The envelope of the harmonics of the train of pulses is readily obtained from a Fourier integral formulation of a single pulse.

The Fourier transform of the function $f(t)$[4,5] is given by

$$g(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt, \quad (1)$$

where $g(\omega)$ describes the spectral density of the waveform. Substituting $f(t)$ for the waveform shown above, (1) becomes

$$g(\omega) = -\frac{1}{2\pi}\int_{-\tau}^{0} A \sin \omega_0 t e^{-j\omega t}dt + \frac{1}{2\pi}\int_{0}^{\tau} A \sin \omega_0 t e^{-j\omega t}dt. \quad (2)$$

Substituting $e^{-j\omega t} = \cos \omega t - j \sin \omega t$ and integrating yields

$$g(\omega) = \frac{A}{2\pi}\left[\frac{1-\cos(\omega_0-\omega)\tau}{(\omega_0-\omega)} + \frac{1-\cos(\omega_0+\omega)\tau}{(\omega_0-\omega)}\right]. \quad (3)$$

[4] J. A. Stratton, "Electromagnetic Theory," 1st ed., McGraw-Hill Book Co., Inc., New York, N. Y.; 1941.

[5] L. A. Pipes, "Applied Mathematics for Engineers and Physicists," 2nd ed., McGraw-Hill Book Co., Inc., New York, N. Y.; 1958.

Eq. (3) can be simplified by neglecting the term containing $(\omega_0+\omega)$ in the denominator and defining a new term α by $\omega = \omega_0 \pm \alpha$ where $\alpha \ll \omega_0$. Thus (3) becomes

$$g(\omega) = \frac{A}{2\pi} \frac{1-\cos(\pm\alpha)\tau}{(\pm\alpha)}. \quad (4)$$

The power spectrum is given by

$$(g(\omega))^2 = \left[\frac{A}{2\pi}\right]^2 \left[\frac{1-\cos(\pm\alpha)\tau}{(\pm\alpha)}\right]^2. \quad (5)$$

Multiplying numerator and denominator by τ, (5) becomes

$$(g(\omega))^2 = \left(\frac{A\tau}{2\pi}\right)^2 \left[\frac{1-\cos(\pm\alpha)\tau}{(\pm\alpha)\tau}\right]^2. \quad (6)$$

Eq. (6) displays power peaks at $\alpha \approx \pm 3/4\pi/\tau$, as shown in Fig. 13. While actual differentiation of (6) yields power peaks at $\alpha = \pm 0.741\ \pi/\tau$, for purposes of this report, the approximation of $3/4\pi/\tau$ will be used.

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Excess Noise in Microwave Detector Diodes*

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Summary—The dependence of available excess noise in type 1N26 microwave crystal-diode rectifiers on applied microwave power was measured. This may be approximated by a power law with constants characteristic of the particular crystal. As a consequence of the dependence of both excess noise and dc rectified power on input-power level, there is a level which minimizes the ratio of these quantities. Similarly, in the case of a modulated microwave carrier there is an input level which minimizes the ratio of excess noise to demodulated power, and so provides optimum detection of small modulation.

I. Introduction

THE NOISE in excess of kT_0B resulting from application of microwave power to a crystal detector is important in many applications of microwaves. It is interesting in itself to know the functional dependence of the excess noise on the input microwave power and the variation of excess noise with change in the various parameters that may be varied. Furthermore, in order to determine operating conditions that result in optimum video detection of signals of specified RF power and modulation factor, it is necessary to know the dependence of both detected signal and excess noise on these parameters. These considerations find direct application in systems dealing with small amplitude, low-frequency modulation on relatively large microwave signals—for example, in detection of Zeeman or Stark modulation in microwave spectroscopy and parmagnetic resonance, or in certain stabilization systems for microwave oscillators in which error modulation is placed on a microwave signal by a stabilizing element such as a reference cavity.

When a crystal diode is used as a detector of microwave power, the average operating point $(\bar{e}, \bar{\imath})$ that results is a point in the current voltage plane that cannot be reached by application of dc voltages to the crystal. Thus, the excess noise produced by application of microwave signals on a crystal detector cannot be

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inferred from dc measurements. In another study,[1] the notation of which is adopted here, measurement techniques were developed that permit measurement of excess video output noise voltage resulting from microwave excitation of crystal detectors. These techniques essentially involved a two-channel detector system in which detected klystron noise common to both channels was rejected by out-of-phase cancellation, permitting the observation of excess noise to lower levels than otherwise. By also measuring the video impedance of the crystal, which varies with the various operating conditions used in the noise measurements, it is possible to express these results in terms of the total available noise power, $S_0 = kT_0B + S_{excess}$, which in turn may be expressed as a noise temperature, $T = T_0 + T_{excess}$, through the relation $S_0 = kTB$ for bandwidth B and standard noise temperature T_0.

II. Experimental

Measurements were made on three 1N26 crystal detectors assumed typical, using two values of load resistance in the rectified crystal current return path, 2000 ohms and 10,000 ohms. The pass band of the measurement system would pass the same power from a white-noise spectrum as a square pass band of width 8 cps; however, the data presented have been reduced to a bandwidth of 1 cps. For all of the measurements reported here, this pass band was centered on 270 cps, where the excess noise predominated over the Johnson and shot noise because of its approximate $1/f$ spectral distribution. The microwave power impressed on the crystal under test was varied from about 10 μw to 100 mw when the 10,000-ohm biasing resistor was used and from 10 μw to 1 mw with the 2000-ohm bias.

The data obtained are shown in Fig. 1 (next page). These curves indicate, for the range investigated, that below a critical input microwave power, the available excess-noise power can be approximated by an expression of the form

$$S_{excess} = CP_1^n, \tag{1}$$

where P_1 is the input microwave power, and both C and n are parameters which depend on the particular crystal under test. The $1/f$ dependence probably can be contained in C. The critical input power above which this expression no longer holds is also a characteristic of the individual crystal. It is interesting to note that for the three crystals tested, the value of C is less with the 2000-ohm bias than with the 10,000-ohm; however, the value of n seems to be essentially independent of bias resistance.

Approximate values for these two parameters are shown in Table I for the three crystals tested.

The noise power S_0 is related to fluctuations in the rectified voltage. A measure of the severity of these fluctuations is an equivalent noise modulation factor m_0, defined by the general relationship.

$$m_0^2 = 2S_0/P_0, \tag{2}$$

where P_0 is the available rectified power from the detector.[2] Fig. 2 shows measured values of m_0^2 plotted as a function of P_1 for the crystals used in this study. It is interesting to note that m_0 has a definite minimum value for a power input of a few db below 1 mw. This is readily understandable, since square-law detection applies at low powers. Thus P_0 varies as P_1^2 while S_0 varies as P_1^n, where $n<2$ from Table I. At high powers the detector law becomes linear or even less strong because of incipient saturation, while S_0 continues to rise approximately linearly because of excess noise. This modulation on the rectified dc output places a lower limit on the smallest detectable RF modulation.

Finally, it is interesting to consider an RF signal of power level P_1 with a small modulation m_{1s} impressed on it. It is desirable to adjust the signal level before detection by attenuation (or by amplification with negligible addition of noise) to a value that will maximize the ratio of available detected signal power S_{0s} to available noise power S_0. We designate the signal sideband power on the RF carrier S_{1s}; thus $S_{1s} = \frac{1}{2}m_{1s}^2P_1$. We further define a small signal demodulation efficiency $G = S_{0s}/S_{1s}$. For any particular crystal, G may be obtained from the slope of the curve of available rectified power vs input power and is a function of P_1. Thus the ratio of demodulated signal power to noise power is

$$S_{0s}/S_0 = GS_{1s}/S_0 = \tfrac{1}{2}Gm_{1s}^2P_1/S_0. \tag{3}$$

The least value of the ratio S_{0s}/S_0 that results in a detectable signal depends on the method of observation used. However, we will adopt the commonly used criterion of detectability that a signal is detectable if the ratio

$$\frac{S_{0s}}{S_0} \geqq 1.$$

Thus we can define a critical value of m_{1s}, which we will label M_{1s}, that produces a signal at the limit of detectability. Hence

$$\tfrac{1}{2}GM_{1s}^2P_1/S_0 = 1, \tag{4}$$

or

$$M_{1s}^2 = 2S_0/GP_1. \tag{5}$$

Fig. 3 displays M_{1s}^2 as a function of P_1. The character of the curves is readily understood, since at low powers square-law detection applies and G is roughly proportional to P_1, so that M_{1s}^2 falls with increasing P_1. At high powers, G becomes constant or even decreases, while S_0 continues to increase so that M_{1s}^2 rises again.

[1] J. M. Richardson and J. J. Faris, "Excess noise in microwave crystal diodes used as rectifiers and harmonic generators," IRE Trans. on Microwave Theory and Techniques, vol. MTT-5, pp. 208–212; July, 1957.

[2] For sinusoidal amplitude modulation of a carrier, it is well known that the total sideband power is $S = m^2P/2$. This relation is easily generalized by defining an effective modulation factor so as to apply to a complex modulating waveform synthesized from several frequencies. The same relationship holds even for zero carrier frequency, so that we identify the dc component of the rectified output with P_0 and the fluctuating component with S_0 in (2).

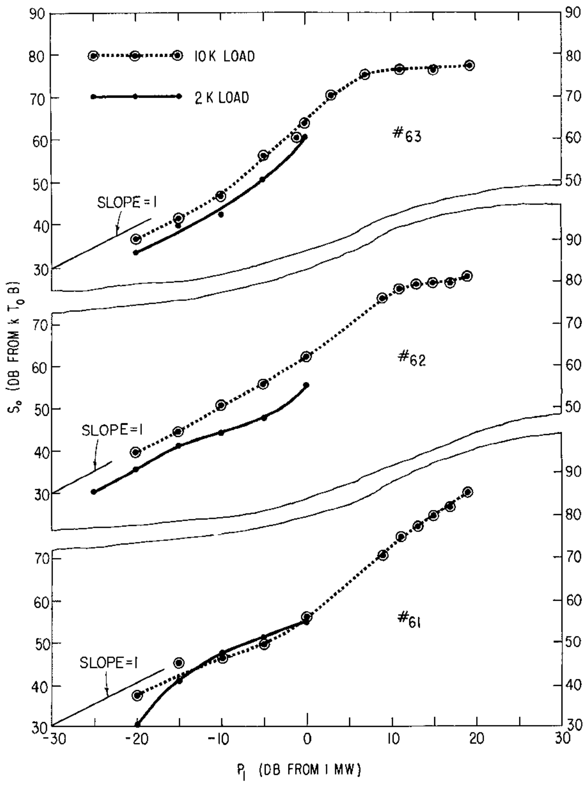

Fig. 1—Available noise power for unit bandwidth at 270 cps as a function of input microwave power; 1N26 detector crystals.

TABLE I

CONSTANTS FOR EQUATION (1)

Crystal No.	n	C (watts)$^{-(n-1)}$	
		2 K load	10 K load
61	0.9	1.25×10^{-12}	1.25×10^{-12}
62	1.0	1.25×10^{-12}	$5.0\ \times10^{-12}$
63	1.5	1.12×10^{-10}	$2.5\ \times10^{-10}$

The range of M_{1s}^2 may be several orders of magnitude. These data demonstrate that there is a signal level which minimizes M_{1s}^2 and thus leads to optimum detection of a small modulation on the RF signal.

III. Conclusions

We have demonstrated that the available excess noise power in a microwave crystal detector can be approximated by the expression

$$S_{\text{excess}} = CP_1^n,$$

where both C and n depend on the characteristics of the particular crystal.

We have further shown that if the excess noise present on the dc rectified output from the crystal studied is represented as a modulation on the dc carrier, there is a signal level that minimizes the modulation coefficient.

Finally we conclude that there is a signal level that results in a maximum ratio of available detected modulation power to available noise power, for small modulation. This signal level provides optimum detection of a small modulation on an RF signal.

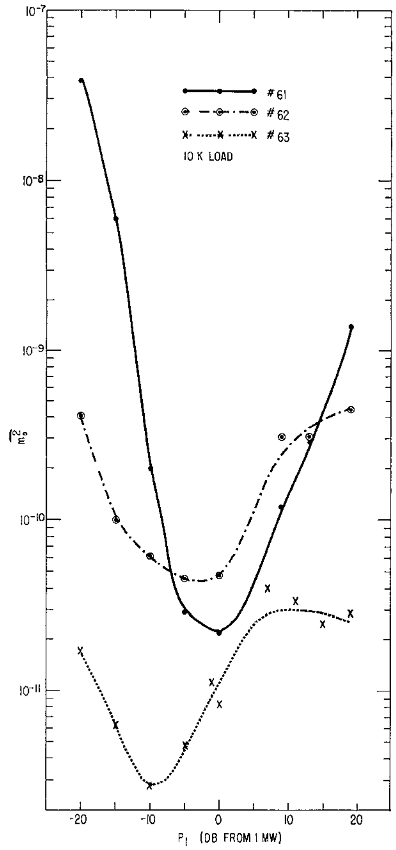

Fig. 2—Square of equivalent noise modulation factor for unit bandwidth at 270 cps as a function of input microwave power.

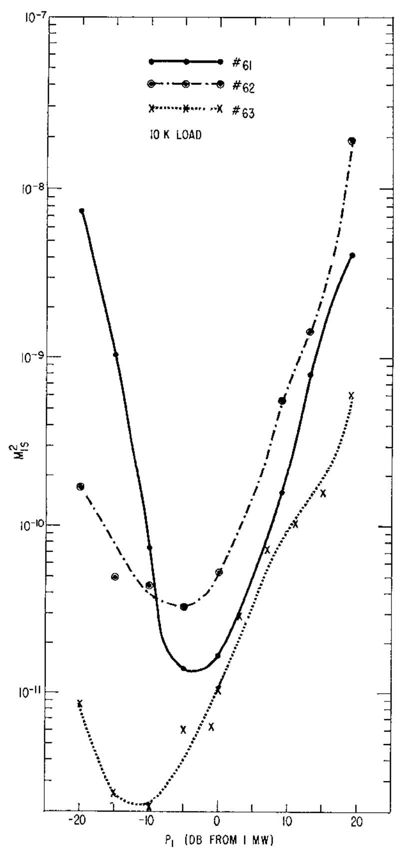

Fig. 3—Square of critical modulation factor to produce demodulated signal power equal to noise power in unit bandwidth at 270 cps.