Excess Noise in Microwave Crystal Diodes Used as Rectifiers and Harmonic Generators*

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Summary—**Excess noise produced by microwave excitation of silicon crystal diodes was studied for operation of the crystal as a detector and as a microwave harmonic generator. The noise appears at the detector terminals and also as noise sidebands of the microwave harmonic, thus degrading the spectral purity of the harmonic relative to that of the fundamental. Possible models of the processes involved are presented. Difficulties and technique of measurement are discussed. Observations for 1N26 crystals, used as detectors, doublers, and triplers, and excited by *X*-band power in the range 8 to 100 mw are presented, showing limitations on spectral purity set by the process of noise production during harmonic generation.**

INTRODUCTION: MOTIVATION AND PROBLEM

IN precision work in microwave measurements, for example, microwave spectroscopy and interferometry, it is important to know the spectral purity of the sources used because the problem is usually not that of the minimum detectable signal but that of the minimum detectable change in a strong signal. If the transmission and detection portions of an experiment have been made sufficiently noise free, the source noise temperature may set the limit of precision in the measurements.

The most important present source of monochromatic waves in the millimeter region is the crystal diode harmonic generator. This device is, for convenience, also frequently used in producing low-level centimetric waves from uhf inputs. Since there were indications that the output harmonics were rather noisy, a study of the multiplication process and the associated noise was undertaken. The existence of excess noise in a crystal detector having the familiar $1/f$ spectral density is already known, and it was asked whether these phenomena were related. For example, the excess noise currents flowing as a result of the fundamental microwave excitation and its rectification may become modulated onto the harmonic by virtue of the nonlinear nature of the crystal and appear as noise sidebands with spectral density and magnitude characteristic of the low-frequency excess noise.

The problem may then be stated as: Given a crystal diode n-harmonic generator excited by power P_1, at angular frequency ω, investigate the noise sideband power associated with the nth harmonic, S_n, if any. S_n may be investigated as a function of P_1 and frequency; and its relationship to S_0, the excess noise power of the crystal used as a detector, may also be investigated. The nature of S_n may be studied in any convenient way, but perhaps the simplest is by demodulation of the spectrum in the neighborhood of $n\omega$.

MODELS

The problem can be considered as a generalized mixer problem in which the exciting microwave may be thought of as the local oscillator.[1] Fig. 1 illustrates the model and Fig. 2 illustrates the spectral relations involved. The harmonic generator is fed at the terminals ω with high level microwave power P_1 in the range 10 to 100 milliwatts. Rectified power or harmonics will appear at the terminals dc, $2\omega, \cdots n\omega$. For any particular harmonic generator, say a tripler, the terminals at the desired harmonic are connected to a matched load and an attempt to reactively terminate the other microwave terminals is made. The dc termination is a resistor providing self bias for the crystal.

In the absence of input power other than at the frequency ω, no other outputs except those just mentioned will occur. If, on the other hand, power is fed to the terminals β, perhaps by the internal production of a component of excess noise in the crystal, converted power will appear at all signal and image terminals, $n\omega \pm \beta$. In Fig. 2, the power at β and $n\omega \pm \beta$ is shown distributed over a small band to emphasize its possible noisy character. The band represents the limiting band of the apparatus. Interest is focused on near-carrier noise components (β small) for two reasons: 1) The severity of the effect is greater for small β due to the $1/f$ excess noise spectrum, and 2) it is desired to analyze experiments which may involve slow signal modulation of the carrier $n\omega$, as for example, by 30 cps Stark or Zeeman modulation of a microwave spectral line. Sidebands due to this signal modulation would then be superimposed on the noise-sideband power, to give some signal-sideband/noise-sideband power ratio.

Although the above description, in terms of generalized mixer theory, is undoubtedly a correct and potentially fruitful model, analysis of the problem was not attempted in these terms. This decision was taken because of the formidable problem of specifying the admittance matrix of the mixer at a particular P_1, and also as a function of P_1, and of specifying the terminating impedances of all the terminals.

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* Manuscript received by the PGMTT, December 19, 1956. Part of this work was carried out under a program of research and development in basic instrumentation cooperatively supported by ONR, AFOSR, AEC, and NBS.

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[1] H. C. Torrey and C. A. Whitmer, "Crystal Rectifiers," Rad. Lab. Ser., vol. 15, McGraw-Hill Book Co., Inc., New York, N. Y.; 1948.

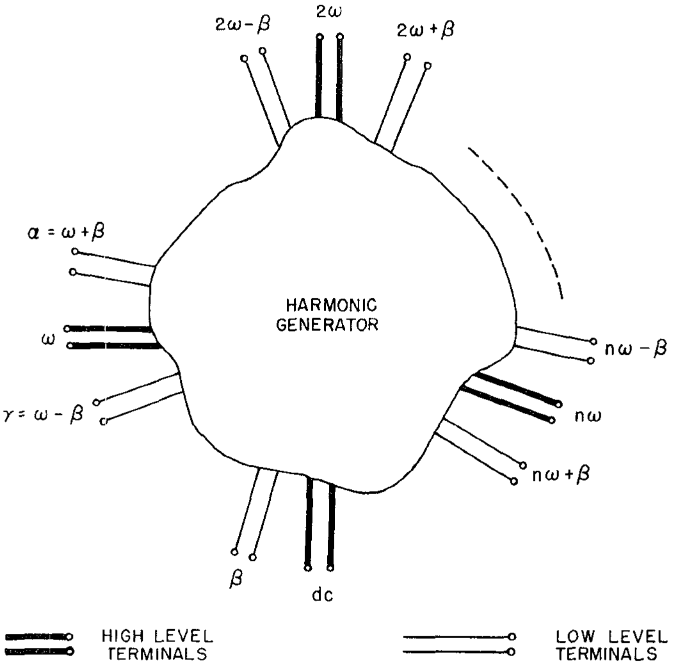

Fig. 1—Generalized mixer showing high- and low-level terminals at various frequencies of interest.

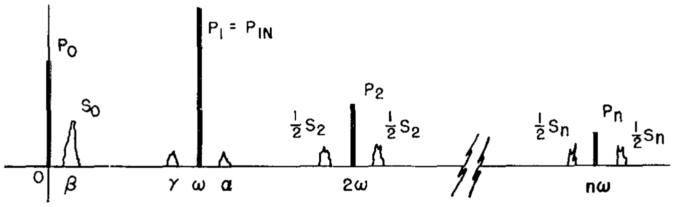

Fig. 2—Spectrum of interest in the detector-harmonic generator problem. The lines at 0, ω, 2ω, $\cdots$, $n\omega$ are spectral lines whose heights have been depicted proportional to the power they represent. The small bands represent spectral densities of noise power whose integrated values over a specified band are denoted by S_0, S_2, etc.

An alternative model was, therefore, used for guidance. Some current-voltage characteristic, $i(e)$, valid at all frequencies of interest,[2] is assumed, and is expanded about the origin in the power series

$$i = \sum a_m e^m. \tag{1}$$

In this series the applied voltage, e, has the form

$$e = E_0 + E_1 \cos \omega t + B(t) \tag{2}$$

where E_0 is the self bias developed across the diode terminals, $E_1 \cos \omega t$ is the applied microwave voltage at angular frequency ω of amplitude E_1 corresponding to P_1, and $B(t)$ is a time dependent virtual applied voltage descriptive of the excess noise generated by the crystal upon excitation. $B(t)$ may be thought of as narrow-band noise near the frequency β, whose magnitude is described by B. E_0 and β are functions of E_1 (or P_1).

Substituting (2) into (1) and retaining several terms in the power series because of the highly nonlinear characteristic and large excursion of e, it is possible to write expressions for P_n and S_n in terms of P_1, E_0, and B, and the coefficients a_m. The dependence of E_0 on P_1 is observable from the experiment. The result is that

$$S_n = G_n B^2$$

and

$$S_n/P_n = H_n B^2 \tag{3}$$

where G_n and H_n are functions of P_1, E_0, and the a_m. Eq. (3) have such form as to permit various dependences of the various S_n with P_1.

Problem of Measurement

Although the source is made as noise free as possible, it is unavoidably noisy to some degree, and care must be taken that the observed noise is not demodulated source noise, S_1, (that is, noise associated with P_1) in the case of the detector experiments, and does not arise from source noise modulated onto the harmonics in the case of the multiplier experiments. Source noise would appear at the detected output of the crystal with relative amplitude determined by S_1/P_1 and the shape of the P_1–P_0 curve. Analysis using the above described models suggests that noise arising from S_1 would appear on all harmonic outputs in relative strength determined by S_1/P_1, and the shape of the P_1–P_n curve.

Thus it was found necessary to carry out the measurements using techniques that balance out the effects of the noise modulation of the source. The rejection ratio required to insure that the measurements obtained were not influenced by the source noise modulation could be estimated by making measurements with the system both balanced and unbalanced.

As a further precaution, demodulation of the harmonics was always carried out using a bolometer, an excess-noise free device, so that it was certain that the bolometer output, if well above its own thermal noise level, consisted essentially of noise demodulated from the harmonic and its noise sidebands.

Technique of Measurement

Although interest was focused on the generation of millimeter waves from centimetric waves, measurements were conducted at X band to enjoy the convenience of complete instrumentation at the fundamental and the first few harmonics. Type 1N26 crystals, a useful type in millimeter wave generation, were used.

The source of X-band power used was a klystron, type SMX-32, operating as the final stage of a frequency multiplying chain. The first stage of this chain was a temperature-controlled quartz crystal oscillator.

Measurements of S_0, the excess noise in detectors, were made with the apparatus of Fig. 3. The detector mount terminating the branch produced by the shunt tee was used for two purposes. First, a bolometer could be placed in this mount to monitor the noise modulation on the output of the source. Second, a crystal

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[2] This assumption is of course not strictly valid, as seen from the usual equivalent circuit of the crystal diode.

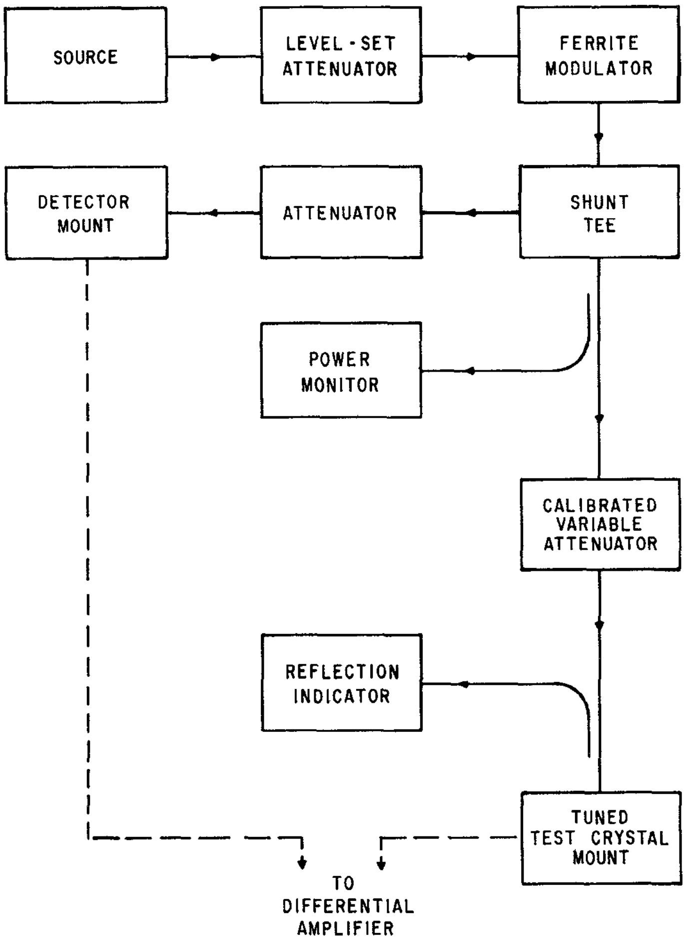

Fig. 3—Microwave circuit used for investigation of excess noise in detector diodes.

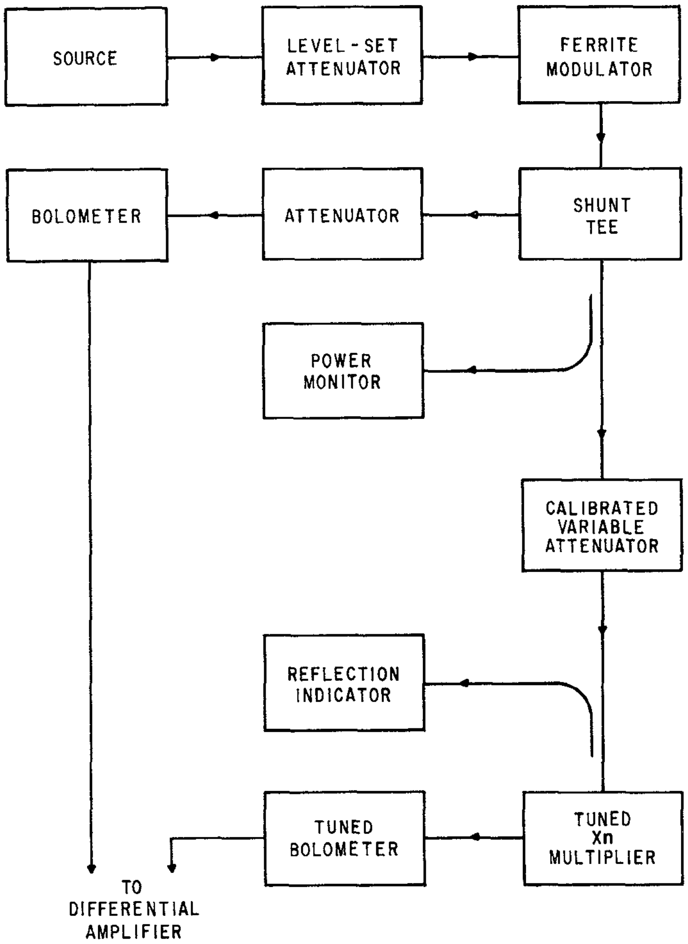

Fig. 4—Microwave circuit used for investigation of noise modulation introduced by crystal diode harmonic generators.

placed in this mount was used to obtain a signal to be introduced into a differential amplifier with the signal from the crystal under test. Thus the input to the differential amplifier consisted of in-phase detected source noise from each crystal and incoherent excess noise from each crystal. The former noise did not appear in the amplifier output because of the differential action, while the latter combined according to an rms rule.

The ferrite modulator was used for two purposes. First, it was used to produce modulation at the center frequency of the narrow-band amplifier to tune the test crystal to eliminate reflections. Second, it was used to produce modulation to enable adjusting the attenuator preceding the balancing detector for maximum discrimination against source modulation. Rejection ratios of over 100 times were easily obtained for the detected voltage due to source modulation. For all data taken, this rejection ratio was adequate.

Because of the balancing technique employed, in order to obtain a single measurement of S_0, the excess noise in the crystal under test, three measurements were needed. First, the noise present in the balanced system described above was measured, then the noise present in the test crystal and in the balancing crystal were each measured separately. From these three measurements, enough information was obtained to find the excess noise generated by the test crystal alone. The crystal under test was always operated with a load resistance of 10,000 ohms.

Measurements of S_2 and S_3 and P_2 and P_3 for doublers and triplers were made with the apparatus of Fig. 4. Again, it was found necessary to eliminate the effects of the source modulation by using a differential method. The bolometer terminating the branch formed by the shunt tee and the bolometer demodulating the multiplied power both produced signals due to the noise on the source output, whereas the bolometer detecting the multiplier output also detected the noise introduced by the process of multiplication. Thus the differential action of the circuit rejected the in-phase noise produced by modulation of the source and recorded only the noise produced by the multiplication. As in the measurements of S_0, it was necessary to adjust the power demodulated at the balancing bolometer to obtain maximum rejection of the in-phase signal. The ferrite modulator was used to produce a modulation for the purpose of making this adjustment. It was found that a rejection of over 50 times could easily be obtained. For all of the data taken, this ratio was determined to be adequate.

Finally the multiplied power could be measured with a power bridge at the bolometer following the multiplier.

The multiplying crystals were always operated with the bias resistance that produced the maximum multiplied power output. When doubling, this resistance was usually in the range from 50 to 200 ohms. For tripling, a broad plateau of maximum power occurred for bias resistance of about 5000 ohms and above, therefore in this case the bias used was always 10,000 ohms.

The measuring apparatus is shown in Fig. 5. The audio-frequency harmonic analyzer was used as a narrow-band af voltmeter, centered at either 270 or 540 cps. Its output was averaged in a circuit with time constant of approximately 25 seconds and recorded with a recording milliammeter. Each measurement was taken by operating the recorder at least three minutes and the resulting trace was averaged visually.

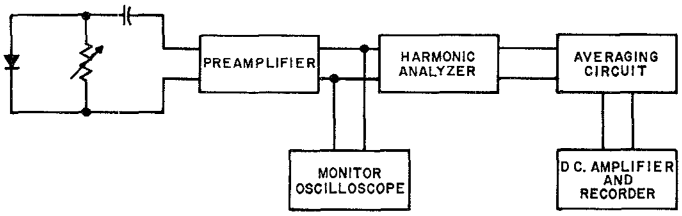

Fig. 5—Block diagram of noise recording apparatus.

RESULTS

Results are presented as the square of a modulation coefficient because this is a measure of the spectral purity of the wave involved, namely the ratio of noise sideband power in a given band, S, to the carrier power, P. As described above, S is the remaining noise after elimination of the effect of source noise. We have

$$m_n^2 = 2S_n/P_n \tag{4}$$

where $n=2, 3$ for the doubler and tripler, and $n=0$ for the detector. The definition of modulation coefficient used agrees with Goldman[3] for a complex waveform, and is on the basis of unit bandwidth. Values of S_n given by (4) may be used to obtain noise temperatures of the multipliers as sources if desired.

Table I shows the variations of spectral purity with crystal, input power, type of operation, and frequency. In general, a crystal which is noisy as a detector (large m_0) is also noisy as a multiplier. This result is expected if the addition of noise in the multiplication process arises from excess noise. The value of m_n^2 usually increases with P_1, for $n=0, 2, 3$ although there are exceptions. This fact substantiates at these high power levels the strong dependence of excess noise production on input power quoted by some authors[4] as P_1^2 at lower power levels. It also is seen that the tripler output is less pure than the doubler output for all crystals and input powers. This fact is understandable in terms of the reduced efficiency of their harmonic production, whereas excess noise production for a given input power is unaltered.

TABLE I
EXCESS NOISE MODULATION COEFFICIENTS TOGETHER WITH OUTPUTS FOR 1N26 CRYSTALS USED AS DETECTORS, DOUBLERS, AND TRIPLERS

Crystal No.	P_1 db below 100mw	$10^{12}m_0^2$ at 270~	$10^{12}m_2^2$ at 270~	$10^{12}m_2^2$ at 540~	$10^{12}m_3^2$ at 270~	V_0 (volts)	P_2 (μw)	P_3 (μw)
2	1	3.2	0.6	0.3	1.2	4.8	850	510
	3	5.3	0.6	0.4	1.2	4.2	550	320
	5	4.8	0.6	0.4	1.0	3.5	330	220
	7	4.8	0.6			3.0		
60	1	25.0	0.8	0.6	18.0	2.4	830	280
	3	13.0	0.5	0.5	8.4	2.4	550	240
	5	14.0	0.8	0.4	5.8	2.3	340	170
	7	13.0			3.6	2.2		100
	9	14.0				2.1		
61	1	200.0	3.6	1.4	140.0	2.3	980	250
	3	94.0	5.8	4.0	23.0	2.4	610	200
	5	59.0	10.2	8.4	27.0	2.4	330	120
	7	40.0	13.0	13.0	30.0	2.2	170	59
	9	30.0	19.0	10.9		2.0	74	
	11	15.0	16.0			1.6	26	
62	1	72.0	2.6	1.7	32.0	2.4	770	280
	3	42.0	2.0	1.4	20.0	2.4	530	220
	5	49.0	1.4	1.0	15.0	2.4	330	150
	7	50.0	1.4	0.6	8.4	2.3	170	83
	9	49.0				2.1		
	11	35.0				1.9		
63	1	29.0	0.4	0.3	19.0	2.7	700	270
	3	26.0	0.3	0.2	19.0	2.6	460	200
	5	24.0	0.2	0.1	19.0	2.5	290	110
	7	29.0			18.0	2.4		46
	9	30.0				2.3		
	11	36.0				2.0		

Table I also shows that the noise modulation at 540 cps on the doubler output is smaller than that at 270 cps. This result can also be explained by the assumption that excess noise is the primary cause of the multiplier noise. It is known that excess noise has a power spectrum that varies approximately as $1/f$, so that modulation sidebands arising from this noise should have the same power spectrum by (3).

Finally, m_3^2 seems to bear a closer relation to m_0^2 than to m_2^2 as to both magnitude and dependence on P_1. This behavior is possibly related to the fact that m_0 and m_3 were observed using the same bias resistance, whereas m_2 was observed with a lower bias resistance.

In the practical case of a fundamental source of modulation coefficient m_1, the total relative noise in the output would probably be given by adding a term of the

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[3] S. Goldman, "Frequency Analysis, Modulation and Noise," McGraw-Hill Book Co., Inc., New York, N. Y.; 1948.

[4] M. W. P. Strandberg, H. R. Johnson, and J. R. Eshbach, "Apparatus for microwave spectroscopy," *Rev. Sci. Instr.*, vol. 25, p. 776; August, 1954.

order of m_1^2 (due to an excess noise-free multiplier) to m_n^2 (due to excess noise only). Thus the amount of degradation of signal purity depends on the characteristics of the particular fundamental source used. In this work, the source had $m_1 = 0.65 \times 10^{-6}$ ($m_1^2 = 0.42 \times 10^{-12}$) so that the degradation was high with all tripler crystals and "noisy" doubler crystals, but not severe with "quiet" doubler crystals.

The rather important question of whether the multiplier noise produces fm sidebands in addition to AM sidebands was not answered by this study, since only the AM sidebands were recovered. However, if as suggested, the mechanism is that of excess noise, the production of fm sidebands should not occur.

CONCLUSION

The results of an investigation into the degradation of spectral purity by the processes of detection and harmonic generation in crystal diodes have been presented. No completely satisfactory model of the process of harmonic generation has been developed, but two possible approaches were presented. The measurements are difficult because of the need to avoid the influence of residual source modulation on the results. An X-band source of equivalent noise modulation index of about 0.7×10^{-6} for unit bandwidth centered at 270 cps from the carrier was satisfactorily used. For 1N26 crystals as detectors, doublers, and triplers excited with X-band input power from 8 to 100 mw, noise dependent on the individual crystal, input power, and frequency is added to the output. The noise modulation introduced by the multiplication process has characteristics similar to the excess noise observed in crystal detectors.

Further investigation, involving the examination of many more crystals, more complete spectral density data of the various noise powers, and fm noise would be desirable as an aid to the designer of precision experiments in the microwave and millimeter wave region.

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