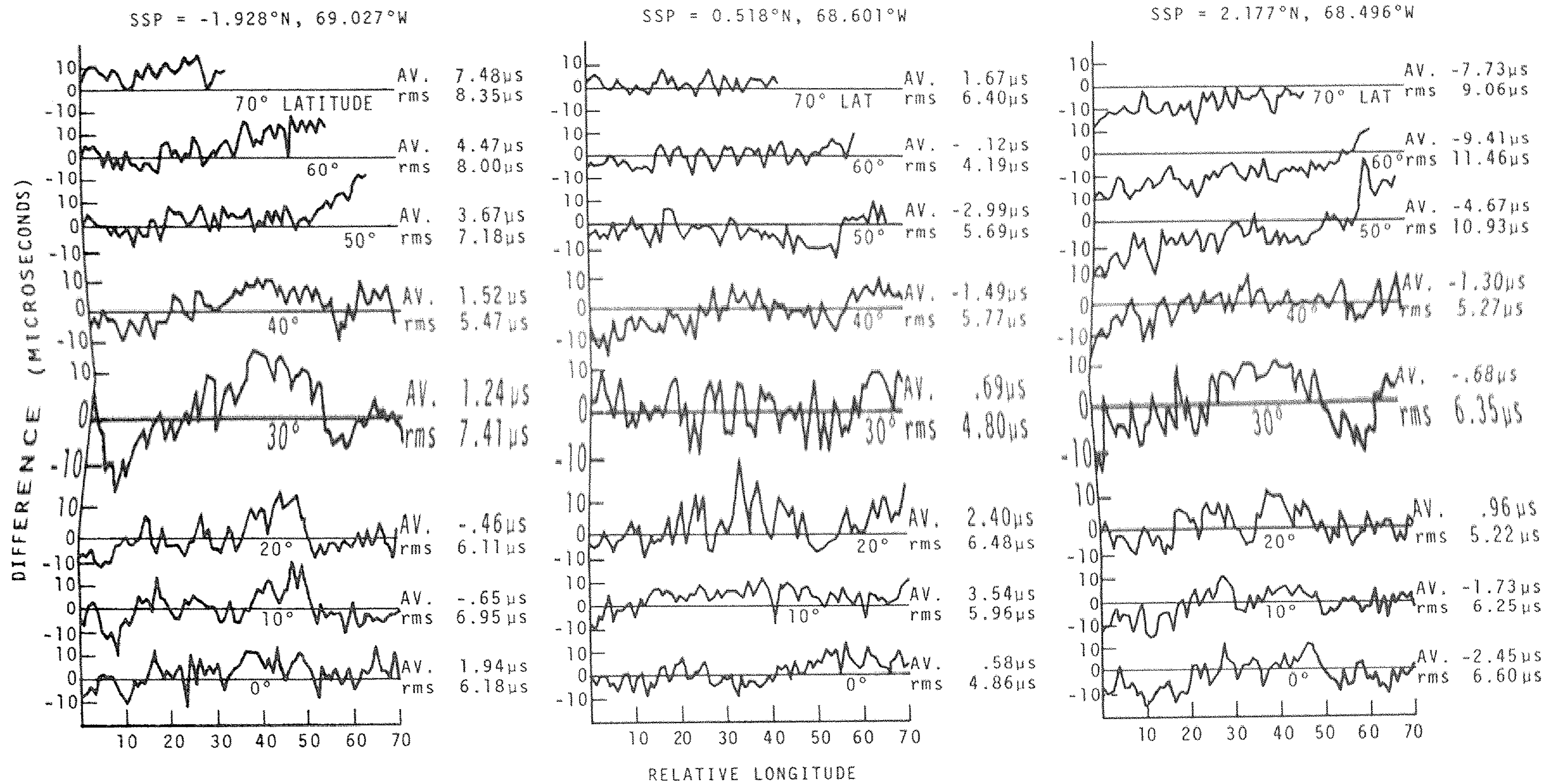

Figure 9. Slide rule delay error with the longitudinal error removed. Theoretical delay minus slide rule delay.

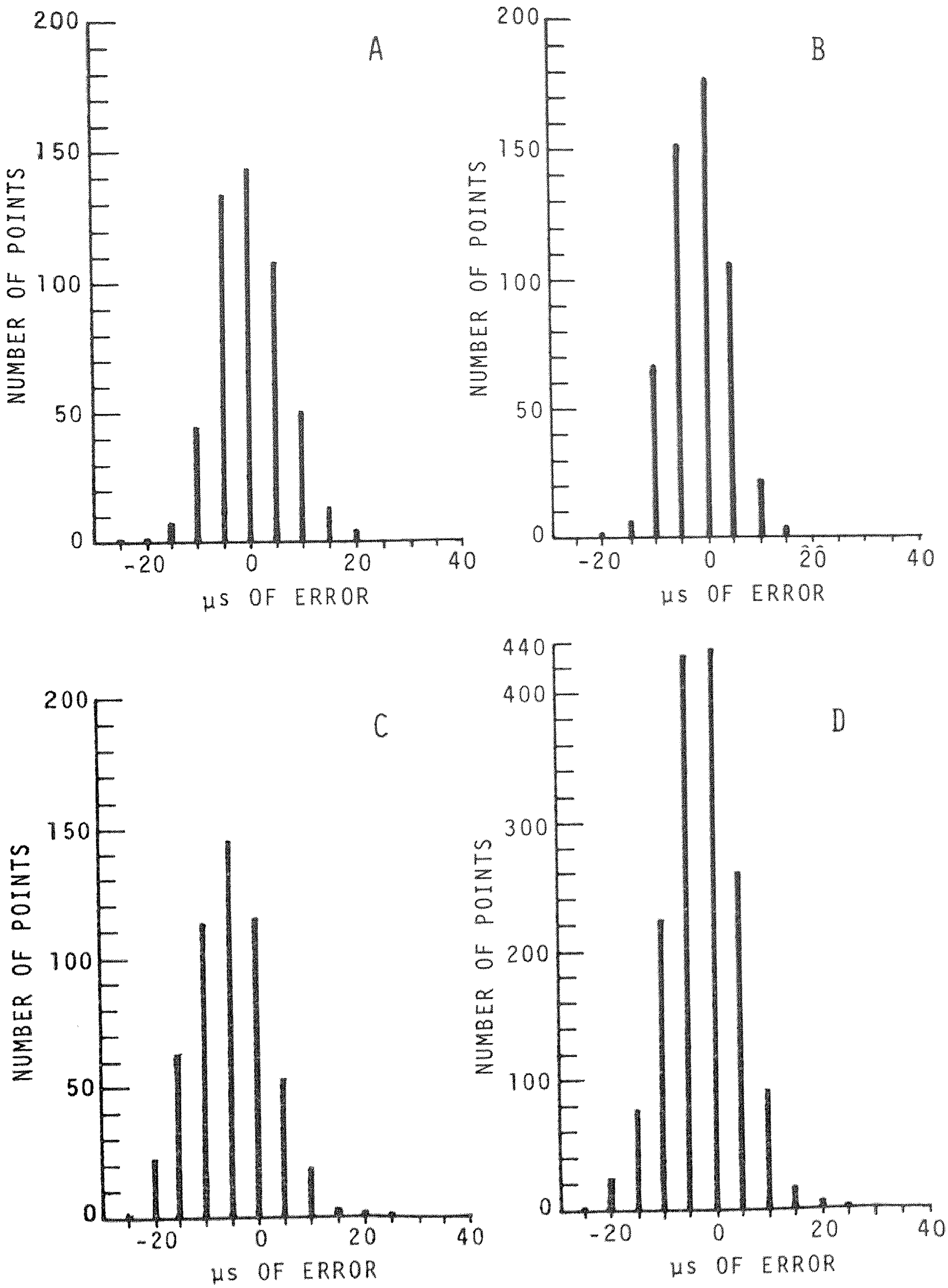

Figure 10. Distribution of the slide rule delay errors with the longitudinal error removed. A, B, C, D.

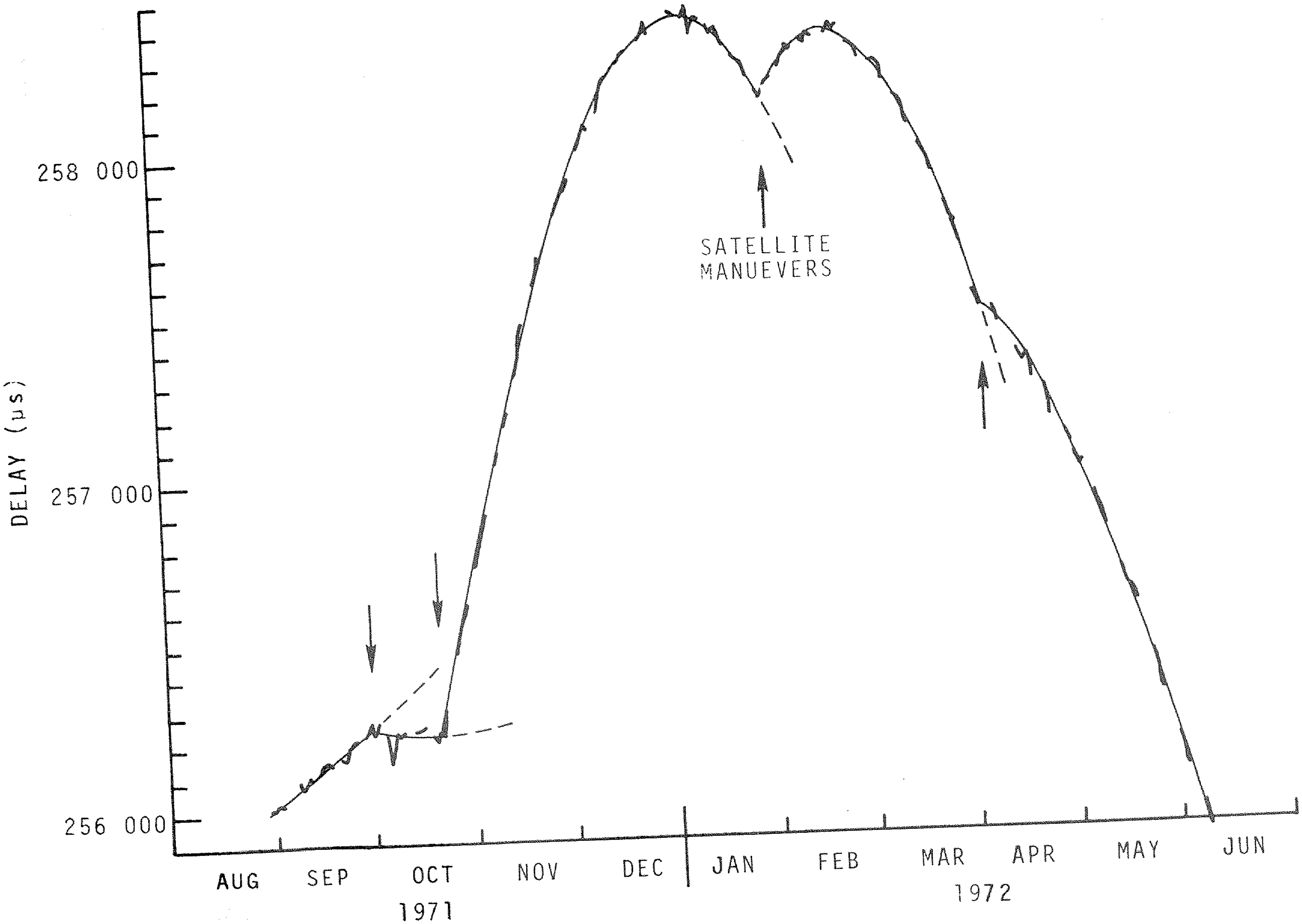

Figure 11. Two-way delay measurements between Boulder and ATS-3 at 1700 GMT.

fall to within an rms value of 10 μs which was approximately the measurement resolution. The particulars of the measurements are reported upon in a later publication [3].

4. OPERATION OF THE SLIDE RULE

For computation of the up-link and down-link delay by this slide rule, several pieces of information are needed. They are:

1. The transmitter and receiver longitude and latitude;
2. The longitude and latitude of the sub-satellite point; and
3. The difference between the actual and nominal value of the radius to the satellite (i.e., the radius correction).

The location of the observing site may be determined from a good quality map to about 0.01°. Referring to figure 3, it may be seen that this position error will not introduce a significant amount of error in the delay computation.

The position of the satellite is constantly changing. If the sub-satellite point is supplied to the user to the nearest 0.01°, updates of positon would have to be supplied once every few minutes for a typical geostationary satellite. An obvious way of disseminating this information is to broadcast, either by voice or code, the satellite position along with the timing signals.

In the explanations and examples of slide rule operation, it may be useful to cut out and assemble the sample slide rule (figures 12 and 13). The steps to calculate delay are the following:

1. Determine the longitude of the receiver and transmitter sites relative to the satellite's longitude (i.e., determine relative longitude). This may be computed as the longitudinal difference between the site and the sub-satellite point, $|\lambda_r - \lambda_s|$.

2. The corrected satellite latitude, α, is determined by setting the appropriate index of scale #1 to the sub-satellite latitude on scale #2 and reading the corrected satellite latitude on scale #2 opposite the relative longitude. Possible decimal point and sign confusion will be clarified in examples 2 and 3. The relative latitude is then computed as the difference between the site latitude and the corrected satellite latitude, $|\varphi_r - \alpha|$.

3. An uncorrected delay is determined by setting the index of scale #3 at the relative longitude on scale #4, positioning the cursor at the relative latitude on scale #3 and reading the delay on scale #5. The central angle, β, is also read on scale #4 under the cursor. Two corrections are then applied to this delay. The oblateness correction, found from the graph using the site's latitude and the relative longitude, is added. The radius correction is also added.

This procedure must be followed for both the transmitter and receiver sites in order to determine the round-trip propagation delay.

For the purposes of the following examples, the transmitter is assumed to be located at NBS, Boulder, 40.00° North Latitude, 105.26° West Longitude.

Example #1: In this example the satellite, transmitter, and receiver are north of the equator. The transmitter is west and the receiver east of the sub-satellite point.

Sub-satellite point	2.25° 70.37°	North Latitude West Longitude
Receiver site	47.85° 56.11°	North Latitude West Longitude
Radius correction	135	Microseconds

	Up-link Delay	Down-link Delay
	105.26°	70.37°
	-70.37°	-56.11°
Relative longitude	34.89°	14.26°
Corrected satellite latitude	2.74°	2.32°
	40.00°	47.85°
	-2.73°	-2.32°
Relative latitude	37.27°	45.53°
Uncorrected delay	127670 µs	127030 µs
Oblateness correction	16 µs	22 µs
Radius correction	135 µs	135 µs
One-way delay	127821 µs	127187 µs
Round-trip delay	255008 µs	

Example #2: The same receiving site is used, but the satellite is south of the equator. Note that the sub-satellite latitude is considered to be a negative number.

Sub-satellite point	2.25° 70.37°	Sough Latitude West Latitude
Receiver site	47.85° 56.11°	North Latitude West Longitude
Radius correction	135	Microseconds

	Up-link Delay	Down-link Delay
Relative longitude	34.89°	14.26°
Corrected satellite latitude	-2.74°	-2.32°
	40.00° $-(-2.73^{\circ})$	47.85° $-(-2.32^{\circ})$
Relative latitude	42.73°	50.17°
Uncorrected delay	128830 µs	128390 µs
Oblateness correction	16 µs	22 µs
Radius correction	135 µs	135 µs
One-way delay	128981 µs	128547 µs
Round-trip delay	257528 µs	

Example #3: Here the receiving site is south of the equator yielding a negative φ_r. The relative latitude, however, is still a positive number. Also, note the procedure for handling sub-satellite latitudes of less than 1°.

Sub-satellite point	0.98°	North Latitude
	69.84°	West Longitude
Receiver site	34.47°	South Latitude
	58.40°	West Longitude
Radius correction	348	Microseconds

	Up-link Delay	Down-link Delay
	105.26°	69.84°
	−69.84°	−58.40°
Relative longitude	35.42°	11.44°
Corrected satellite latitude	1.20°	1.00°
	40.00°	−34.47°
	−1.19°	1.00°
Relative latitude	38.81°	33.47°
Uncorrected delay	128085 µs	123745 µs
Oblateness correction	16 µs	18 µs
Radius correction	348 µs	348 µs
One-way delay	128449 µs	124111 µs
Round-trip delay	252560 µs	

Example #4: This example delineates the procedure for correcting the longitudinal error introduced by setting the relative longitude $|\lambda_s - \lambda_r|$ equal to δ in eq (7). The receiver is assumed to be located on Tristan da Cunha in the south Atlantic.

Sub-satellite point	2.50°	North Latitude
	74.67°	West Longitude
Receiver site	37.15°	South Latitude
	12.30°	West Longitude
Radius correction	-176	Microseconds

	Up-link Delay	Down-link Delay
	105.26°	74.67°
	-74.67°	-12.30°
Relative longitude	30.59°	62.37°
Corrected satellite latitude	2.90°	5.40°
	40.00°	-37.15°
	-2.90°	-5.40°
Relative latitude	37.10°	42.55°
Uncorrected delay	126880 μs	134765 μs
Oblateness correction	17 μs	6 μs
Radius correction	-176 μs	-176 μs
	126721 μs	134595 μs

Before determining the round-trip delay, the down-link should be corrected for the longitudinal error since the relative longitude and satellite latitude combinations fall into the shaded area of table 1. Interpolating from this table, we find that δ should be 62.26°. Using this value there instead of 62.37° to compute the down-link delay yields an uncorrected delay of 134740 μs, a one-way delay of 134570 μs, and a round-trip delay of 261291 μs.

An alternate procedure for correcting this down-link delay requires figures 3 and 4 in addition to table 1. The longitudinal error determined from table 1 is found on figure 4. This shows an error in central angle of 0.075°. The delay error resulting from this error in β is found on figure 3 to be about 27 μs using a central angle of 70°. This value does not differ significantly from the 25 μs error found by recomputing the delay with the slide rule.

5. SUMMARY

The special purpose slide rule developed in this report offers a simple, fast, convenient and accurate method of computing the free space propagation delay to a geostationary satellite. A satellite time synchronization system utilizing this slide rule would require that the operating agency compute the satellite's position coordinates and relay them to the user along with the rest of the timing format. Following the five-step procedure of section 4, the user enters his own longitude and latitude along with the satellite coordinates on the slide rule to determine the theoretical free space propagation delay for comparison with his delay measurement. The slide rule delay has been shown to agree with the delay as computed using a high speed digital computer to within 10 μs rms. A satellite time and frequency system which requires only an easily operated slide rule for the computation of the propagation delay should find expanded applications for its use.

6. REFERENCES

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